EE301 Exam 3 Solns

SP 04

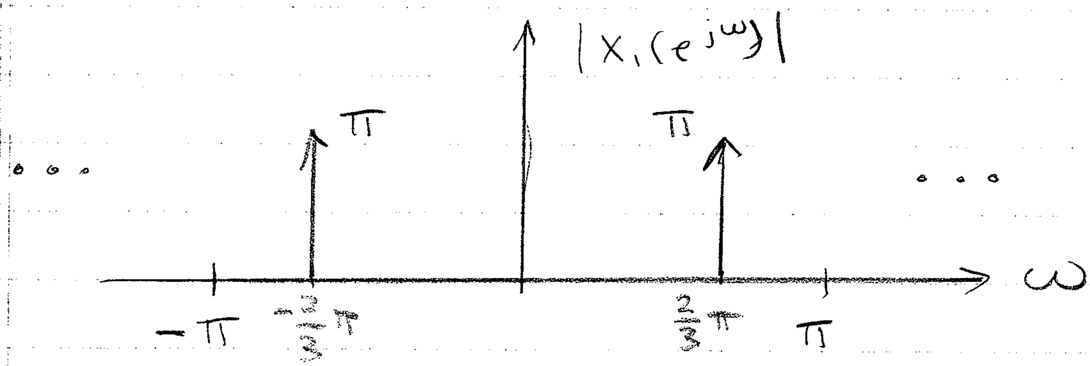
1

Prob. (a):

$$x_1[n] = \cos\left(4n\frac{2\pi}{6}\right)$$
$$= \cos\left(\frac{4}{3}\pi n\right)$$
$$= \cos\left(\frac{4}{3}\pi n - \frac{6}{3}\pi n\right)$$
$$= \cos\left(\frac{2}{3}\pi n\right)$$

Nyquist rate = 8 > 6

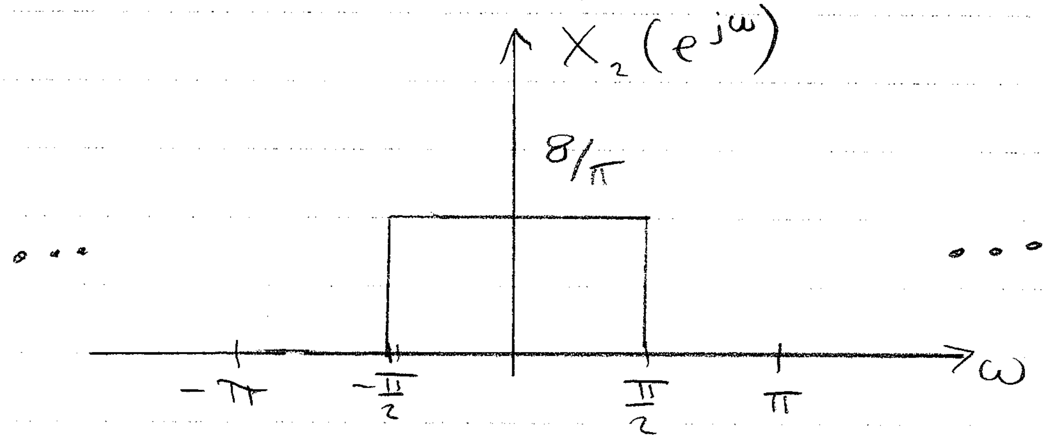
$\Rightarrow$ aliasing



Prob. (b)

$$x_2[n] = \frac{\sin\left(4n\frac{2\pi}{16}\right)}{\pi n\frac{2\pi}{16}}$$

$$= \frac{8}{\pi}\frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n}$$



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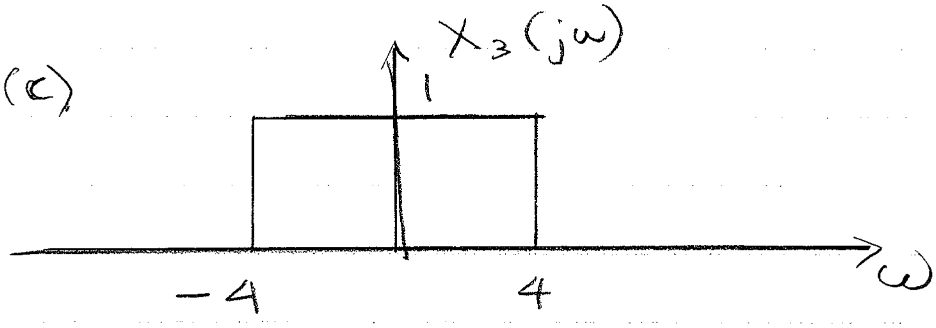
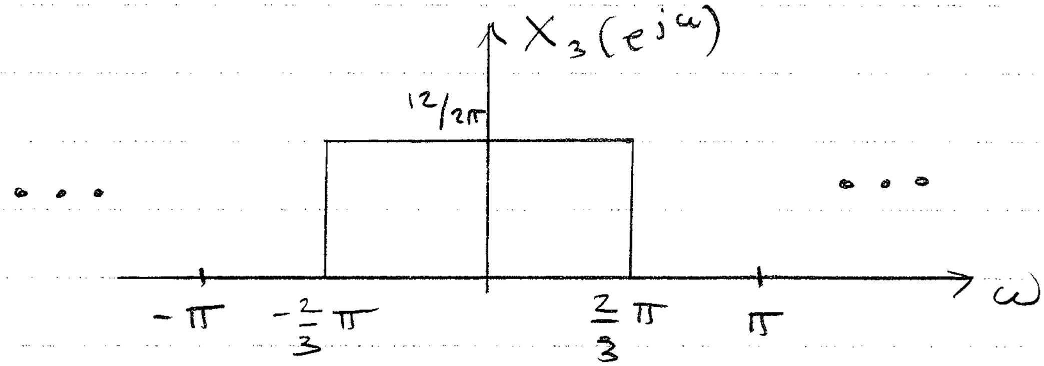
Prob. (c)



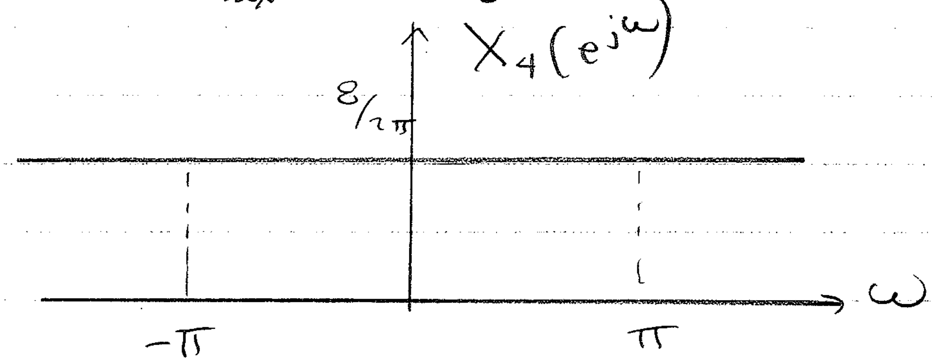
$\Downarrow$

$$\omega_d = \omega_a T_s = \omega_a \frac{2\pi}{12}$$

$$\omega_{d_{max}} = 4 \, \frac{2\pi}{12} = \frac{2}{3}\pi < \pi$$ no aliasing



Prob. (d) $\omega_{d_{max}} = 4 \, \frac{2\pi}{8} = \pi$ $\Rightarrow$ Nyquist rate



$$x_4[n] = \frac{\sin\left(4n\frac{2\pi}{8}\right)}{\pi n \frac{2\pi}{8}} = \frac{8}{2\pi}\frac{\sin(\pi n)}{\pi n}$$

$$= \frac{8}{2\pi}\,\delta[n]$$

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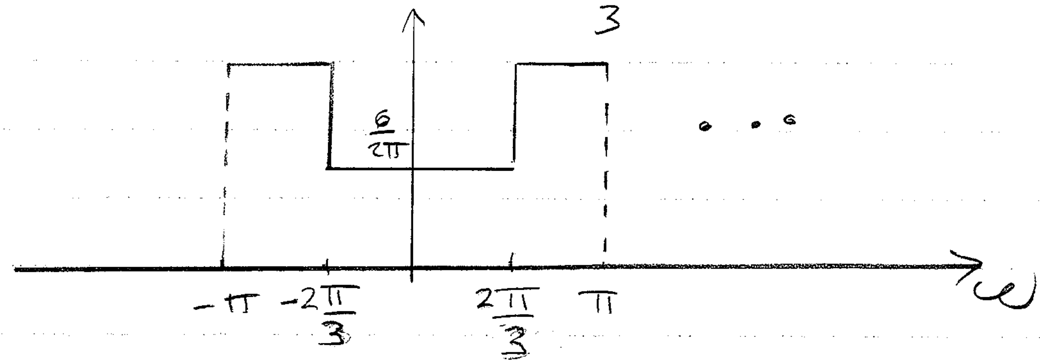
Prob. (e)

