Continuous Time

Laplace Transform:

$$X_a(s) = \int_{-\infty}^{\infty} x_a(t) e^{-st} dt$$

CT Fourier Transform:

$$X_a(\omega) = \int_{-\infty}^{\infty} x_a(t) e^{-j\omega t} dt$$

Inverse CTFT:

$$x_a(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_a(\omega) e^{j\omega t} d\omega$$

$$X_a(\omega) = X_a(s)\Big|_{s=j\omega}$$

for finite energy signals

Discrete-Time

Z-Transform:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

DT Fourier Transform:

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

periodic: period = 2π

Inverse DTFT:

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega$$

$$X(\omega) = X(z)\Big|_{z=e^{j\omega}}$$

for finite-energy signals

If $x[n] = x_a(nT_s) = x_a(t)\Big|_{t=nT_s}$ ②

and $-\infty < n < \infty$

$x_a(t) \overset{\mathcal{F}}{\longleftrightarrow} X_a(\omega)$ $\quad$ $x[n] \overset{DTFT}{\longleftrightarrow} X(\omega)$

Then: $X(\omega) = X_s(F_s\omega)$ $\quad$ $\boxed{F_s = \frac{1}{T_s}}$

where: $X_s(\omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X_a\left(\omega - k\frac{2\pi}{T_s}\right)$

If $\omega_s = \frac{2\pi}{T_s} > 2\,\omega_M$ $\quad$ where: $X_a(\omega) = 0$ for $|\omega| > \omega_M$

Then: $X(\omega) = F_s\, X_a(F_s\omega)$ for $-\pi < \omega < \pi$

$\Rightarrow$ compression by sampling rate: $\omega_d = \frac{\omega_a}{F_s}$