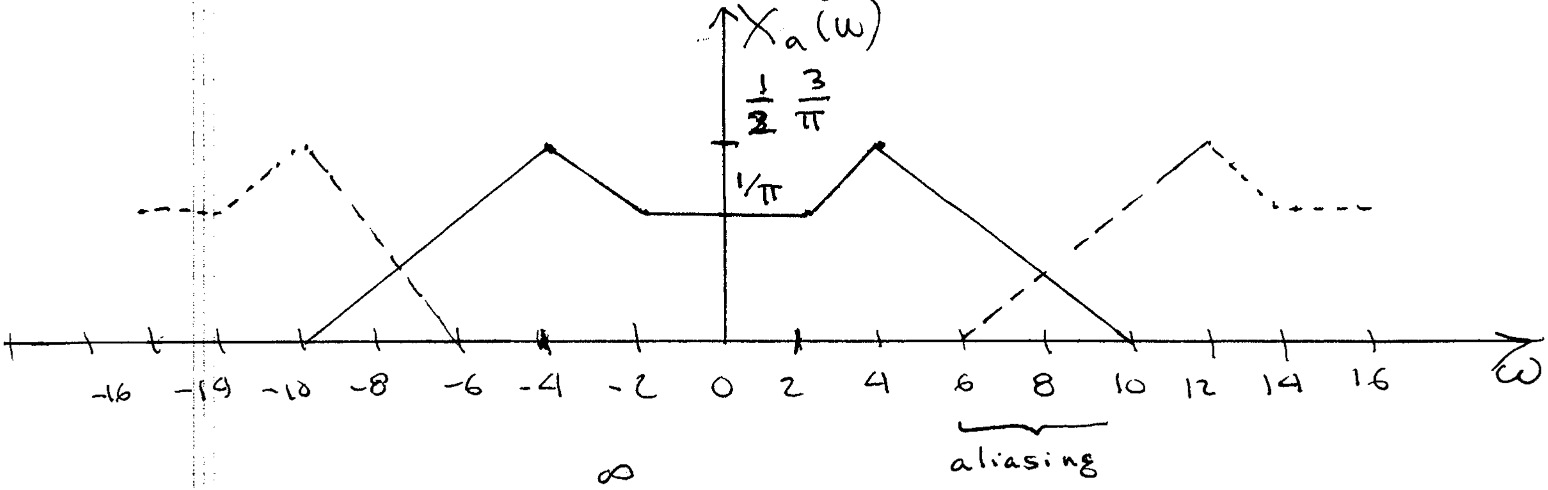
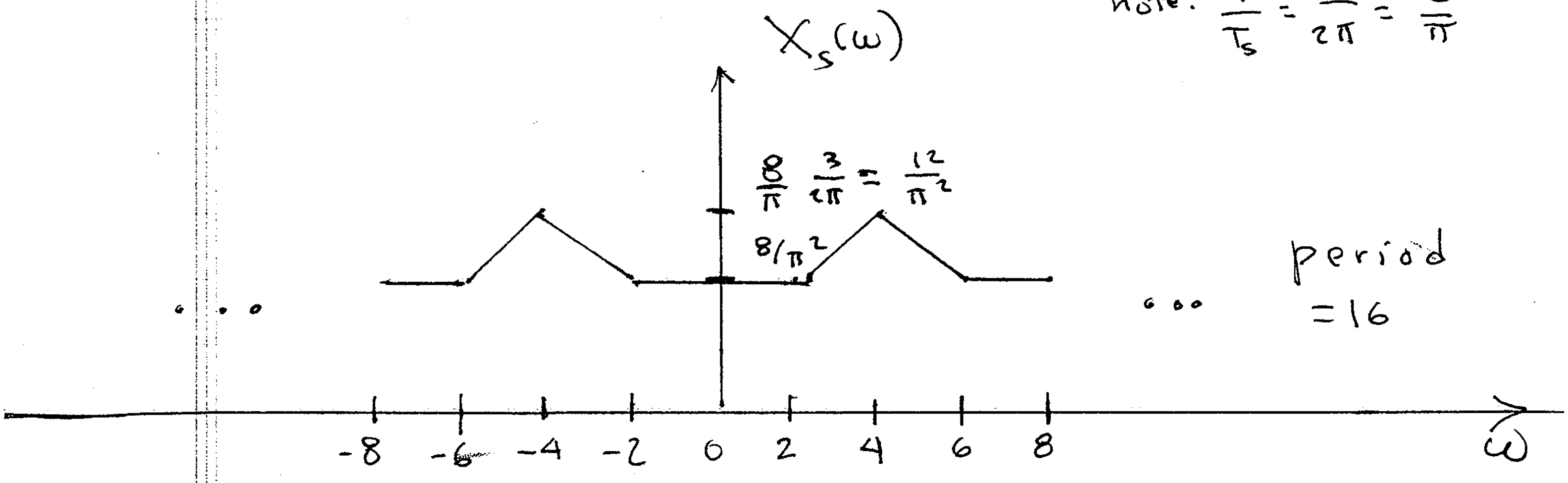
Exam 3 Solution SP 2008

