EE301 Signals and Systems **Exam 3**
Exam 3 **Thursday, April 23, 2009**

Cover Sheet

Test Duration: 75 minutes.
Coverage: Chaps. 7 and 5, but will need Chap. 4 material
Open Book but Closed Notes. One two-sided handwritten sheet.
Calculators NOT allowed.
This test contains **one** long problem, with many parts.
All work should be done in the blue books provided.
You must show all work for each problem to receive full credit.
Do **not** return this test sheet, just return the blue books.

Problem 1.

(a) Consider the continuous-time signal $x_1(t)$ below A discrete-time signal is created by sampling $x_1(t)$ according to $x_1[n] = x_1(nT_s)$ for $T_s = \frac{2\pi}{160}$. Plot the magnitude of the DTFT of $x_1[n]$, $|X_1(\omega)|$, over $-\pi < \omega < \pi$.

$$x_1(t) = T_s \frac{d}{dt} \left\{ \frac{\sin(10t)}{\pi t} \right\}$$

(b) Consider the continuous-time signal $x_2(t)$ below A discrete-time signal is created by sampling $x_2(t)$ according to $x_2[n] = x_2(nT_s)$ for $T_s = \frac{2\pi}{160}$. Plot the magnitude of the DTFT of $x_2[n]$, $|X_2(\omega)|$, over $-\pi < \omega < \pi$.

$$x_2(t) = T_s \frac{1}{2} \left\{ \frac{\sin(10(t - \frac{\pi}{20}))}{\pi(t - \frac{\pi}{20})} + \frac{\sin(10(t + \frac{\pi}{20}))}{\pi(t + \frac{\pi}{20})} \right\}$$

(c) Given $x_1[n]$ and $x_2[n]$ defined above, the signal $x[n]$ is created as shown below. Determine the DTFT, $X(\omega)$, of $x[n]$ and plot the magnitude $|X(\omega)|$ over $-\pi < \omega < \pi$ showing as much detail as possible.

$$x[n] = 2x_1[n] \cos\left(\frac{3\pi}{8} n\right) + 2x_2[n] \cos\left(\frac{7\pi}{8} n\right)$$

For EACH of parts (d) thru (g) of this problem, the signal $x[n]$ from part (c) above is input to a DT LTI system whose impulse response is given. For EACH part, you must do EACH of the following THREE steps. You MUST show all your work.

(i) Plot the magnitude $|H_i(\omega)|$ of the DTFT of impulse response $h_i[n]$ over $-\pi < \omega < \pi$.

(ii) Plot the magnitude $|Y_i(\omega)|$ of the DTFT of the output signal $y_i[n]$ over $-\pi < \omega < \pi$.

(iii) Determine a simple, closed-form expression for the time-domain output $y_i[n]$.

(d) $h_1[n] = 8(-1)^n \left\{ \frac{\sin\left(\frac{\pi}{8} n\right)}{\pi n} \frac{\sin\left(\frac{3\pi}{8} n\right)}{\pi n} \right\}$

(e) $h_2[n] = 32 \left\{ \frac{\sin\left(\frac{\pi}{16} n\right)}{\pi n} \frac{\sin\left(\frac{3\pi}{16} n\right)}{\pi n} \right\} \cos\left(\frac{3\pi}{8} n\right)$

(f) $h_3[n] = 8 \left\{ \frac{\sin\left(\frac{\pi}{8} n\right)}{\pi n} \right\}^2$

(g) $h_4[n] = 2 \frac{\sin\left(\frac{\pi}{8} n\right)}{\pi n} \cos\left(\frac{3\pi}{8} n\right) + 2 \frac{\sin\left(\frac{\pi}{8} n\right)}{\pi n} \cos\left(\frac{7\pi}{8} n\right)$

(h) Determine and plot the magnitude of the DTFT $Z(\omega)$ of the signal $z[n]$ defined below, where $x[n] = 2x_1[n]\cos\left(\frac{3\pi}{8}n\right) + 2x_2[n]\cos\left(\frac{7\pi}{8}n\right)$ as defined in part (c). The trig identity $2\cos(\theta)\cos(\phi) = \cos(\theta+\phi) + \cos(\theta-\phi)$ should be useful.

$$z[n] = 2x[n]\cos\left(\frac{3\pi}{8}n\right)$$

(i) The signal $w[n]$ is the output obtained with $z[n] = 2x[n]\cos\left(\frac{3\pi}{8}n\right)$ from part (h) as the input to the DT lowpass filter with impulse response $h[n]$ defined below. That is, $w[n] = z[n] * h[n]$, where $z[n] = 2x[n]\cos\left(\frac{3\pi}{8}n\right)$ and

$$h[n] = 8\left\{\frac{\sin\left(\frac{\pi}{8}n\right)}{\pi n}\,\frac{\sin\left(\frac{\pi}{4}n\right)}{\pi n}\right\}$$

(i) Plot the magnitude of the DTFT $H(\omega)$ of $h[n]$ over $-\pi < \omega < \pi$.

(ii) Plot the magnitude of the DTFT $W(\omega)$ of output signal $w[n]$ over $-\pi < \omega < \pi$.

(iii) Is $w[n] = 2x_1[n]$?

(iii) Compute the energy of the signal at the output:

$$E_w = \sum_{-\infty}^{\infty} |w[n]|^2$$