$$\omega_{d\,max} = 4\,\frac{2\pi}{6} = \frac{8}{6}\pi$$

$$= \frac{4}{3}\pi > \pi \Rightarrow \text{aliasing!}$$

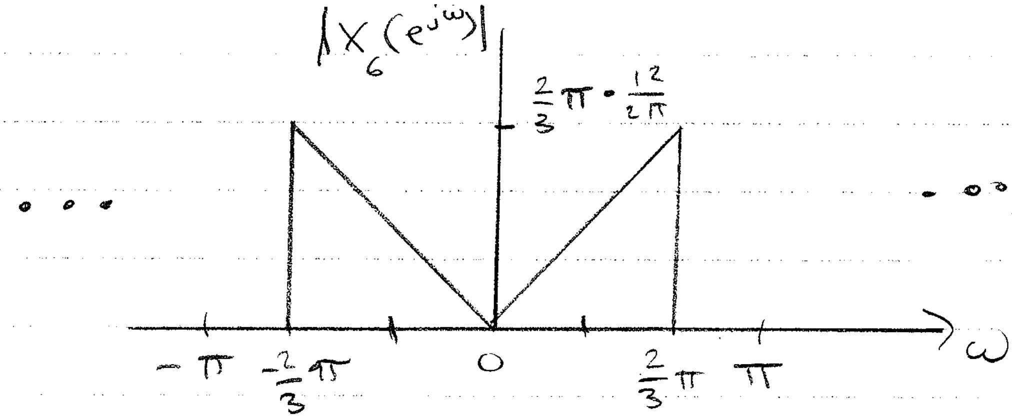


$$2\pi - \frac{4}{3}\pi = \frac{6}{3}\pi - \frac{4}{3}\pi = \frac{2\pi}{3}$$

Prob. (f):

$$\frac{d}{dt}\,\frac{\sin(4t)}{\pi t} \overset{F}{\longleftrightarrow} j\omega\left\{u[\omega+4] - u[\omega-4]\right\}$$

$$\omega_{d\,max} = 4\,\frac{2\pi}{12} = \frac{2}{3}\pi < \pi \Rightarrow \text{no aliasing!}$$



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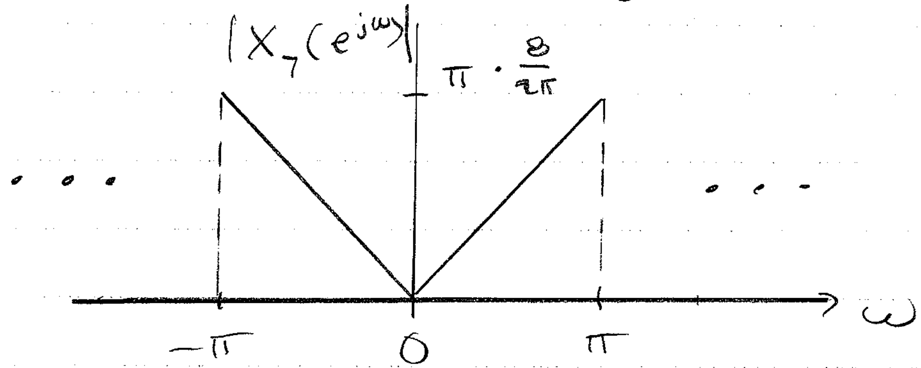
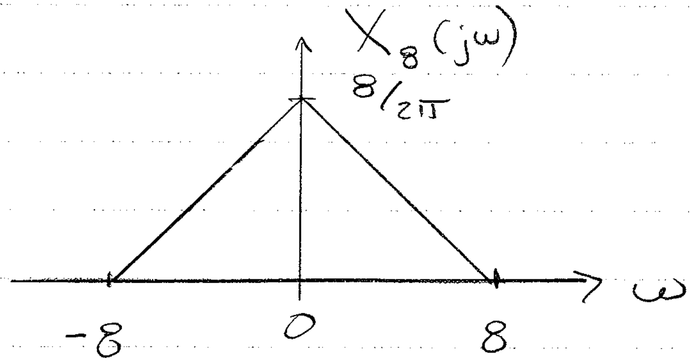
Part (g): $\omega_{d_{max}} = 4 \, \frac{2\pi}{8} = \pi \Rightarrow$ Nyquist rate



