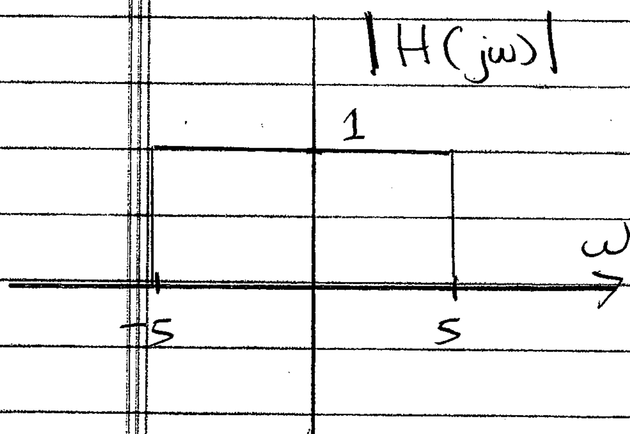
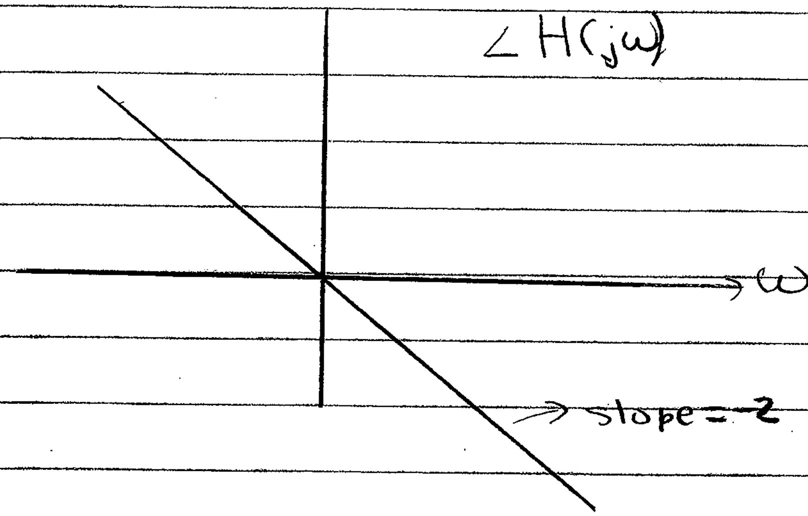
EE 301 Sol'n to Exam 2 SP'04

(a) $h(t) = \dfrac{\sin[5(t-2)]}{\pi(t-2)}$

$H(j\omega) = \{u(\omega+5) - u(\omega-5)\}\, e^{-j2\omega}$

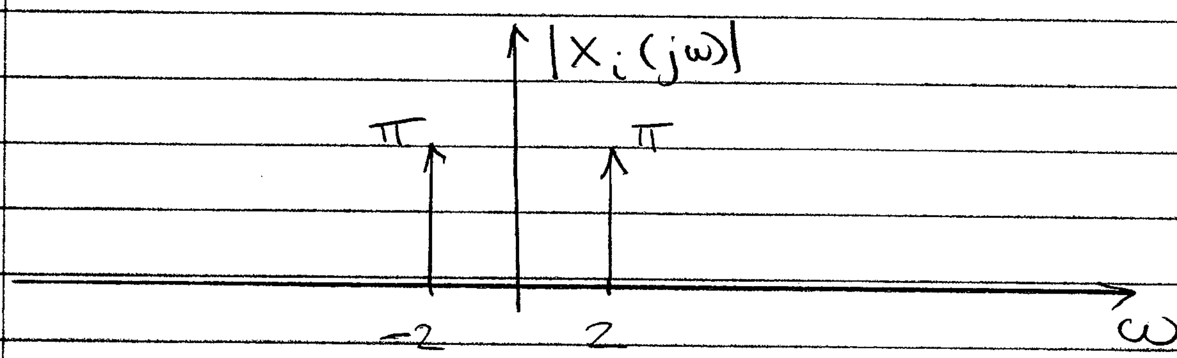




(b)

