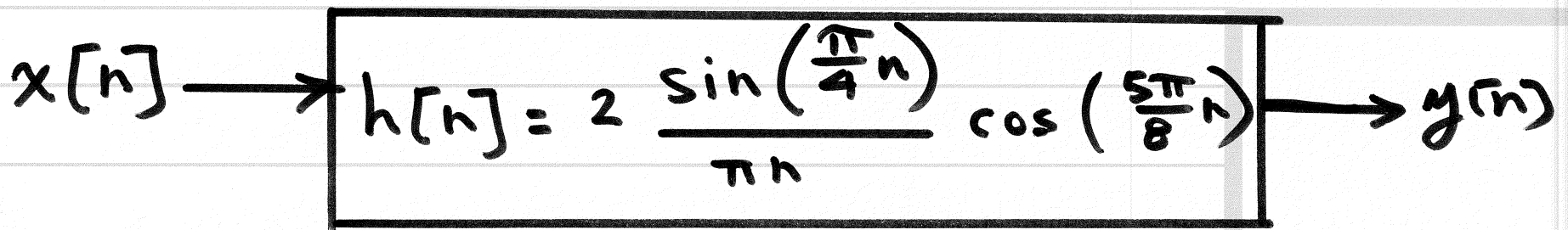
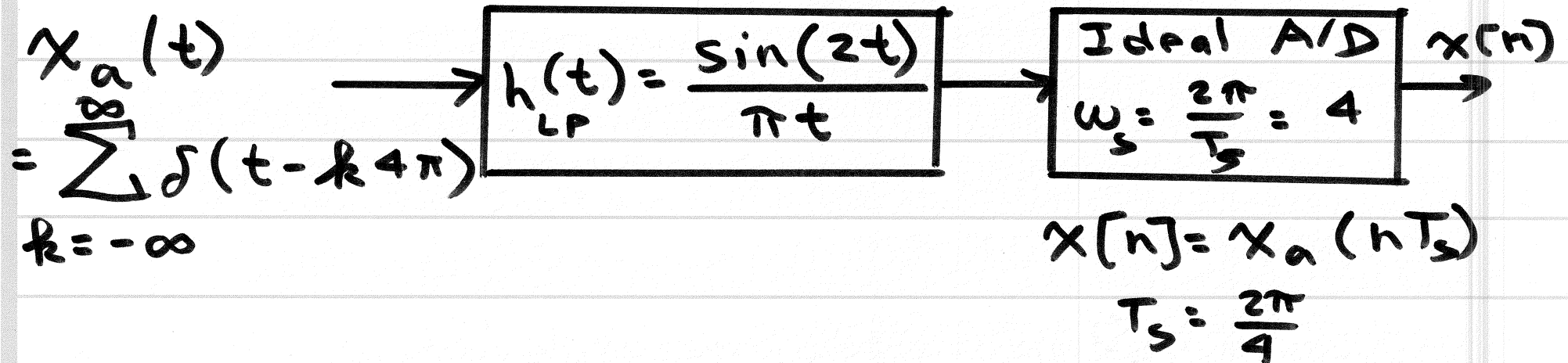


Exam 3 Help Problem:

①

Prob. 5 from Final Exam Spring 2008

Problem:

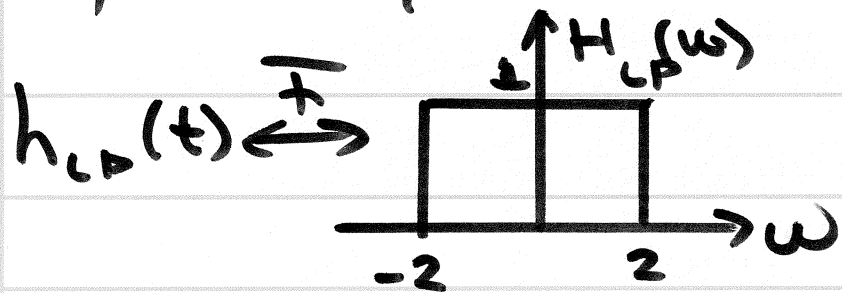


$y[n] = ?$ Plot $|Y(\omega)|$ over $-\pi < \omega < \pi$

From Table 9.2, we have FS expansion:

$$\begin{aligned}
 x_a(t) &= \sum_{k=-\infty}^{\infty} \delta(t - k4\pi) = \sum_{k=-\infty}^{\infty} \frac{1}{4\pi} e^{j k \frac{2\pi}{4\pi} t} \quad (2) \\
 &= \frac{1}{4\pi} \sum_{k=-\infty}^{\infty} e^{j k \frac{1}{2} t}
 \end{aligned}$$

