NAME:
EE301 Signals and Systems
Exam 3

NAME
In-Class Exam
Apr. 23, 2015

Cover Sheet

Test Duration: 75 minutes.
Coverage: Chaps. 5,7
Open Book but Closed Notes.
One 8.5 in. x 11 in. crib sheet
Calculators NOT allowed.
All work should be done on the sheets provided.
You must show all work for each problem to receive full credit.
For Problem 4, plot your answers on the graphs provided.

Problem 1. Short answer questions.

(a) Briefly write and explain one of the main advantages of digital over analog, in terms of storage, transmission, and processing.

Error control coding enables digital to automatically detect and correct for bit errors made due to noise and other imperfections.

Flexibility and Programmability of DSP over analog signal processing.

Also, regeneration for reliable wireless transmission

(b) If you sample at a rate ω_s in terms of radians/sec, what discrete-time frequency is the analog frequency $\frac{\omega_s}{3}$ mapped to?

$$\omega_d = \omega_a T_s \qquad \omega_s = \frac{2\pi}{T_s}$$

$$\omega_d = \frac{\omega_s}{3} T_s = \frac{2\pi}{3} \Leftarrow \qquad \Rightarrow \omega_s T_s = 2\pi$$

(a) The DT signal $x[n]$ is obtained by sampling the sinewave $x_{a_1}(t) = \cos(10t + \phi)$, where the value of the phase is $\phi = \pi/\sqrt{2}$, at a rate of $\omega_s = 30$ radians/sec. Specify the frequency ω_{a2} of another sinewave $x_{a_2}(t) = \cos(\omega_{a_2} t)$ at a higher frequency that will yield the exact same DT signal $x[n]$ when sampled at the same rate, $\omega_s = 30$ radians/sec.

$10 + l\,30$ $\quad l$, integer

Problem 2 (a). The signal $x_a(t) = \{u(t+2) - u(t-2)\}$ is sampled every $T_s = 0.5$ seconds to form $x[n] = x_a(nT_s)$, where, again, T_s is a half of a second. Determine a closed-form expression for the DTFT $X(\omega)$ of the $x[n]$ thus obtained. Assume that the signal is turned on equal to 1 at both ends (edges), that is, at both $t = -2$ secs and $t = +2$ secs.

$$x[n] = x_a(nT_s) \quad \text{where: } T_s = 0.5 \text{ secs} \quad \text{and} \quad x_a(t) = \{u(t+2) - u(t-2)\}$$

Sampling rate $= F_s = \dfrac{1}{T_s} = 2$ samples/sec

Duration of signal $= 4$ secs

Total no. of samples $= 1 \Rightarrow F_s \cdot 4 =$

$2 \dfrac{\text{samples}}{\text{sec}} \times 4$ secs

$= 8$

but assume "turned on" at both ends $\Rightarrow$ 9 samples all equal to 1

$x[n] = u[n+4] - u[n-5]$

$$X(\omega) = \frac{\sin\left(\frac{9}{2}\omega\right)}{\sin\left(\frac{\omega}{2}\right)}$$

Problem 2 (b). The signal $x_a(t) = t\{u(t+2) - u(t-2)\}$ is sampled every $T_s = 0.5$ seconds to form $x[n] = x_a(nT_s)$, where, again, T_s is a half of a second. Determine a closed-form expression for the DTFT $X(\omega)$ of the $x[n]$ thus obtained. Assume that the signal is turned on equal to 1 at both ends (edges), that is, at both $t = -2$ secs and $t = +2$ secs.

$$x[n] = x_a(nT_s) \quad \text{where: } T_s = 0.5 \text{ secs} \quad \text{and} \quad x_a(t) = t\{u(t+2) - u(t-2)\}$$

From previous problem

$$x[n] = nT_s \{u[n-4] - u[n-5]\}$$

$$= \frac{1}{2} n \, x_{2b}[n]$$

Thus:

$$X(\omega) = \frac{1}{2} j \frac{d}{d\omega} \left\{ \frac{\sin\left(\frac{9}{2}\omega\right)}{\sin\left(\frac{1}{2}\omega\right)} \right\}$$

Problem 3. Consider the input signal $x_p(t)$ below.

