ECE301 Signals and Systems
Spring 2005
Exam II
Solutions

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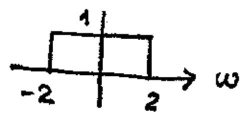
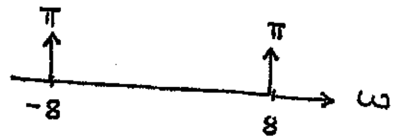
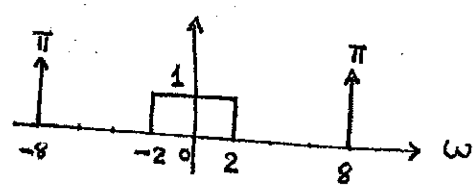
(a) $\dfrac{\sin(2t)}{\pi t} \stackrel{\mathcal{F}}{\longleftrightarrow}$ 

$\cos(8t) \stackrel{\mathcal{F}}{\longleftrightarrow}$ 

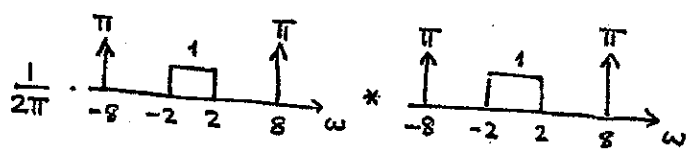
By linearity,

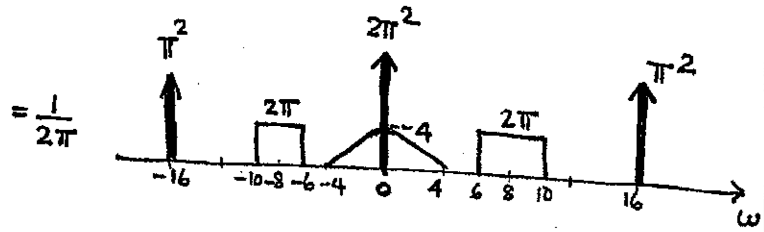
$\dfrac{\sin(2t)}{\pi t} + \cos(8t) \stackrel{\mathcal{F}}{\longleftrightarrow}$ 

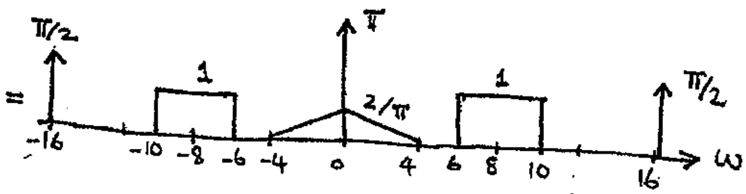
Then by multiplication property,

$$\left\{\frac{\sin(2t)}{\pi t} + \cos(8t)\right\}\left\{\frac{\sin(2t)}{\pi t} + \cos(8t)\right\}$$

$$= \left\{\frac{\sin(2t)}{\pi t} + \cos(8t)\right\}^2 \stackrel{\mathcal{F}}{\longleftrightarrow}$$





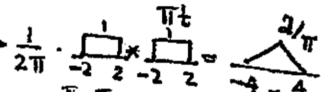


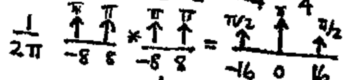
plot of $|X(\omega)|$

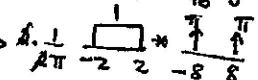
where $x(t) = \left\{\dfrac{\sin(2t)}{\pi t} + \cos(8t)\right\}^2$

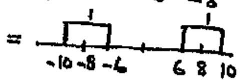
Alternatively,

$$\left[\frac{\sin(2t)}{\pi t} + \cos(8t)\right]^2 = \left[\frac{\sin(2t)}{\pi t}\right]^2 + \left[\cos(8t)\right]^2 + 2\,\frac{\sin(2t)}{\pi t}\cdot\cos(8t)$$

$\left[\dfrac{\sin(2t)}{\pi t}\right]^2 \stackrel{FT}{\longleftrightarrow}$ 

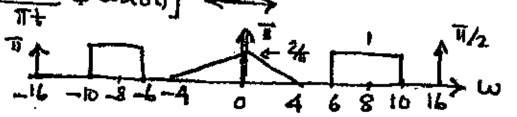
$\left[\cos(8t)\right]^2 \stackrel{FT}{\longleftrightarrow}$ 

$2\dfrac{\sin(2t)}{\pi t}\cdot\cos(8t) \stackrel{FT}{\longleftrightarrow}$ 



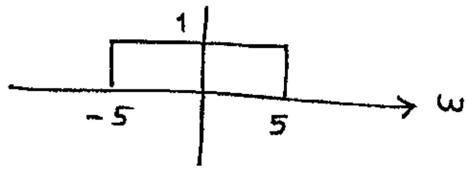
Then by linearity,

$\left[\dfrac{\sin(2t)}{\pi t} + \cos(8t)\right]^2 \stackrel{F.T}{\longleftrightarrow}$