$$x_a(t) = \left\{ \frac{\sin(3t)}{\pi t} \right\}^2 \cos(4t)$$



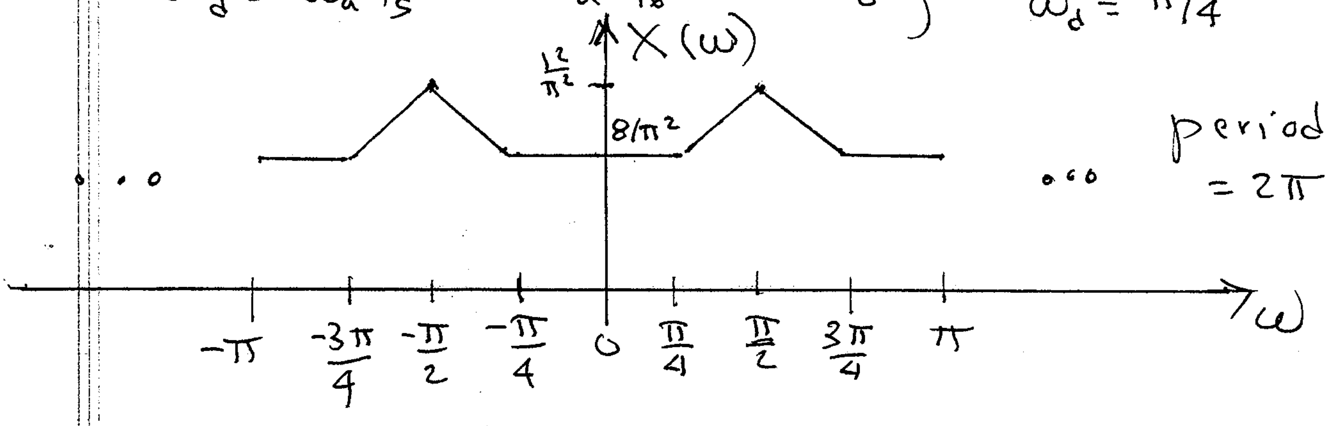
$$X_s(\omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X_a\left(\omega - k\frac{2\pi}{T_s}\right) \qquad \frac{2\pi}{T_s} = 16$$