$$x_0(t) = e^{-j25t} + e^{-j20t} + e^{-j15t} + e^{-j10t} + e^{-j5t} + 1 + e^{j5t} + e^{j10t} + e^{j15t} + e^{j20t} + e^{j25t}$$

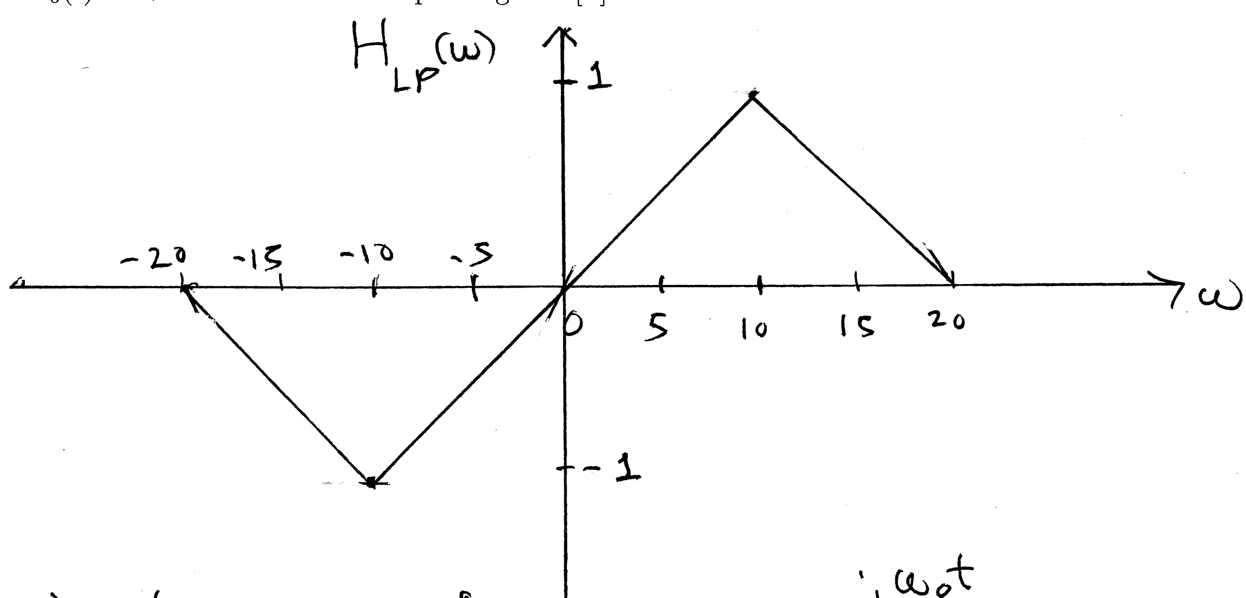
This signal is first input to an analog filter with impulse response

$$h_{LP}(t) = \frac{\pi}{5}\left\{\frac{\sin(5t)}{\pi t}\right\}^2 2j\sin(10t)$$

to form $x(t) = x_0(t) * h_{LP}(t)$, and then $x(t)$ is sampled at a rate of $\omega_s = 40$ to form $x[n]$, so that the time between samples is $T_s = \frac{2\pi}{40}$. The DT signal $x[n]$ thus obtained is then input to a DT LTI system with impulse response

$$h[n] = e^{j\pi n}\left\{\frac{\sin\left(\frac{5\pi}{8}n\right)}{\pi n}\right\} \tag{1}$$

Show all work. Write your expression for the output $y[n] = x[n] * h[n]$ in the space below. Plot both the Fourier Transform of $h_{LP}(t)$ and the DTFT of $h[n]$ to help solve the problem, but since the input is a sum of sinewaves, it is not necessary to plot the Fourier Transform of $x_0(t)$ or the DTFT of the sampled signal $x[n]$.



$$e^{j\omega_0 t} \rightarrow \boxed{h(t)} \rightarrow H(\omega_0)e^{j\omega_0 t}$$

$$H(-25) = H(25) = 0 = H(-20) = H(20)$$

$$H(-15) = H(-5) = -\frac{1}{2} \qquad H(+5) = H(15) = \frac{1}{2}$$

$$H(-10) = 1 = H(+10) \qquad H(0) = 0$$

Thus:

$$x(t) = -\frac{1}{2}e^{-j15t} - e^{-j10t} - \frac{1}{2}e^{-j5t} + \frac{1}{2}e^{j5t} + e^{j10t} + \frac{1}{2}e^{j15t}$$

Problem 3. You can continue your work for 3 here.

$$x[n] = x(nT_s) \qquad T_s = \frac{2\pi}{40} = \frac{\pi}{20} \qquad \omega_d = \omega_a T_s$$

$$\omega_a = 5\,\frac{2\pi}{40} = \frac{\pi}{4} \qquad\qquad = \omega_a \frac{2\pi}{40}$$

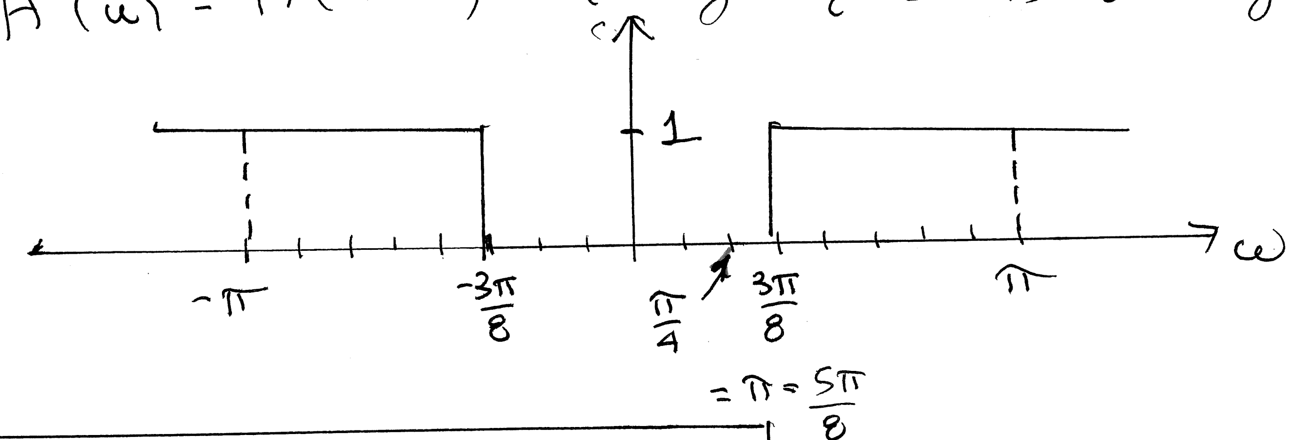
Thus: $\underbrace{k5}_{\text{analog}}$ mapped to $\underbrace{k\frac{\pi}{4}}_{\text{digital}}$

$$x[n] = -\frac{1}{2}e^{-j\frac{3\pi}{4}n} - e^{-j\frac{\pi}{2}n} - \frac{1}{2}e^{-j\frac{\pi}{4}n}$$

$$+ \frac{1}{2}e^{j\frac{3\pi}{4}n} + e^{j\frac{\pi}{2}n} + \frac{1}{2}e^{j\frac{\pi}{4}n}$$

$$h[n] = e^{j\pi n}\,h_{LP}(n) \qquad h_{LP}(n) = \frac{\sin\left(\frac{5}{8}\pi n\right)}{\pi n}$$

$$H(\omega) = H(\omega - \pi)$$ everything shifts over by π



$$\boxed{e^{j\omega_0 n} \longrightarrow \boxed{h[n]} \longrightarrow H(\omega_0)\,e^{j\omega_0 n}}$$

$$H\left(-\frac{\pi}{4}\right) = H\left(\frac{\pi}{4}\right) = 0$$

$$y[n] = -e^{-j\frac{\pi}{2}n} - \frac{1}{2}e^{-j\frac{3\pi}{4}n} + e^{j\frac{\pi}{2}n} + \frac{1}{2}e^{j\frac{3\pi}{4}n}$$

Workout Problem 4. Consider the continuous-time signal $x_a(t)$ below. Note that the multiplication by the scalar j is included to make the Fourier Transform $X_a(\omega)$ be purely real-valued, and the multiplication by the scalar T_s is intended to offset the amplitude-scaling by the sampling rate $F_s = \frac{1}{T_s}$ that inherently occurs in the process of sampling.

$$x_a(t) = T_s \frac{\pi}{5} \left\{ \frac{\sin(5t)}{\pi t} \right\}^2 2j \sin(10t)$$

(a) A discrete-time signal is created by sampling $x_a(t)$ according to $x[n] = x_a(nT_s)$ for $T_s = \frac{2\pi}{40}$. Plot the DTFT of $x[n]$, $X(\omega)$, over $-\pi < \omega < \pi$. Show your work on this page and the next page, and do your plot in the space provided on the next page.

(b) Repeat part (a) for $T_s = \frac{2\pi}{25}$. Plot the new DTFT of $x[n]$, $X(\omega)$, over $-\pi < \omega < \pi$. Show your work and do your plot in the space provided on the sheets attached.

$\omega_M = 20$ $\quad\quad$ $\omega_s = 40 \geq 2\omega_M$ $\Rightarrow$ No aliasing

since

$X(20) = 0$

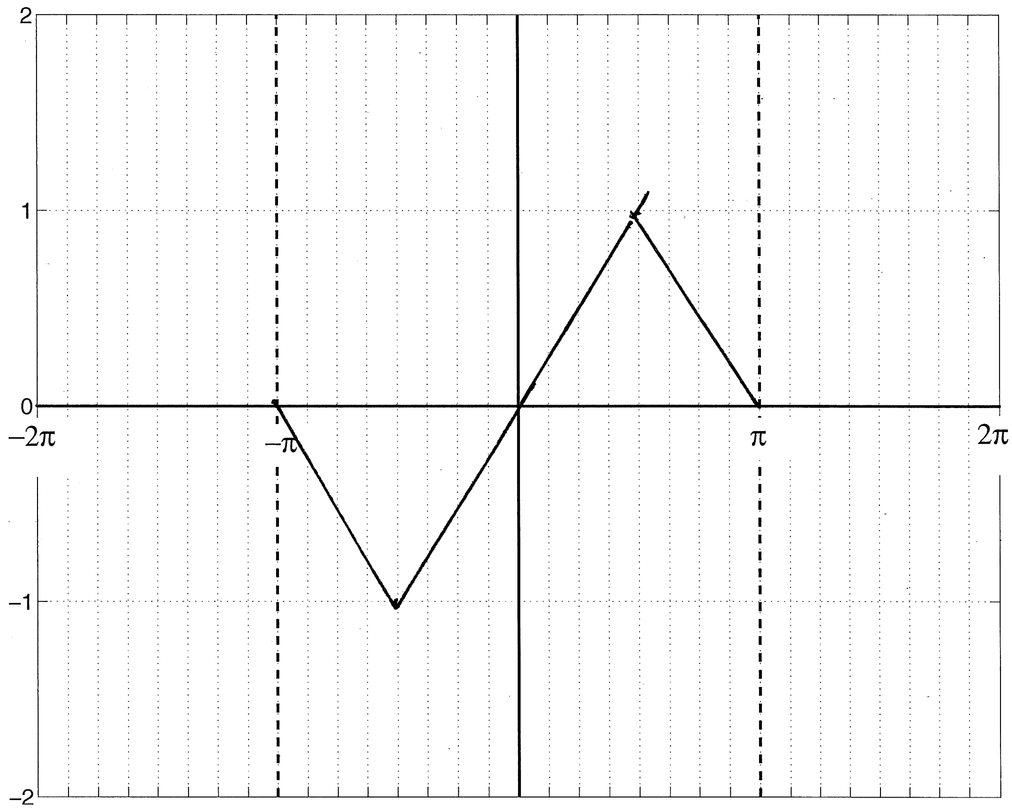
$\omega_s = 25$ $\Rightarrow$ aliasing

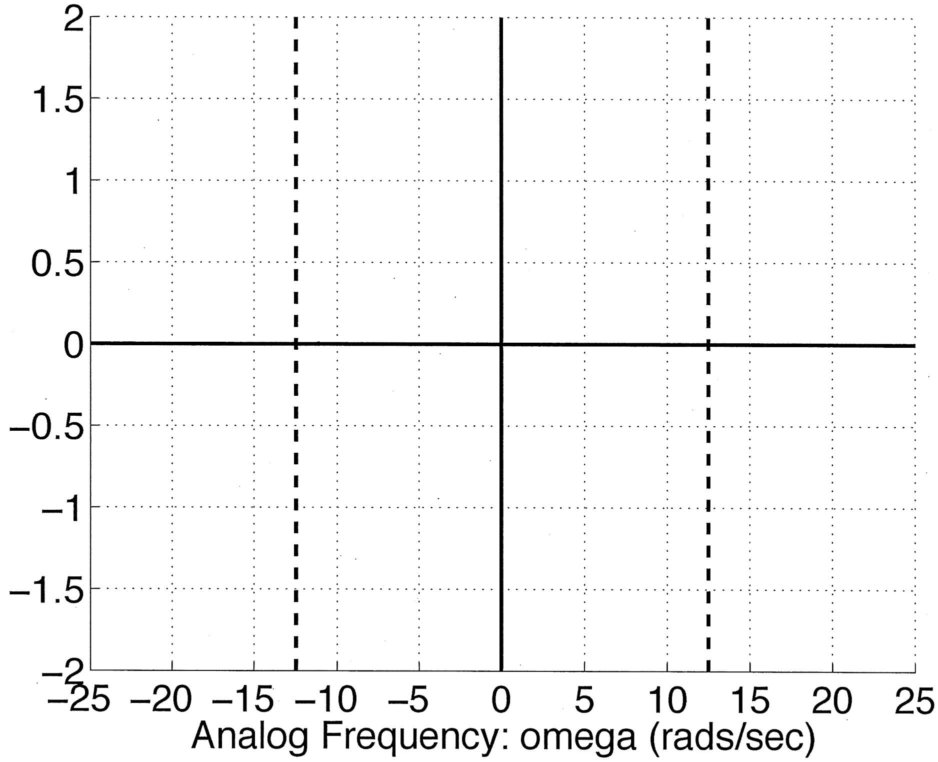
starts at $\omega_s - \omega_M = 25 - 20 = 5$

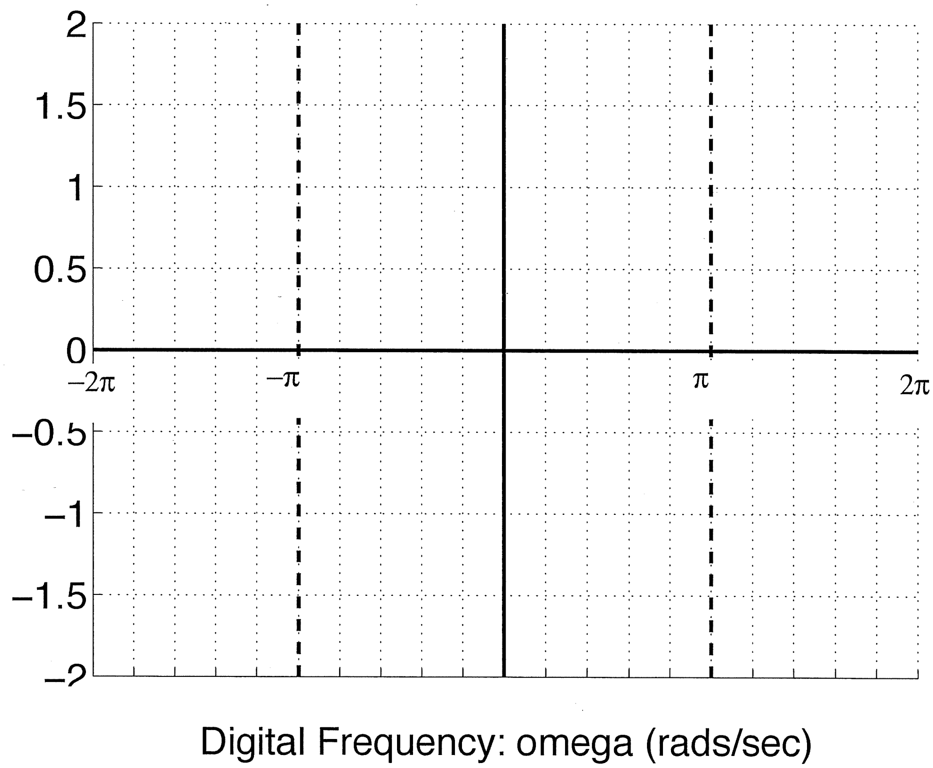
digital frequency: $5 T_s = 5 \frac{2\pi}{25} = \frac{2\pi}{5}$