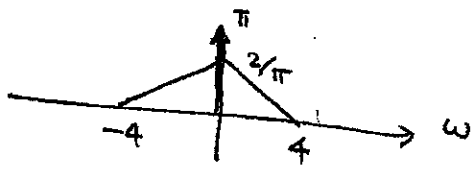


(b) $h_1(t) = \dfrac{\sin(5t)}{\pi t}$

(i) $|H_1(\omega)|$

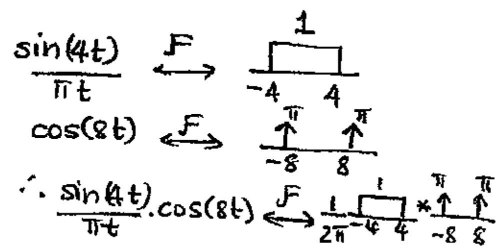


(ii) $|Y_1(\omega)|$

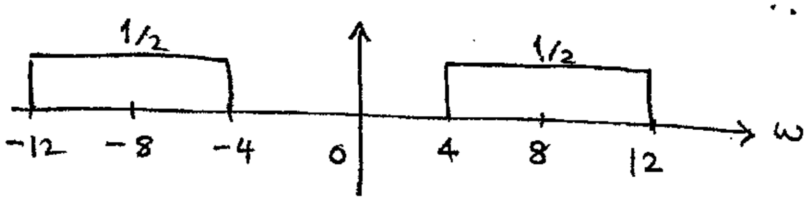


(iii) $y_1(t) = \left[\dfrac{\sin(2t)}{\pi t}\right]^2 + \dfrac{1}{2}$ ←

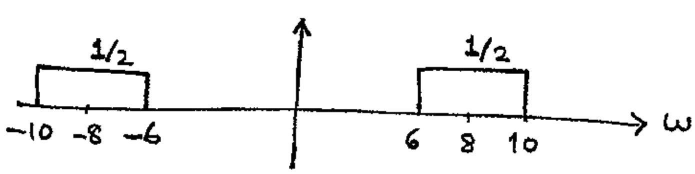
(c) $h_2(t) = \dfrac{\sin(4t)}{\pi t}\cos(8t)$



(i) $|H_2(\omega)|$



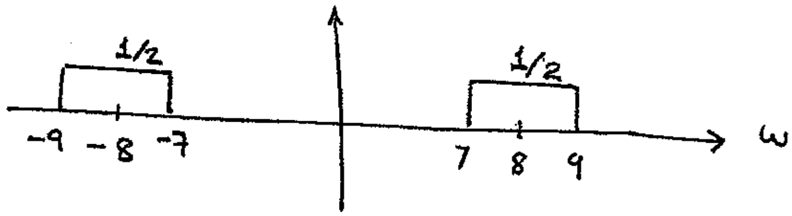
(ii) $|Y_2(\omega)|$



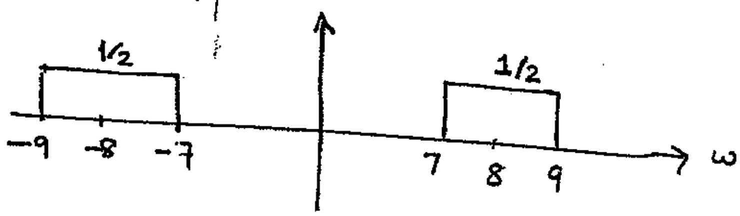
(iii) $y_2(t) = \dfrac{\sin(2t)}{\pi t}\cos(8t)$ ←

