# ECE301 Signals and Systems Exam 3

## Exam 3 Friday, April 18, 2008

## Cover Sheet

Test Duration: 55 minutes. Coverage: Emphasis on Chaps. 5 and 7 Open Book but Closed Notes. One loose sheet allowed. Calculators allowed. This test contains **two** problems. All work should be done in the blue books provided. You must show all work for each problem to receive full credit. Do **not** return this test sheet, just return the blue books.

For EACH of the part of this problem:

- You need only plot the magnitude of the DTFT over  $-\pi < \omega < \pi$ , but it is very important to keep in mind that a DTFT is always periodic with period  $2\pi$ .
- You must clearly label the DTFT magnitude plot requested and show as much detail as possible, clearly pointing out regions over  $-\pi < \omega < \pi$  for which the DTFT is zero.
- You MUST show all work and explain how you got your answer concisely but with sufficient detail to receive full credit.
- The unit of  $T_s$  is seconds for all parts.

### Problem 1.

Consider the continuous-time signal  $x(t) = \left\{\frac{\sin(3t)}{\pi t}\right\}^2 \cos(4t)$ . A discrete-time signal is created by sampling  $x_1(t)$  according to  $x[n] = x(nT_s)$  for  $T_s = \frac{2\pi}{16}$ .

- (a) Plot the magnitude of the DTFT of x[n],  $|X(\omega)|$ , over  $-\pi < \omega < \pi$ .
- (b) x[n] is passed through a DT linear system with impulse response  $h_a[n] = \left\{\frac{\sin(\frac{\pi}{8}n)}{\pi n}\right\} \cos\left(\frac{7\pi}{8}n\right)$ yielding the output  $y_a[n]$ . Plot the magnitude of the DTFT of  $h_a[n]$ ,  $|H_a(\omega)|$
- (c) Plot magnitude of the DTFT of  $y_a[n]$ ,  $|Y_a(\omega)|$ , over  $-\pi < \omega < \pi$ .
- (d) Determine the numerical values of the energy  $\sum_{n=-\infty}^{\infty} y_a^2[n]$
- (e) x[n] is passed through a DT linear system with impulse response  $h_b[n] = \frac{\sin(\frac{\pi}{4}n)}{\pi n}$  yielding the output  $y_b[n]$ . Plot the magnitude of the DTFT of  $h_b[n]$ ,  $|H_b(\omega)|$ .
- (f) Plot magnitude of the DTFT of  $y_b[n]$ ,  $|Y_b(\omega)|$ , over  $-\pi < \omega < \pi$ .
- (g) Determine the numerical values of the energy  $\sum_{n=-\infty}^{\infty} y_b^2[n]$

#### Problem 2.

Consider the continuous-time signal below equal to a sum of four sinewaves.

$$x(t) = \cos(3t) + \cos(6t) + \cos(7.5t) + \cos(9t) + \cos(10.5t) + \cos(12t)$$

A discrete-time signal is created by sampling x(t) according to  $x[n] = x(nT_s)$  for  $T_s = \frac{2\pi}{12}$ .

- (a) Plot the magnitude of the DTFT of x[n],  $|X(\omega)|$ , over  $-\pi < \omega < \pi$ .
- (b) x[n] is passed through a DT linear system described by the difference equation

$$y[n] = -x[n+1] + 2x[n] - x[n-1]$$

Plot the magnitude of the DTFT of the impulse response of the system, i.e., the frequency response,  $|H(\omega)|$ , over  $-\pi < \omega < \pi$ .

(c) Plot magnitude of the DTFT of the output y[n],  $|Y(\omega)|$ , over  $-\pi < \omega < \pi$ .