(i) $\cos(2t+\pi) \overset{\mathcal{F}}{\longleftrightarrow} \pi e^{j\pi}\delta(\omega-2) + \pi e^{-j\pi}\delta(\omega+2)$

(ii)



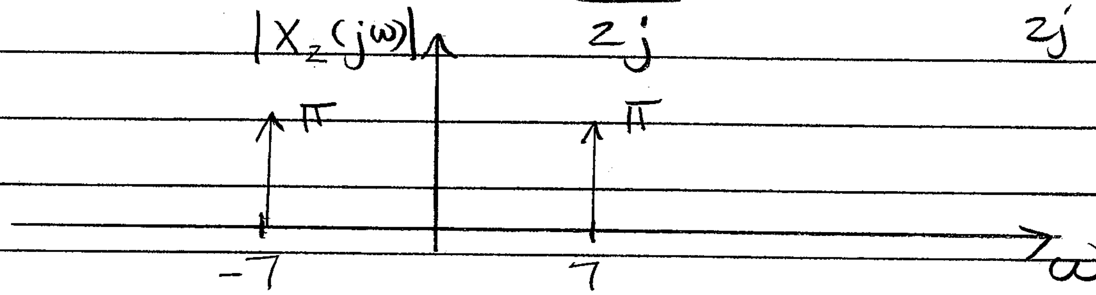
(iii) $y_i(t) = |H(j2)| \cos(2t + \pi + \angle H(j2))$

$= \cos(2(t-2) + \pi)$

(c)

(i) $\sin(7t+3\pi) \overset{\mathcal{F}}{\longleftrightarrow} \frac{e^{j3\pi}}{2j} 2\pi\delta(\omega-7) - \frac{e^{-j3\pi}}{2j} 2\pi\delta(\omega+7)$

(ii)

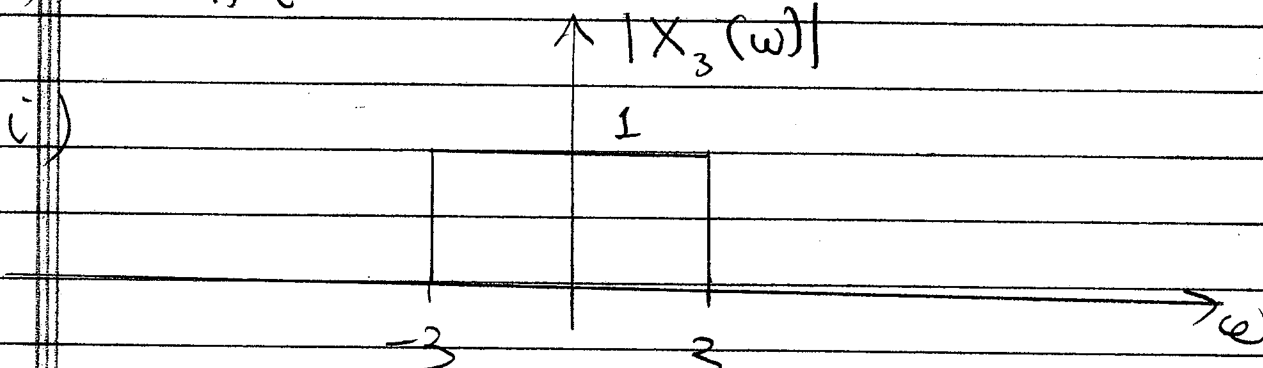


(iii) $y_2(t)=0$ since $|H(j7)|=0$

(d)

(i) $\frac{\sin(3t)}{\pi t} \overset{\mathcal{F}}{\longleftrightarrow} \{u[\omega+3]-u[\omega-3]\}$

(ii)