input freqs: $k \frac{1}{2}$, $-\infty < k < \infty$



passes:

$$-2, -\frac{3}{2}, -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, \frac{3}{2}, 2 \quad k=4$$

recall, when you sample complex sine wave

$$\left. \begin{aligned}
 e^{j\omega_a t} \\
 \left. \begin{aligned}
 & \Big|_{t=nT_s} = e^{j(\omega_a T_s) n} \\
 & \Big\} \begin{aligned}
 & \text{DT frequencies} \\
 & = T_s \times \text{analog} \\
 & \text{frequencies}
 \end{aligned}
 \end{aligned}
 \right\}
 \end{aligned}$$

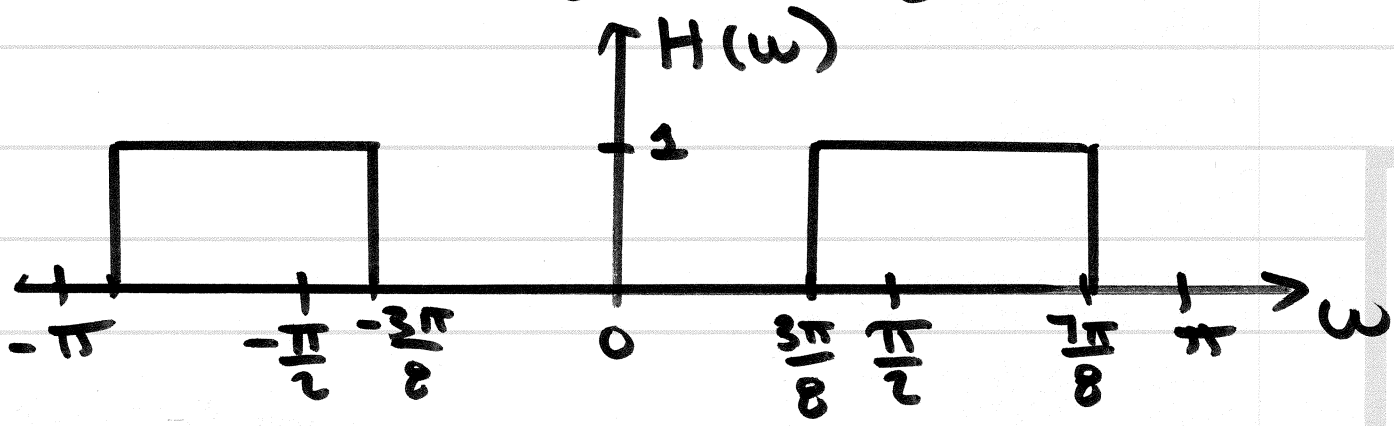
Thus, $x[n] = \frac{1}{4\pi} \sum_{k=-4}^4 e^{j k \frac{\pi}{2}}$ (3)

$$\frac{1}{4\pi} \sum_{k=-4}^4 e^{j k \frac{\pi}{2}}$$

 } freqs:
 $-\pi, -\frac{\pi}{2}, \frac{\pi}{2}, \pi, 0$
 $\pi, \frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2}, 0$

DTFT of $h[n]$: $\frac{1}{4} - \frac{\pi}{4} = \frac{1}{4} - \frac{\pi}{2} = \frac{1}{4}$

$\frac{1}{4} + \frac{\pi}{4} = \frac{1}{4} + \frac{\pi}{2} = \frac{1}{4}$



passes: $-\frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2}$

$\frac{\pi}{4}$ is rejected since $\frac{\pi}{4} = \frac{2\pi}{8} > \frac{\pi}{8}$ (4)

$\frac{3\pi}{4}$ is passed since $\frac{3\pi}{4} = \frac{6\pi}{8} > \frac{\pi}{8}$

$$y[n] = \frac{1}{4\pi} \left\{ e^{-j\frac{3\pi}{4}n} + e^{j\frac{\pi}{2}n} + e^{j\frac{\pi}{2}n} + e^{j\frac{3\pi}{4}n} \right\}$$

recall: $e^{j\omega_0 n} \xleftrightarrow{\text{DTFT}} 2\pi \delta(\omega - \omega_0)$
over $-\pi < \omega < \pi$
assuming $-\pi < \omega_0 < \pi$

Thus:

