KEY

EE301 Signals and Systems — In-Class Exam

Exam 2 — Tuesday, Mar. 29, 2011

Cover Sheet

Test Duration: 75 minutes.
Coverage: Chaps. 3,4
Open Book but Closed Notes.
One 8.5 in. x 11 in. crib sheet
Calculators NOT allowed.
All work should be done on the sheets provided.
You must show all work for each problem to receive full credit.
Plot your answers on the graphs provided.

Multiple Choice Question 1. Circle the best answer below. Let $X(\omega)$ be the Fourier Transform of a signal $x(t)$; as you vary ω, a plot of $|X(\omega)|^2$ reveals

(a) how the amplitude and phase of a sinewave change as it passes thru an LTI system.

(b) how multiplying by a high-frequency cosine makes the signal radiate from an antenna.

(c) how the energy of a periodic signal is concentrated at continuous frequencies.

(d) how the energy of the signal is distributed as a function of frequency.

(e) how two different signals can have the same energy and phase distribution

Multiple Choice Question 2. Circle the best answer below.

(a) $x(t)$ and $x(t - t_o)$ have the same energy distribution as a function of time.

(b) $x(t - t_o)$ can be obtained by passing $x(t)$ thru an LTI system with impulse response $h(t) = \delta(t - t_o)$.

(c) The respective Fourier Transforms of $x(t)$ and $x(t - t_o)$ differ only by a linear phase term.

(d) $x(t)$ and $x(t - t_o)$ have the same energy distribution as a function of frequency.

(e) (a), (b), (c), and (d) are all true

(f) only (b), (c), and (d) are true

Multiple Choice Question 3. Circle the best answer below. Forming the product of a baseband signal with a high-frequency sinewave

(a) changes the energy distribution as a function of frequency

(b) causes a shift in the frequency-domain

(c) makes the signal radiate from an antenna and thereby propagate through space

(d) places the signal in another frequency band

(e) (a), (b), (c), and (d) are all true

(f) only (b), (c), and (d) are true

(g) only (c), and (d) are true

Multiple Choice Question 4. Circle the best answer below. Multiplying by the independent variable in one domain (either time or frequency)

(a) keeps the energy distribution the same in both domains

(b) always causes instability

(c) causes differentiation with respect to the independent variable in the other domain

(d) changes the frequency band that the signal occupies

(e) causes multiplication by the independent variable in the other domain

Multiple Choice Question 5. Circle the best answer below. Let $x(t)$ be a **periodic** signal whose average value over one period is zero.

(a) In the frequency domain, the energy is distributed over a continuous band of frequencies.

(b) In the frequency domain, the energy is concentrated at discrete frequencies equal to the fundamental frequency and all of its harmonics.

(c) In the frequency domain, the energy is distributed over both discrete and continuous frequencies.

(d) In the frequency domain, the energy distribution is periodic as a function of frequency.

(e) In the frequency domain, the energy is concentrated at $\omega = 0$.

Multiple Choice Question 6. Circle the best answer below. One of the most important practical implications of the convolution (in time) property of the Fourier Transform (convolution in time leads to multiplication in the frequency domain) is

(a) that it makes multiplication as easy as convolution

(b) that it makes the energy distribution be the same at both the input and output of an LTI system

(c) that it distributes the energy the same way in both the time and frequency domains

(d) frequency selective linear filtering

(e) it stabilizes an LTI system by moving the poles to the right-half-plane

Problem 2. Short Workout Problems Using Fourier Transform Properties.

Problem 2 (a). You are given that the Fourier Transform of a Gaussian pulse $x(t) = e^{\frac{-t^2}{2}}$ is $X(\omega) = \sqrt{2\pi}\, e^{\frac{-\omega^2}{2}}$. That is,

$$x(t) = e^{\frac{-t^2}{2}} \overset{\mathcal{F}}{\longleftrightarrow} X(\omega) = \sqrt{2\pi}\, e^{\frac{-\omega^2}{2}}$$

Determine the Fourier Transform of

$$y(t) = e^{\frac{-t^2}{2\sigma^2}} \quad = e^{-\frac{1}{2}\left(\frac{t}{\sigma}\right)^2}$$

Write your expression for $Y(\omega)$ in the space directly below:

$$x(at) \longleftrightarrow \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$$

Thus: $a = \frac{1}{\sigma}$

$$Y(\omega) = \sigma X(\sigma\omega) = \sqrt{2\pi}\,\sigma e^{-\frac{1}{2}(\sigma\omega)^2}$$

$$= \sigma\sqrt{2\pi}\; e^{-\frac{\sigma^2}{2}\omega^2}$$

Problem 2 (b). You are given the Fourier Transform pair below

$$x(t) = \cos\left(\frac{\pi t}{2}\right) \text{rect}\left(\frac{t}{2}\right) \overset{\mathcal{F}}{\longleftrightarrow} X(\omega) = \frac{4\pi\cos(\omega)}{\pi^2 - 4\omega^2}$$

Determine the Fourier Transform of

$$y(t) = \frac{4\pi\cos(t)}{\pi^2 - 4t^2}$$

Write your expression for $Y(\omega)$ in the space directly below. Simplify as much as possible.

Duality Property dictates:

$$Y(\omega) = 2\pi\, x(-\omega)$$

$$= 2\pi \cos\left(\frac{\pi}{2}(-\omega)\right) \text{rect}\left(\frac{-\omega}{2}\right)$$

$$= 2\pi \cos\left(\frac{\pi}{2}\omega\right) \text{rect}\left(\frac{\omega}{2}\right)$$

Problem 2 (c). Consider an LTI system with impulse response

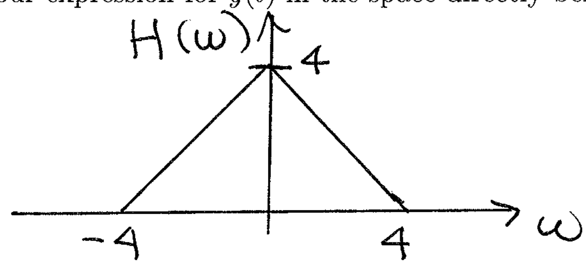
$$h(t) = 2\pi \left\{ \frac{\sin(2t)}{\pi t} \right\}^2$$

Determine the output y(t) for the input $x(t)$ given by

$$x(t) = \sum_{k=-\infty}^{-1} \frac{1}{k} e^{jkt} + e^{j0t} + \sum_{k=1}^{\infty} \frac{1}{k} e^{jkt}$$

$$x(t) = \sum_{k=-\infty}^{-1} \frac{1}{k} e^{jkt} + 1 + \sum_{k=1}^{\infty} \frac{1}{k} e^{jkt}$$

Show work and write your expression for $y(t)$ in the space directly below.



any frequency above 4 (and including 4) is rejected by the filter ⇒ not passed

$H(0) = 4$ $\quad H(1) = 3 = H(-1)$ $\quad H(2) = 2 = H(-2)$ $\quad H(3) = 1 = H(-3)$ $\quad H(4) = 0 = H(-4)$

Basic result: $e^{j\omega_0 t} \rightarrow \boxed{H(\omega)} \rightarrow H(\omega_0) e^{j\omega_0 t}$

Thus:

$$y(t) = H(-3)\left(-\tfrac{1}{3}\right)e^{-j3t} + H(-2)\left(-\tfrac{1}{2}\right)e^{-j2t} + H(-1)(-1)e^{-jt}$$
$$+ H(0)e^{j0t} + H(1)(1)e^{jt} + H(2)\left(\tfrac{1}{2}\right)e^{j2t} + H(3)\left(\tfrac{1}{3}\right)e^{j3t}$$

$$= -\frac{1}{3}e^{-j3t} - e^{-j2t} - 3e^{-jt} + 4$$
$$+ \frac{1}{3}e^{j3t} + e^{j2t} + 3e^{jt}$$

Workout Problem 3 Let $H_{LP}(\omega)$ be the Fourier Transform of the impulse response $h_{LP}(t)$ defined below.

$$h_{LP}(t) = \frac{\pi}{5} \frac{\sin(5t)}{\pi t} \frac{\sin(15t)}{\pi t} \tag{1}$$

(a) Note that $h_{LP}(t)$ is both real-valued and even-symmetric as a function of time. Thus, $H_{LP}(\omega)$ is both real-valued and symmetric as a function of frequency. Plot $H_{LP}(\omega)$ in the space provided. Show as much detail as possible.

(b) $h(t)$ is defined in terms of $h_{LP}(t)$ as:

$$h(t) = 20\, h_{LP}(t)\, \sin(20t) \tag{2}$$

Note that $h(t)$ is odd-symmetric as a function of time. Thus, $H(\omega)$ is purely imaginary for all frequencies. Plot $H(\omega)$ in the space provided. Note that the vertical axis values have the multiplicative scalar $j = \sqrt{-1}$ factored into them.

(c) Consider the input signal $x(t)$ below.

$$x_1(t) = \left\{ \frac{\sin(10t)}{\pi t} \right\}$$

Determine and plot the Fourier Transform $X_1(\omega)$ of the signal $x_1(t)$ in the space provided.

(d) Determine a closed-form analytical expression for the output $y_1(t)$ when the signal $x_1(t)$ in part (c) is the input to the LTI system with impulse response $h(t)$ in part (b) defined by eqn. (2). Write your answer in the space provided. *HINT:* look carefully at the frequency response $H(\omega)$ over $-10 < \omega < 10$ and relate to one of the properties of the Fourier Transform.

(e) Next, consider the input signal $x(t)$ below.

$$x(t) = \frac{2\pi}{5} \left\{ \frac{\sin(5t)}{\pi t} \right\}^2 \cos(20t)$$

Determine and plot the Fourier Transform $X(\omega)$ of the signal $x(t)$ in the space provided.

(f) Denote the output $y(t)$ when the signal in part (e) directly above is the input to the LTI system with impulse response $h(t)$ in part (b) defined by eqn. (2). Determine and plot $Y(\omega)$ in the space provided.

(g) Create a complex-valued signal as

$$z(t) = 10x(t) + jy(t)$$

Determine and plot $Z(\omega)$ in the space provided. Show as much detail as possible.

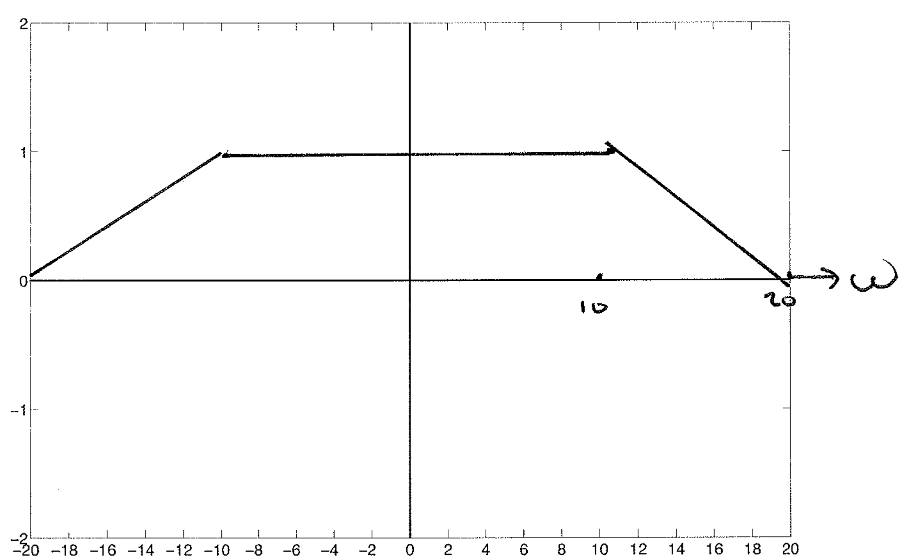
$\omega_1 = 5 \qquad \omega_2 = 15$

"height" $= \frac{\pi}{5} \frac{\omega_1}{\pi} = 1$

flat up to $\omega_2 - \omega_1 = 10$

rolls to zero at $\omega_1 + \omega_2 = 20$

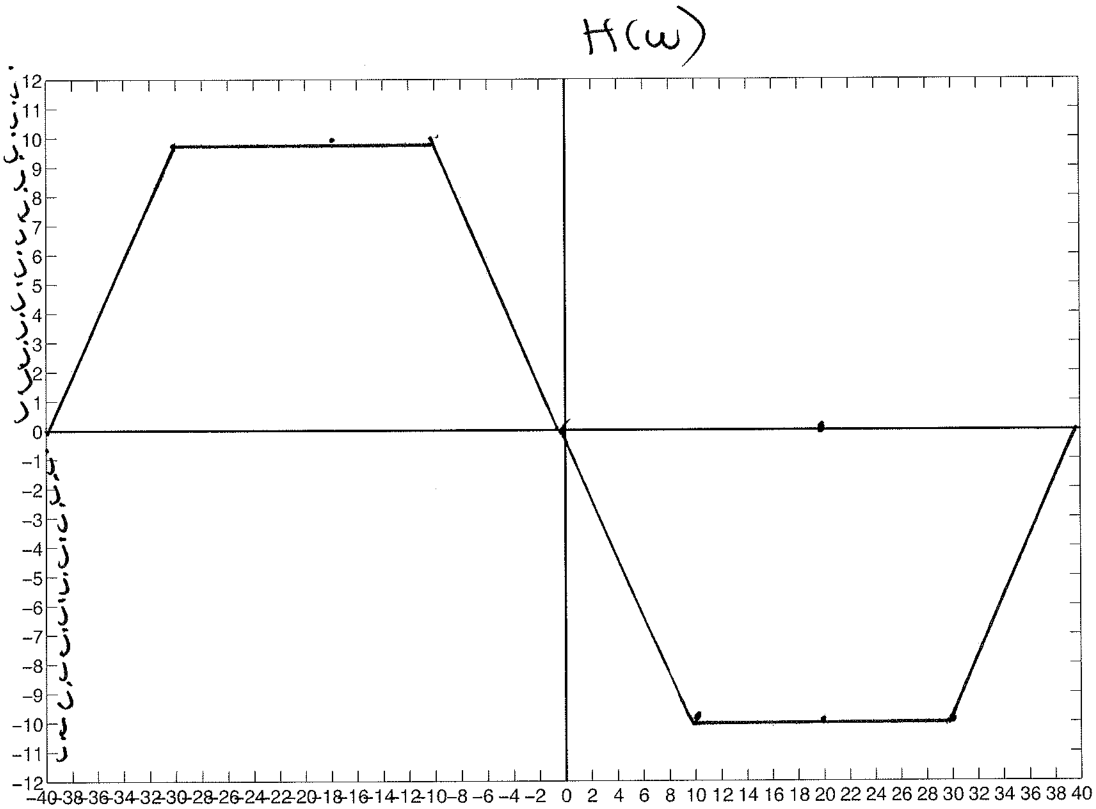
Plot your answer to Problem 3 (a) here. Show work above.



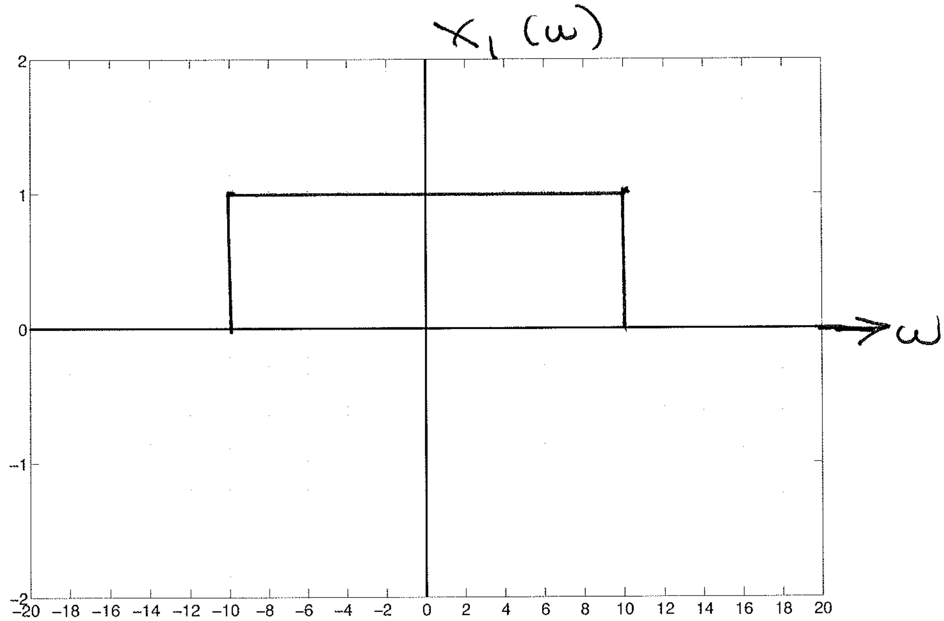
$$h(t) = 20\, h_{LP}(t) \frac{e^{j20t}}{2j} - 20\, h_{LP}(t) \frac{e^{-j20t}}{2j}$$

$$H(\omega) = -j10 H_{LP}(\omega - 20) + j10 H_{LP}(\omega + 20)$$

Plot your answer to Problem 3 (b) here. Show work above.



Plot your answer to Problem 3 (c) here.

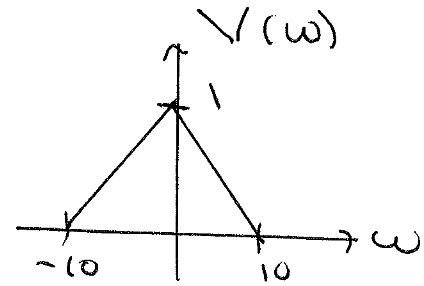


Show your work and write your answers to Problem 3, parts (d) below.

Over the frequency band that $X(\omega)$ occupies, $H(\omega) = -j\omega$ $\Rightarrow$ Comparing with Table of FT properties, we have $Y_1(\omega) = -j\omega X_1(\omega)$

and
$$y_1(t) = -\frac{d}{dt} x(t)$$
$$= -\frac{d}{dt}\left\{\frac{\sin(10t)}{\pi t}\right\}$$
$$= \frac{-10\cos(10t)\pi t + \pi \sin(10t)}{(\pi t)^2}$$

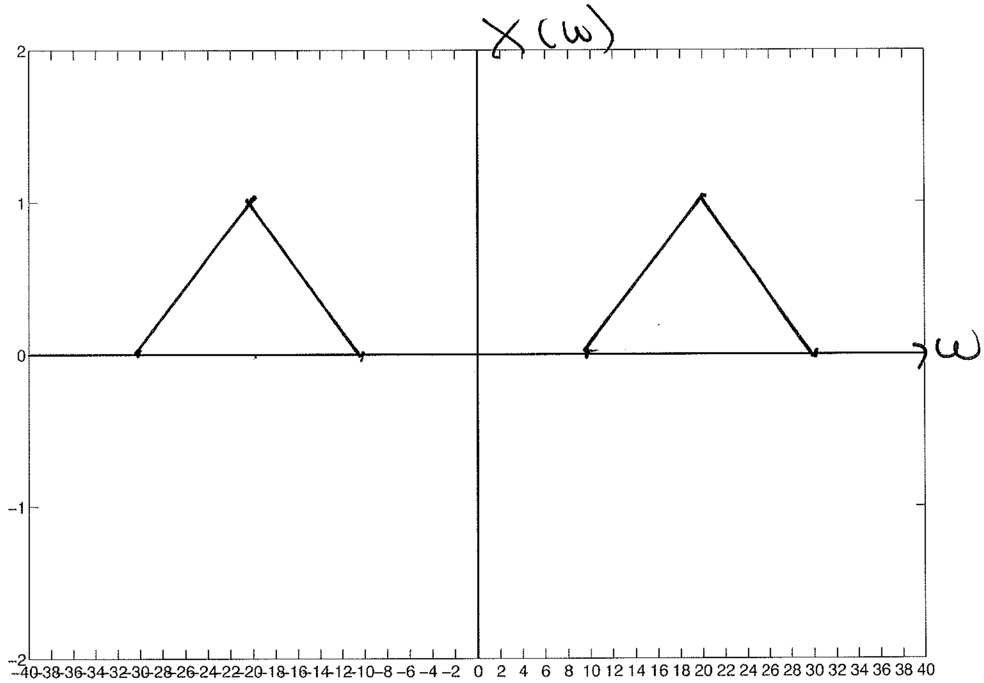
$$\frac{\pi}{5} \quad \frac{\sin(5t)}{\pi t} \quad \frac{\sin(5t)}{\pi t} \xleftrightarrow{\mathcal{F}}$$



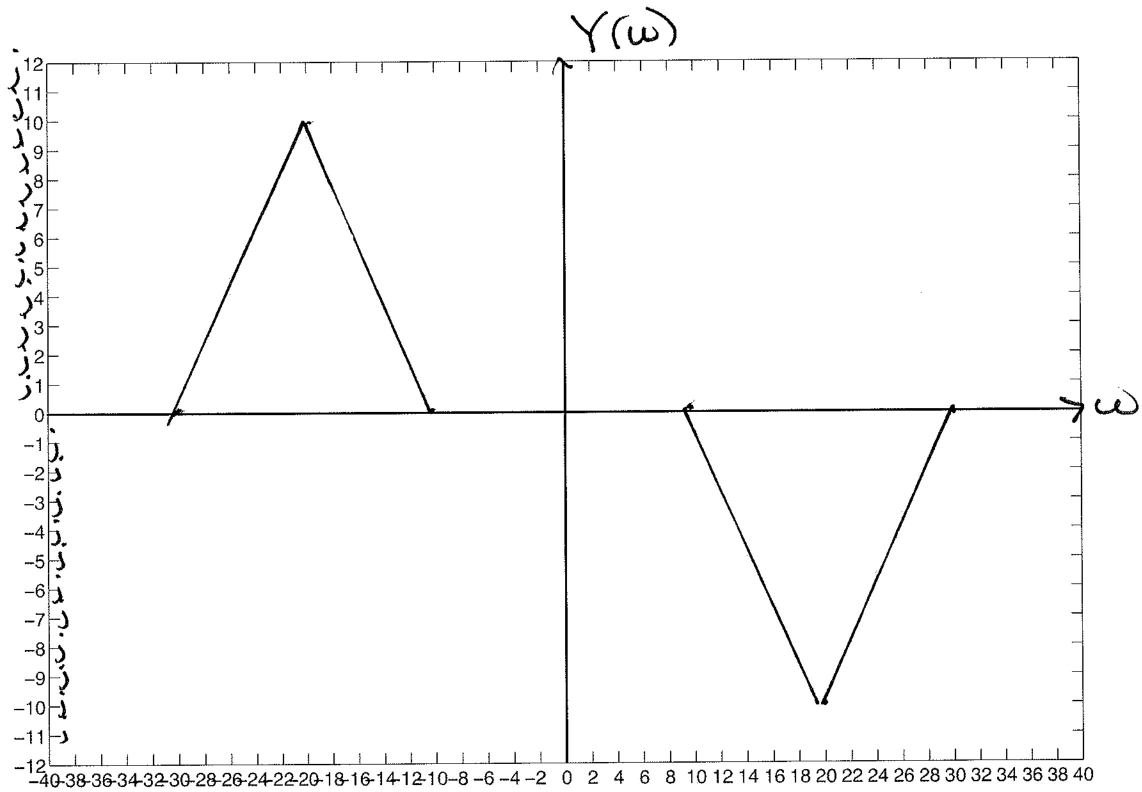
$$X(\omega) = V(\omega + 20) + V(\omega - 20)$$

This is used for $V(\omega)$ in parts (f) + (g)

Plot your answer to Problem 3 (e) here. Show work above.



Plot your answer to Problem 3 (f) here. Show work above.



$$Z(\omega) = 10X(\omega) + jY(\omega)$$

For $\omega < 0$: $Z(\omega) = 10V(\omega+20) + j\left(10jV(\omega+20)\right)$

$$= 10(1-1)V(\omega+20) = 0$$

For $\omega > 0$: $Z(\omega) = 10V(\omega-20) + j\left(-10jV(\omega-20)\right)$

$$= 10(1+1)V(\omega-20)$$

$$= 20V(\omega-20)$$

$\Rightarrow$ Negative frequency content is zeroed out, i.e., gone

Plot your answer to Problem 3 (g) here. Show work above.