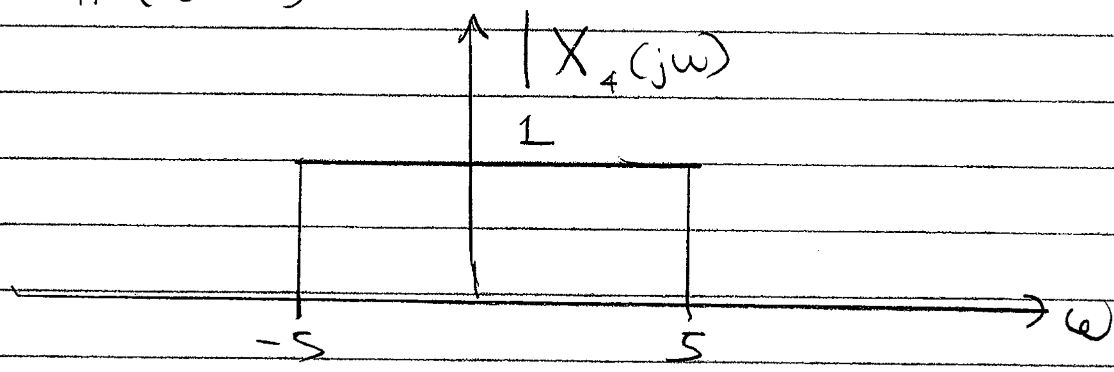
(iii) $y_3(t) = x_3(t) = \frac{\sin(3(t-2))}{\pi(t-2)}$

passes thru system unaltered except for delay by 2

(e)

(i) $$\frac{\sin[5(t+2)]}{\pi(t+2)} \overset{\mathcal{F}}{\longleftrightarrow} \{u[\omega+5]-u[\omega-5]\}e^{+j2\omega}$$

(ii)



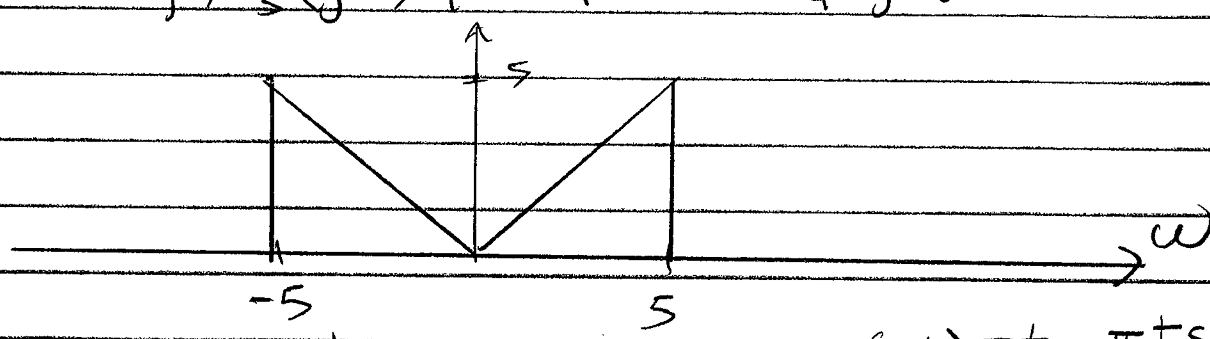
(iii) $$y_4(t) = \mathcal{F}^{-1}\{u[\omega+5]-u[\omega-5]\ \underbrace{e^{-j2\omega}e^{j2\omega}}_{\text{cancel}}\}$$

$$= \frac{\sin[5t]}{\pi t}$$

(f) $$x_5(t) = \frac{d}{dt}x_4(t) \overset{\mathcal{F}}{\longleftrightarrow} j\omega\, X_4(j\omega)$$

(i) $$|X_5(j\omega)| = |\omega|\,|X_4(j\omega)|$$

(ii)



(iii) $$y_5(t) = \frac{d}{dt}y_4(t) = \frac{5\cos(5t)\pi t - \pi\sin[5t]}{(\pi t)^2}$$

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(f) – (iii) (4)