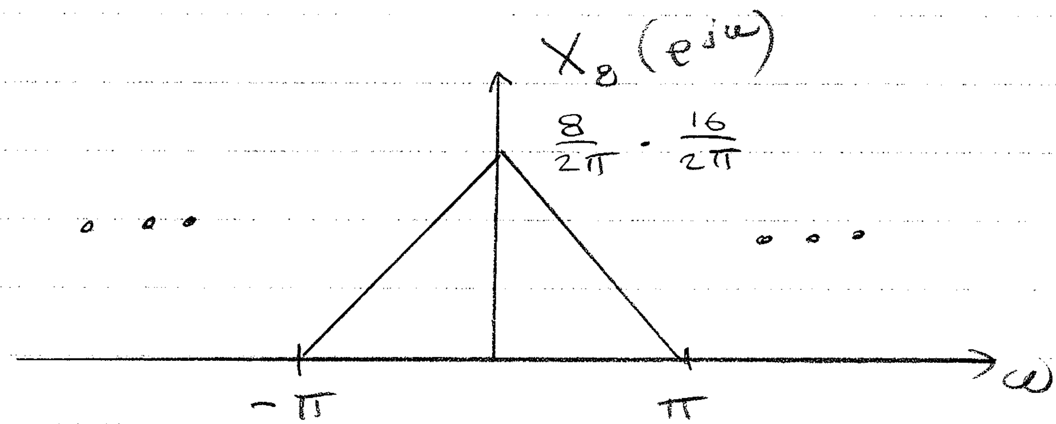
Part (h):



$\Downarrow$

$\omega_d = \omega_a T_s = \omega_a \frac{2\pi}{16}$

$\omega_{d_{max}} = 8 \, \frac{2\pi}{16} = \pi \Rightarrow$ Nyquist rate



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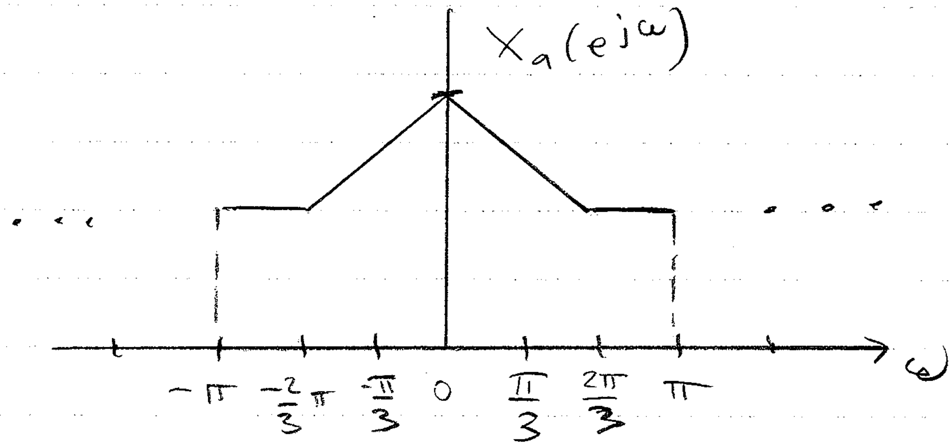
Part (i) $\omega_{d_{max}} = 8 \frac{2\pi}{12} = \frac{4}{3}\pi > \pi$

aliasing!

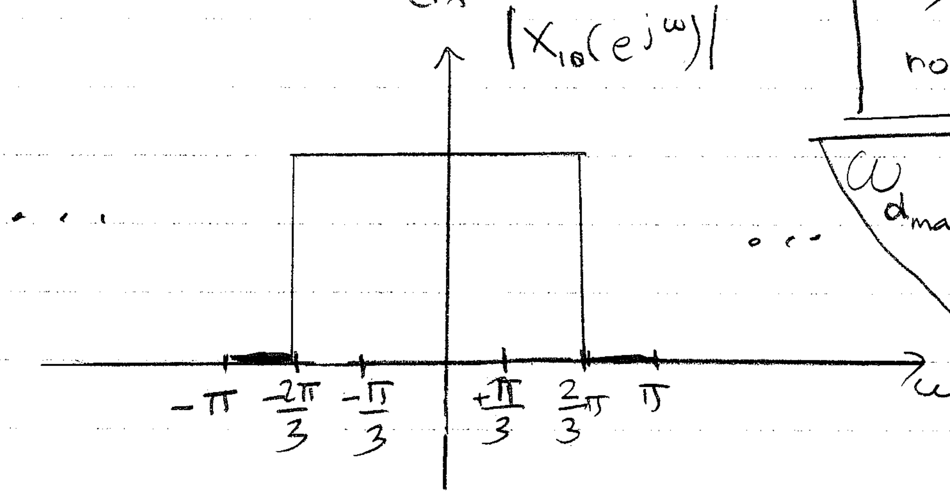
$$2\pi - \frac{4}{3}\pi = \frac{6\pi}{3} - \frac{4}{3}\pi = \frac{2}{3}\pi$$