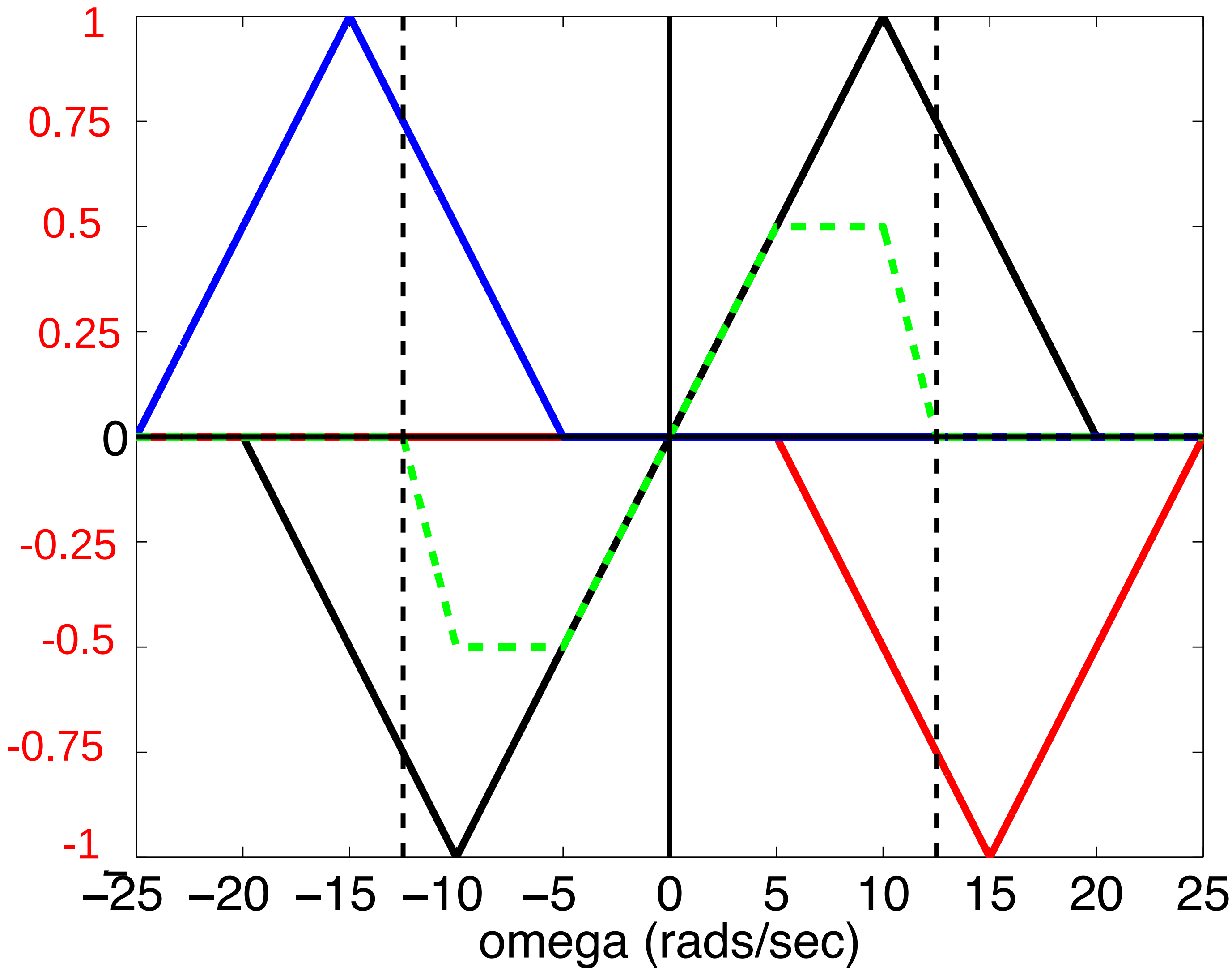
aliasing starts

Plot your answer to Problem 4 (a) here. Show work .
Plot your answer to Problem 4 (b) on next page .




2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−25 −20 −15 −10 −5 0 5 10 15 20 25
Analog Frequency: omega (rads/sec)


2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−2π
−π
π
2π
Digital Frequency: omega (rads/sec)



1
0.75
0.5
0.25
0
-0.25
-0.5
-0.75
-1
−25
−20
−15
−10
−5
0
5
10
15
20
25
omega (rads/sec)