$$Y_5(j\omega) = H(j\omega)\, j\omega\, X_4(j\omega)$$

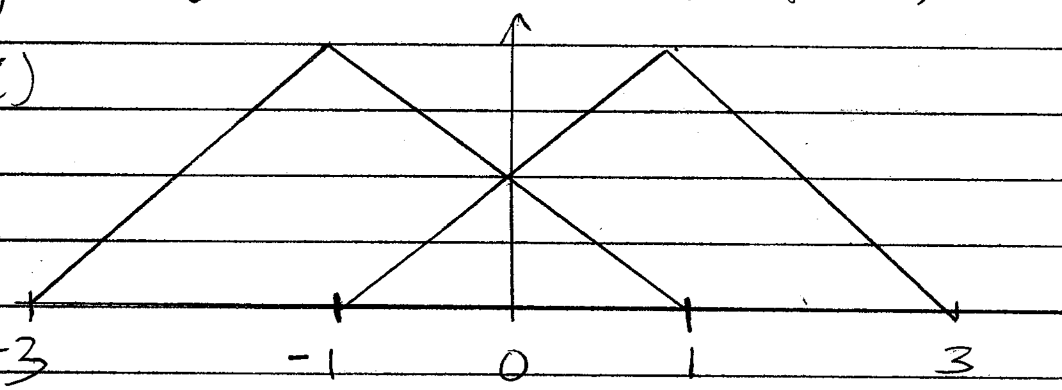
$$= j\omega \left\{ H(j\omega)\, X_4(j\omega) \right\}$$

$$= j\omega\, Y_4(j\omega)$$

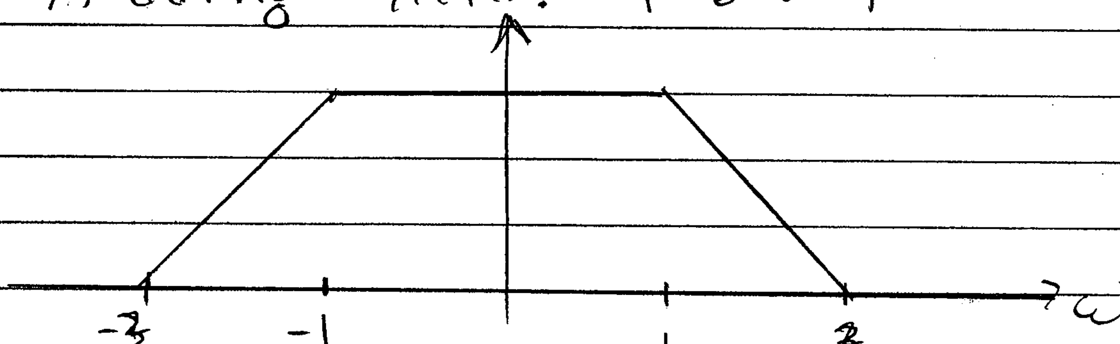
$$y_5(t) = \frac{d}{dt}\left\{ y_4(t) \right\}$$

(g) (i) Using modulation property

(ii)



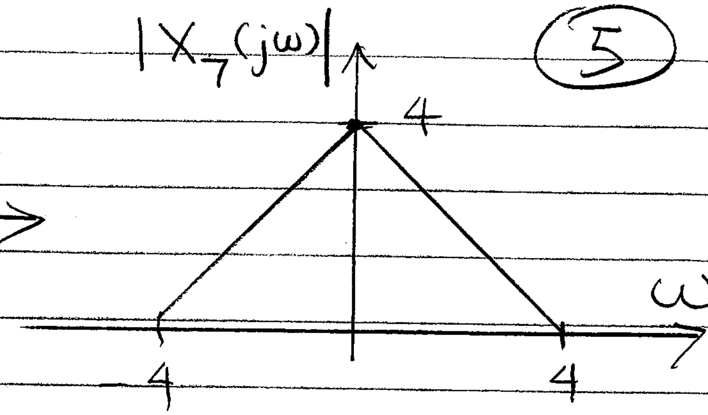
Adding yields: $|X_6(j\omega)|$



(iii) $y_6(t) = x_6(t-2) = \left\{ \dfrac{\sin(t-2)}{\pi(t-2)} \right\}^2 \cos(t-2)$

(h)

$$\left\{\frac{\sin(2t)}{\pi t}\right\}^2 \overset{\mathcal{F}}{\longleftrightarrow}$$