Part (j): $x_{10}(t) = t \left\{\frac{\sin(4t)}{\pi t}\right\}^2$

$$t x(t) \longleftrightarrow +j \frac{d}{dx} X(j\omega)$$

$$\boxed{\begin{array}{l} 24 > 2(8) \\ \Rightarrow 24 > 16 \\ \text{no aliasing} \end{array}}$$



$$\omega_{d_{max}} = \omega_{a_{max}} T = 8 \frac{2\pi}{24} = \frac{2}{3}\pi$$

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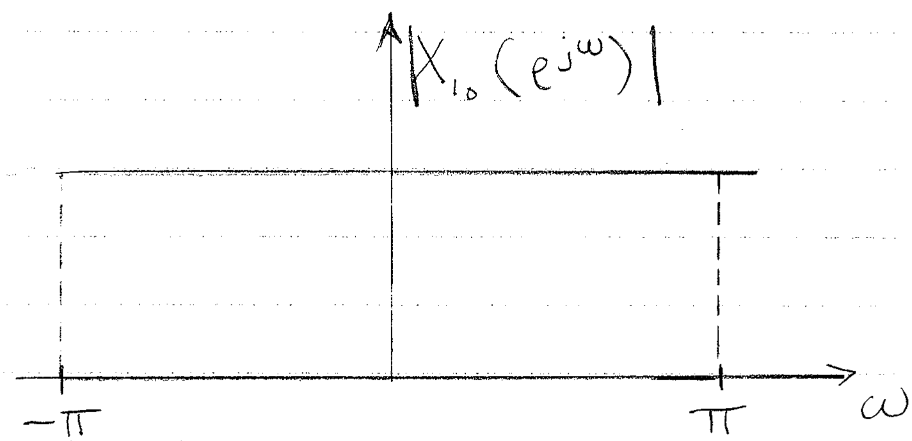
(k) $x_{10}(t) = t \left\{ \frac{\sin(4t)}{\pi t} \right\}^2$

$\omega_{a_{max}} = 8$ (multiplying by t does not change bandwidth)

$\omega_s = 16 = 2\,\omega_{a_{max}} \Rightarrow$ Nyquist rate

$\Rightarrow$ no aliasing

$t\,x(t) \overset{\mathcal{F}}{\longleftrightarrow} j \frac{d}{d\omega} X(j\omega)$



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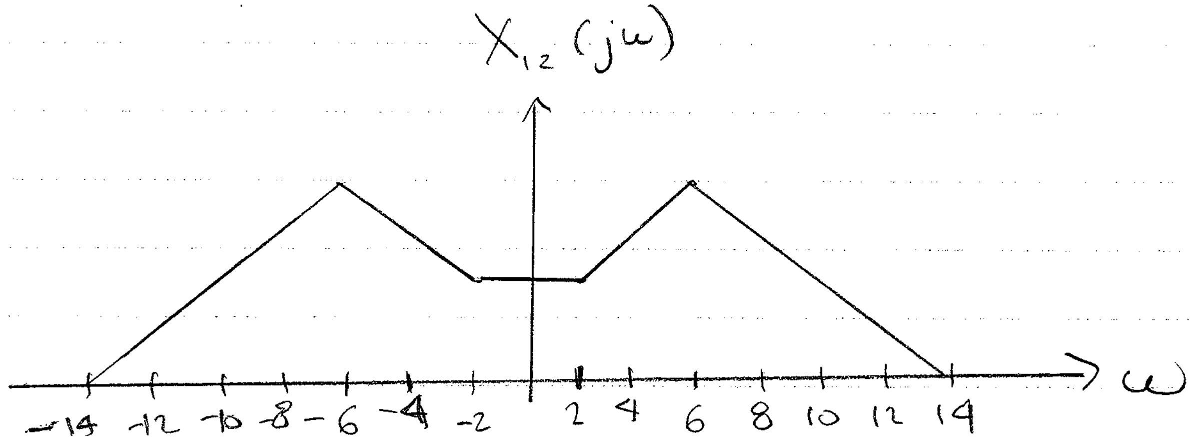
(l)

$$x_{12}(t) = \left\{\frac{\sin(4t)}{\pi t}\right\}^2 \cos(6t) \qquad T_s = \frac{2\pi}{24}$$