note: $\frac{1}{T_s} = \frac{16}{2\pi} = \frac{8}{\pi}$

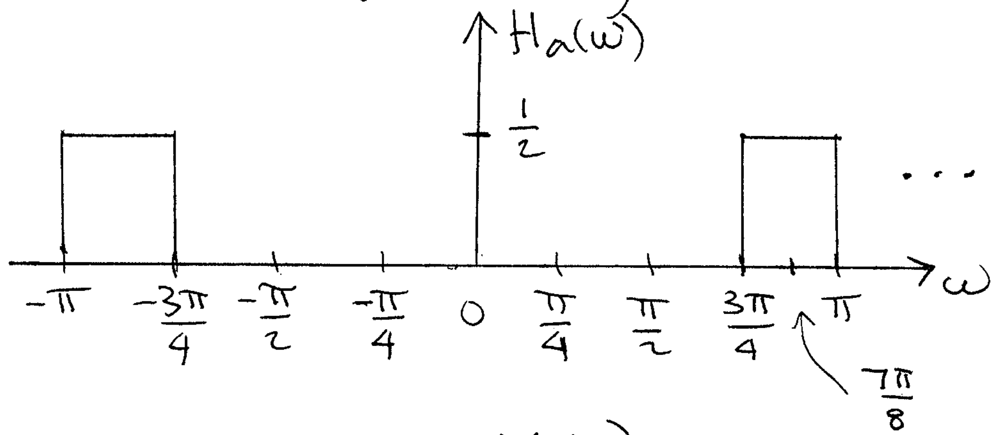


$$\omega_d = \omega_a T_s = \omega_a \frac{2\pi}{16} = \omega_a \frac{\pi}{8}$$

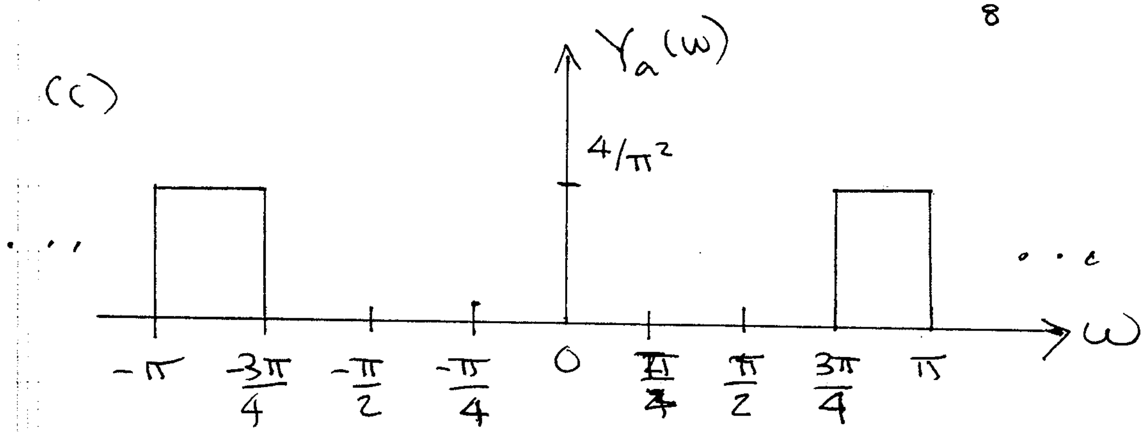
example $\omega_a = 2$ mapped to $\omega_d = \pi/4$



(b) $h_a[n] = \left\{ \dfrac{\sin\left(\frac{\pi}{8}n\right)}{\pi n} \right\} \cos\left(\dfrac{7\pi}{8}n\right)$



(c)

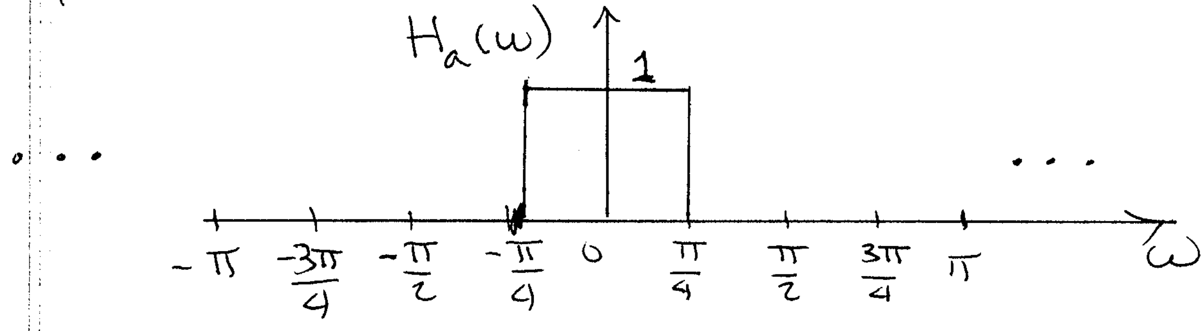


(d) $E_a = \sum_{n=-\infty}^{\infty} y_a^2[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} |Y_a(\omega)|^2 \, d\omega$

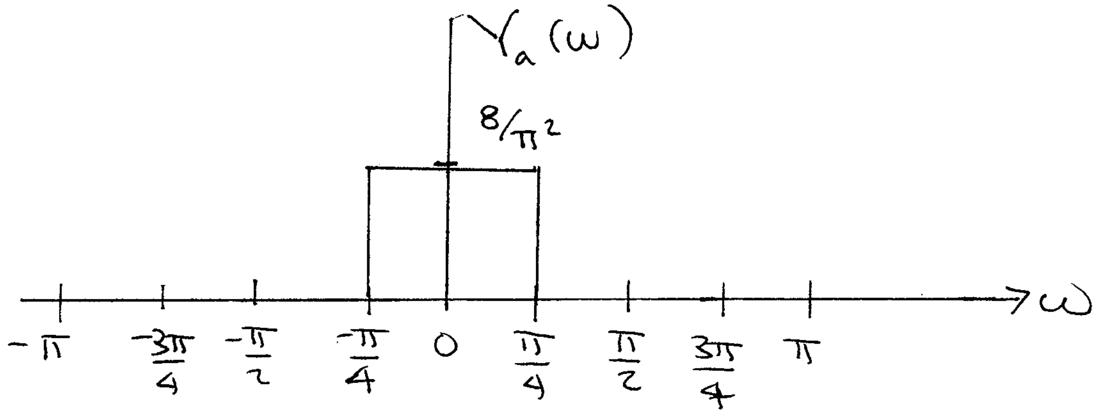
$$= \frac{1}{2\pi}(2)\underbrace{\left(\frac{\pi}{4}\right)}_{\text{width}}\underbrace{\left(\frac{4}{\pi^2}\right)^2}_{\text{height}^2}$$

$$= \frac{4}{\pi^4}$$

(e) $h_b[n] = \dfrac{\sin\left(\frac{\pi}{4}n\right)}{\pi n}$



(f)



(g) $E_b = \sum_{n=-\infty}^{\infty} y_b^2[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} |Y_b(\omega)|^2 d\omega$

$$= \frac{1}{2\pi}\left(\frac{\pi}{2}\right)\left(\frac{8}{\pi^2}\right)^2$$

$$= \frac{1}{4}\,\frac{64}{\pi^4}$$

$$= \frac{16}{\pi^4}$$

Prob. 2 Solution:

(a) $T_s = \frac{2\pi}{12}$ $\quad \omega_s = \frac{2\pi}{T_s} = 12$

- highest frequency we can "see" is 6
- any frequency above $\frac{\omega_s}{2} = 6$ is aliased to a frequency below 6
- easiest approach for the aliased sinewaves:

$$\cos(6t)\Big|_{t=nT_s=n\frac{2\pi}{12}} = \cos\left(n\frac{12\pi}{12}\right) = \cos(\pi n)$$

$$\cos(7.5t)\Big|_{t=n\frac{2\pi}{12}} = \cos\left(n\frac{15}{2}\frac{2\pi}{12}\right) = \cos\left(\frac{5}{4}\pi n\right)$$

$$= \cos\left(\frac{5}{4}\pi n - \frac{8\pi}{4}n\right) = \cos\left(-\frac{3\pi}{4}n\right) = \cos\left(\frac{3\pi}{4}n\right)$$

$$\cos(9t)\Big|_{t=n\frac{2\pi}{12}} = \cos\left(n\,9\,\frac{2\pi}{12}\right) = \cos\left(\frac{3}{2}\pi n\right)$$

$$= \cos\left(\frac{3}{2}\pi n - \frac{4}{2}\pi n\right) = \cos\left(-\frac{\pi}{2}n\right) = \cos\left(\frac{\pi}{2}n\right)$$

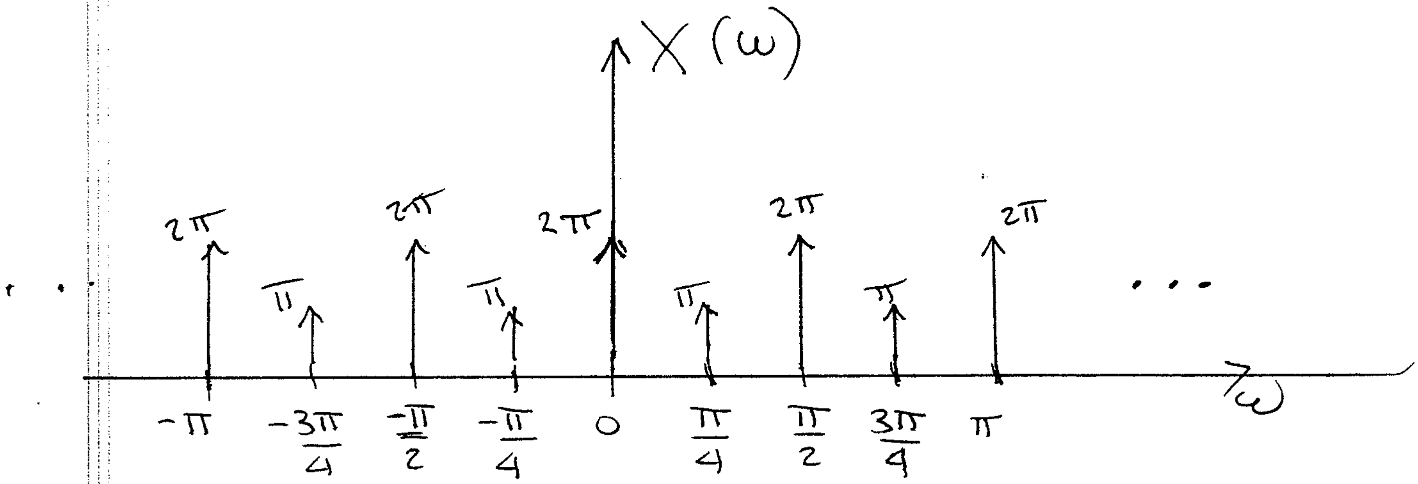
$$\cos(10.5t)\Big|_{t=n\frac{2\pi}{12}} = \cos\left(n\frac{21}{2}\frac{2\pi}{12}\right) = \cos\left(n\frac{7\pi}{4}\right)$$

$$= \cos\left(\frac{7}{4}\pi n - \frac{8}{4}\pi n\right) = \cos\left(-\frac{\pi}{4}n\right) = \cos\left(\frac{\pi}{4}n\right)$$

$$\cos(12t)\Big|_{t=n\frac{2\pi}{12}} = \cos\left(n\,12\,\frac{2\pi}{12}\right)$$

$$= \cos(2\pi n) = \cos(0\cdot n)$$

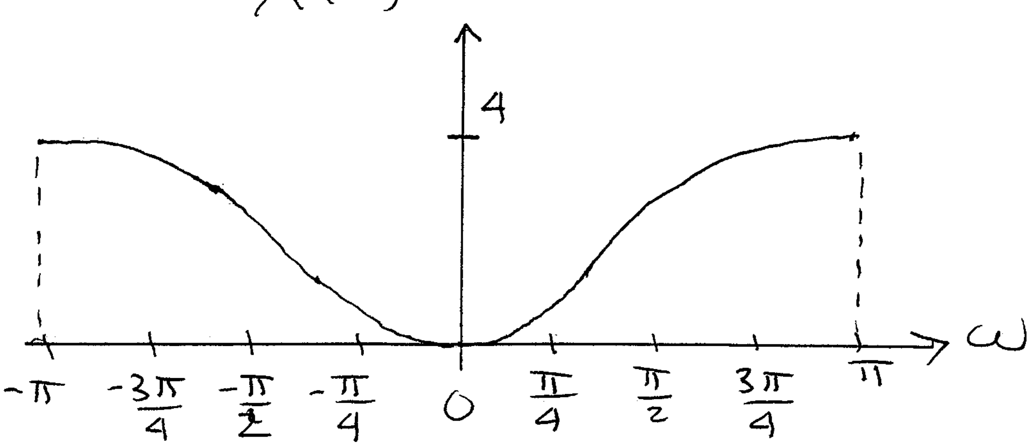
$$\cos(3t)\Big|_{t=n\frac{2\pi}{12}} = \cos\left(n\,3\,\frac{2\pi}{16}\right) = \cos\left(\frac{\pi}{2}n\right)$$



(b) $y[n] = -x[n+1] + 2x[n] - x[n-1]$

$$Y(\omega) = +X(\omega)\left\{-e^{j\omega} + 2 - e^{-j\omega}\right\}$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = 2 - 2\cos(\omega) = 2(1-\cos(\omega))$$



(c) H_a