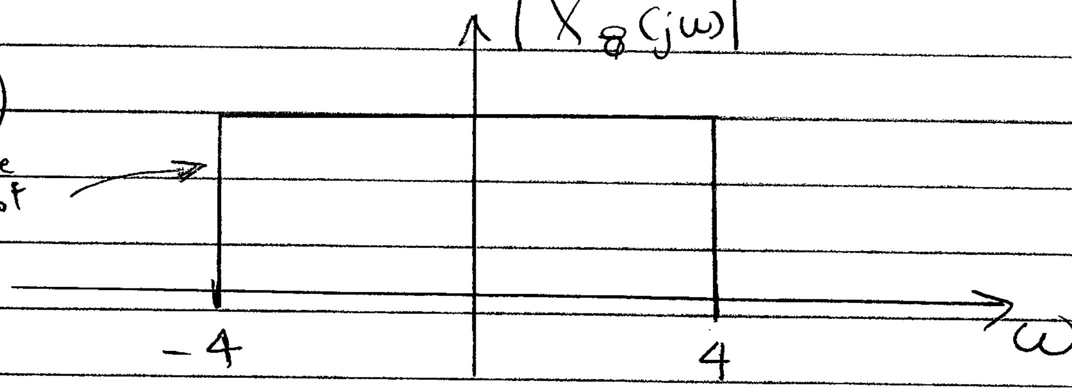


(iii) $y_7(t) = x_7(t-2)$ passes thru system except for delay

(i)-(ii) $$t\left\{\frac{\sin(2t)}{\pi t}\right\}^2 \overset{\mathcal{F}}{\longleftrightarrow} j\frac{d}{d\omega}\left\{X_7(j\omega)\right\}$$

(ii)

absolute value of -1 slope



(iii) $$Y_8(j\omega) = H(j\omega)\, j\frac{d}{d\omega}\left\{X_7(j\omega)\right\}$$

$$= j\frac{d}{d\omega}\left\{X_7(j\omega)\right\} e^{-j2\omega}$$

$$y_8(t) = (t-2)\left\{\frac{\sin[2(t-2)]}{\pi(t-2)}\right\}^2$$

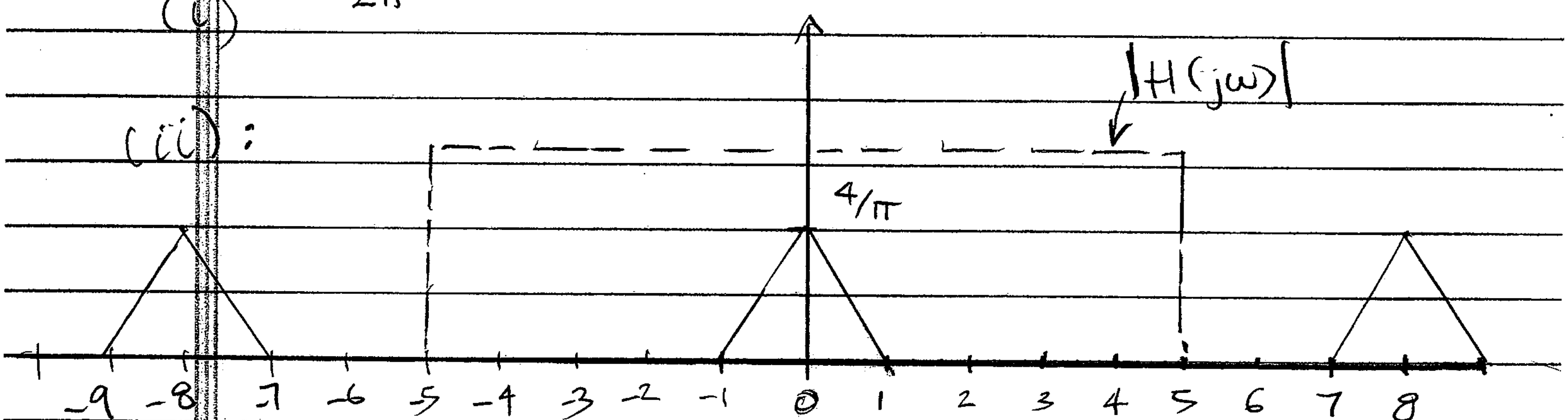
(j) $\Rightarrow$ fundamental frequency: $\omega_0 = \frac{2\pi}{\pi/4} = 8$

$$\left\{\frac{\sin(t)}{\pi t}\right\}^2 \left\{\sum_{n=-\infty}^{\infty} \delta\left(t - n\frac{\pi}{4}\right)\right\}$$

$$\overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2\pi}\left\{1 - |\omega|\right\}\left\{u(\omega+1) - u(\omega-1)\right\} * \frac{1}{\pi/4} \sum_{k} 2\pi\,\delta(\omega - k8)$$