$24 < 2(14) = 28 \Rightarrow$ aliasing! $\quad 24-14$

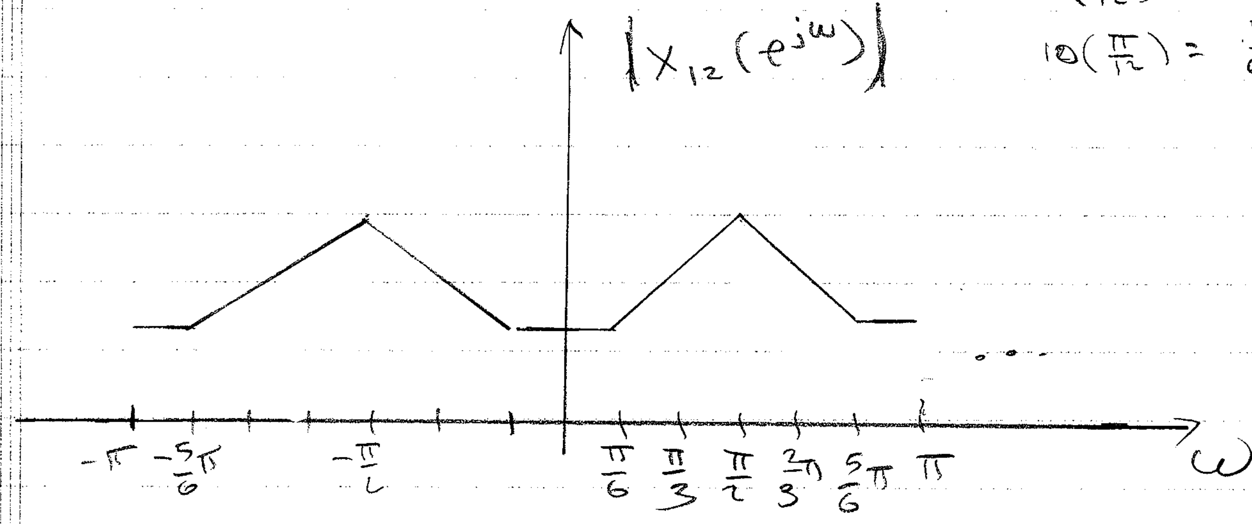
$\frac{1}{2}(24) = 12 \Rightarrow$ no aliasing up 'til $\omega = 10$

$$2\left(\frac{\pi}{12}\right) = \frac{\pi}{6}$$

$$\omega_d = \omega_a T = \omega_a \frac{2\pi}{24}$$

$$6\left(\frac{\pi}{12}\right) = \frac{\pi}{2}$$

$$10\left(\frac{\pi}{12}\right) = \frac{5}{6}\pi$$



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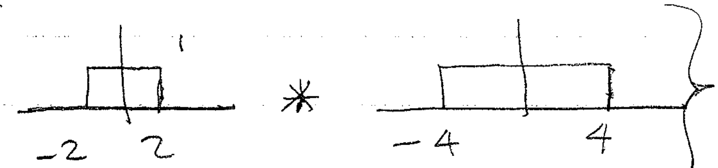
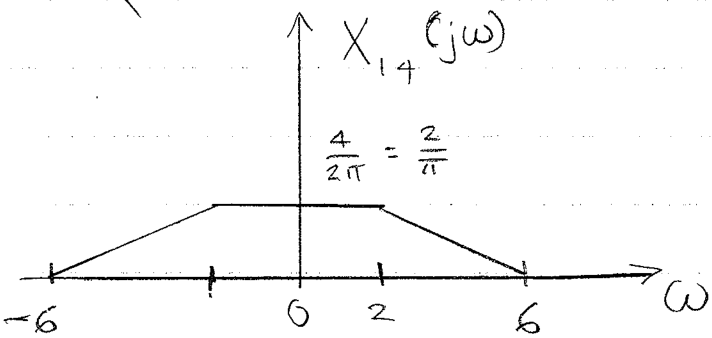
Part (n):

$$x_{14}(t) = \left\{ \frac{\sin(2t)}{\pi t} \right\} \left\{ \frac{\sin(4t)}{\pi t} \right\}$$