(i)

(ii):



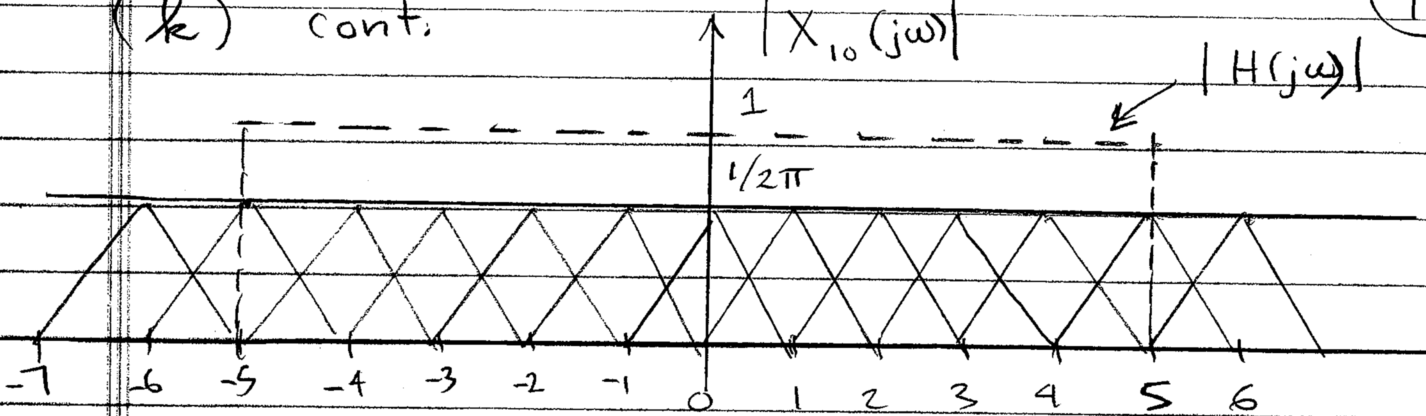
(iii):

$$y_q(t) = \frac{4}{\pi} \left\{\frac{\sin[t-2]}{\pi(t-2)}\right\}^2$$

(k) The difference relative to part (j) is period is now 2π so that fundamental freq $= \frac{2\pi}{2\pi} = 1$

$$X_{10}(j\omega) = \frac{1}{2\pi}\left\{1 - |\omega|\right\}\left\{u(\omega+1) - u(\omega-1)\right\} * \frac{1}{2\pi} \sum_{k} 2\pi\,\delta(\omega - k)$$

(k) cont.



(iii): $y_{10}(t) = h(t) = \dfrac{\sin[5(t-2)]}{\pi(t-2)}$

(l) $x(t)$ is periodic with period $= T = 3$

$$X_{11}(j\omega) = \frac{1}{3}\sum_{k=-\infty}^{\infty} 2\pi a_k\, \delta\left(\omega - k\frac{2\pi}{3}\right)$$

$$a_k = \frac{\sin\left[k\pi \frac{1}{3}\right]}{\pi k} e^{-j\frac{2\pi}{3}\left(\frac{3}{2}\right)k}$$

$$= (-1)^k \frac{\sin\left[k\frac{\pi}{3}\right]}{k\pi}$$

- frequencies passed are: $-\frac{4\pi}{3}, -\frac{2\pi}{3}, 0, \frac{2\pi}{3}, \frac{4\pi}{3}$

Thus:

$$y_{11} = a_{-2}e^{-j\frac{4\pi}{3}(t-2)} + a_{-1}e^{-j\frac{2\pi}{3}(t-2)} + a_0$$

$$+ a_{+2}e^{j\frac{4\pi}{3}(t-2)} + a_1 e^{j\frac{2\pi}{3}(t-2)}$$