$$X_{14}(j\omega) = \frac{1}{2\pi} \left\{ \quad * \quad \right\}$$



$=$

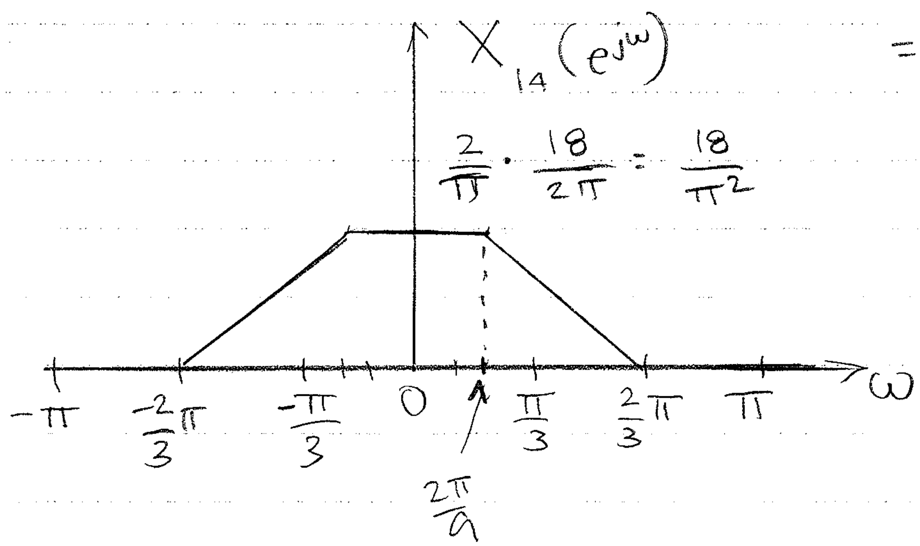


$$x_{14}[n] = x_{14}(nT_s) \qquad T_s = \frac{2\pi}{18}$$

$$\omega_s = 18 > 2\omega_{max} = 2(6) = 12 \Rightarrow \text{No aliasing}$$

$$2T_s = 2\,\frac{2\pi}{18} = \frac{2}{9}\pi \qquad 6T_s = 6\,\frac{2\pi}{18} = \frac{2}{3}\pi$$

$$= \frac{6}{9}\pi$$



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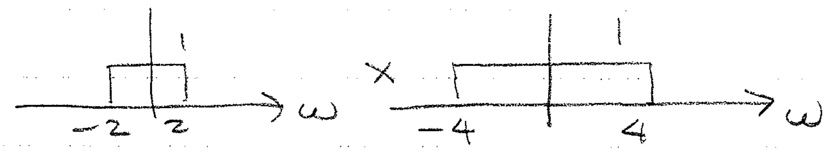
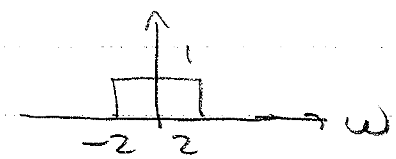
Part (c):

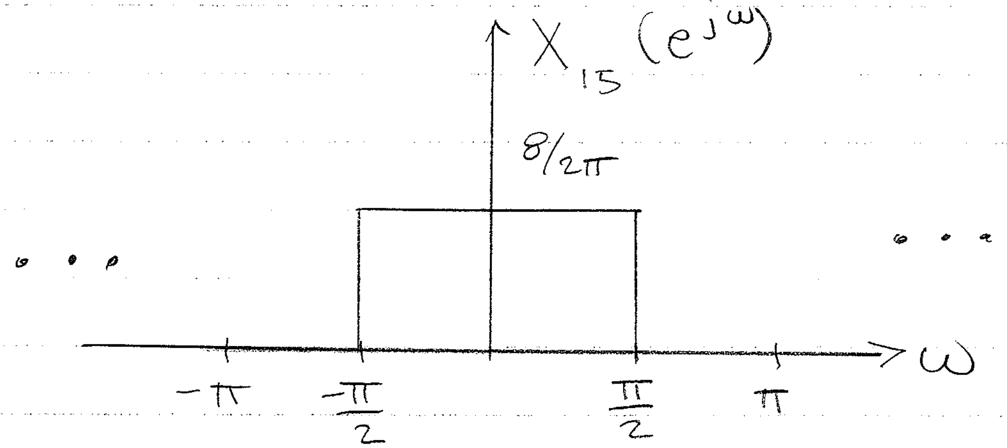
$$x_{15}(t) = \left\{\frac{\sin(2t)}{\pi t}\right\} * \left\{\frac{\sin(4t)}{\pi t}\right\}$$

$X_{15}(j\omega) =$ 

$=$ 

$$\omega_{a_{max}} = 2 \Rightarrow \omega_s = 8 > 2(2) = 4$$

$$\omega_{d_{max}} = 2\,T_s = 2\,\frac{(2\pi)}{8} = \frac{\pi}{2}$$



(a) $x_1(t) = \cos(4t)$. Plot the magnitude of the DTFT of $x_1[n] = x_1(nT_s)$ for $T_s = \frac{2\pi}{6}$.

(b) $x_2(t) = \dfrac{\sin(4t)}{\pi t}$. Plot the magnitude of the DTFT of $x_2[n] = x_2(nT_s)$ for $T_s = \frac{2\pi}{16}$.

(c) $x_3(t) = \dfrac{\sin(4t)}{\pi t}$. Plot the magnitude of the DTFT of $x_3[n] = x_3(nT_s)$ for $T_s = \frac{2\pi}{12}$.

(d) $x_4(t) = \dfrac{\sin(4t)}{\pi t}$. Plot the magnitude of the DTFT of $x_4[n] = x_4(nT_s)$ for $T_s = \frac{2\pi}{8}$.

(e) $x_5(t) = \dfrac{\sin(4t)}{\pi t}$. Plot the magnitude of the DTFT of $x_5[n] = x_5(nT_s)$ for $T_s = \frac{2\pi}{6}$.

(f) $x_6(t) = \dfrac{d}{dt}\left\{\dfrac{\sin(4t)}{\pi t}\right\}$. Plot the magnitude of the DTFT of $x_6[n] = x_6(nT_s)$ for $T_s = \frac{2\pi}{12}$.

(g) $x_7(t) = \dfrac{d}{dt}\left\{\dfrac{\sin(4t)}{\pi t}\right\}$ Plot the magnitude of the DTFT of $x_7[n] = x_7(nT_s)$ for $T_s = \frac{2\pi}{8}$.

(h) $x_8(t) = \left\{\dfrac{\sin(4t)}{\pi t}\right\}^2$. Plot the magnitude of the DTFT of $x_8[n] = x_8(nT_s)$ for $T_s = \frac{2\pi}{16}$.

(i) $x_9(t) = \left\{\dfrac{\sin(4t)}{\pi t}\right\}^2$. Plot magnitude of the DTFT of $x_9[n] = x_9(nT_s)$ for $T_s = \frac{2\pi}{12}$.

(j) $x_{10}(t) = t\left\{\dfrac{\sin(4t)}{\pi t}\right\}^2$. Plot magnitude of the DTFT of $x_{10}[n] = x_{10}(nT_s)$ for $T_s = \frac{2\pi}{24}$.

(k) $x_{11}(t) = t\left\{\dfrac{\sin(4t)}{\pi t}\right\}^2$. Plot magnitude of the DTFT of $x_{11}[n] = x_{11}(nT_s)$ for $T_s = \frac{2\pi}{16}$.

(ℓ) $x_{12}(t) = \left\{\dfrac{\sin(\overset{3}{\cancel{4}}t)}{\pi t}\right\}^2 \cos(\underset{4}{\cancel{6}}t)$. Plot magnitude of DTFT of $x_{12}[n] = x_{12}(nT_s)$ for $T_s = \frac{2\pi}{\cancel{24}}$.
~~18~~ 16

(m) $x_{13}(t) = \left\{\dfrac{\sin(4t)}{\pi t}\right\}^2 \cos(6t)$. Plot magnitude of DTFT of $x_{13}[n] = x_{13}(nT_s)$ for $T_s = \frac{2\pi}{16}$.

(n) $x_{14}(t) = \left\{\dfrac{\sin(2t)}{\pi t}\right\}\left\{\dfrac{\sin(4t)}{\pi t}\right\}$. Plot the magnitude of the DTFT of $x_{14}[n] = x_{14}(nT_s)$ for $T_s = \frac{2\pi}{18}$.

(o) $x_{15}(t) = \left\{\dfrac{\sin(2t)}{\pi t}\right\} * \left\{\dfrac{\sin(4t)}{\pi t}\right\}$, where $*$ denotes convolution. Plot the magnitude of the DTFT of $x_{15}[n] = x_{15}(nT_s)$ for $T_s = \frac{2\pi}{8}$.