(d) $h_3(t) = \dfrac{\sin(t)}{\pi t} \cos(8t)$

(i) $|H_3(\omega)|$

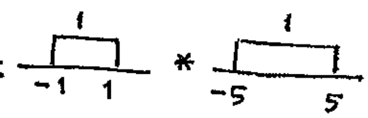


(ii) $|Y_3(\omega)|$

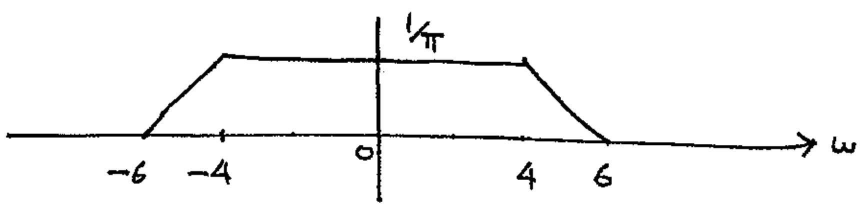


(iii) $y_3(t) = \dfrac{\sin(t)}{\pi t} \cos(8t) \longleftarrow$

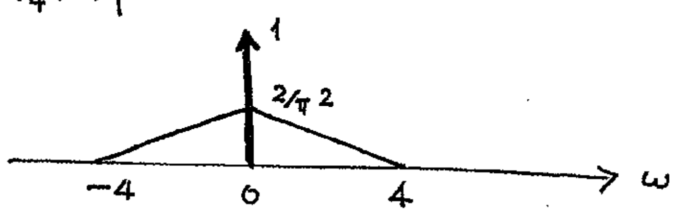
(e) $h_4(t) = \dfrac{\sin(t)}{\pi t} \dfrac{\sin(5t)}{\pi t} \overset{F}{\longleftrightarrow} \dfrac{1}{2\pi}$



(i) $|H_4(\omega)|$



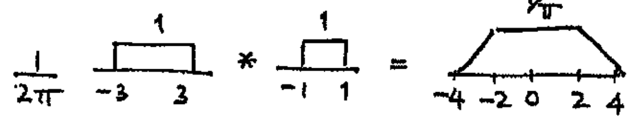
(ii) $|Y_4(\omega)|$



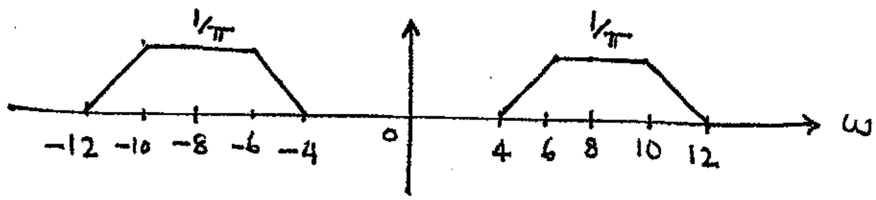
(iii) $y_4(t) = \dfrac{1}{\pi}\left[\dfrac{\sin(2t)}{\pi t}\right]^2 + \dfrac{1}{2\pi} \longleftarrow$

(f) $h_5(t) = \left\{ \frac{\sin(3t)}{\pi t} \frac{\sin(t)}{\pi t} \right\} \cos(8t)$

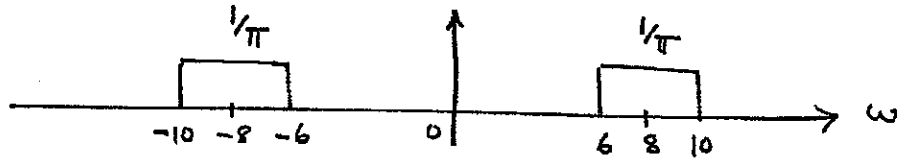
$\frac{\sin(3t)}{\pi t} \frac{\sin(t)}{\pi t} \overset{\mathcal{F}}{\longleftrightarrow}$



(i) $\therefore |H_5(\omega)|$



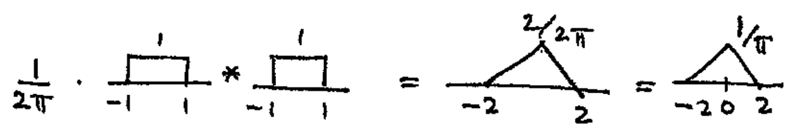
(ii) $|Y_5(\omega)|$



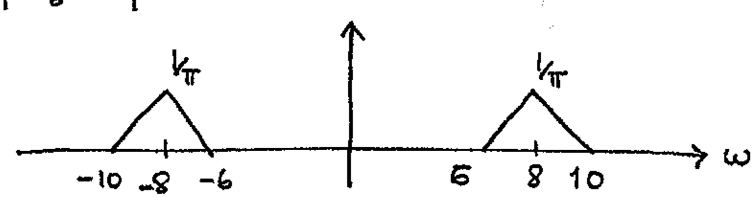
(iii) $y_5(t) = \frac{2}{\pi}\left(\frac{\sin(2t)}{\pi t}\right)(\cos(8t)) \longleftarrow$

(g) $h_6(t) = \left\{\frac{\sin(t)}{\pi t}\right\}^2 \cos(8t)$

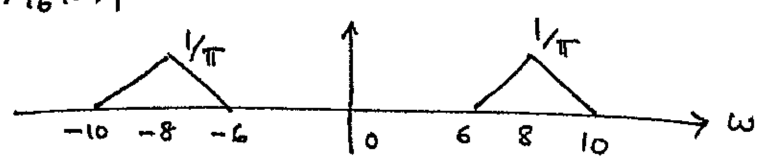
$\left\{\frac{\sin(t)}{\pi t}\right\}^2 \overset{\mathcal{F}}{\longleftrightarrow}$



(i) $\therefore |H_6(\omega)|$



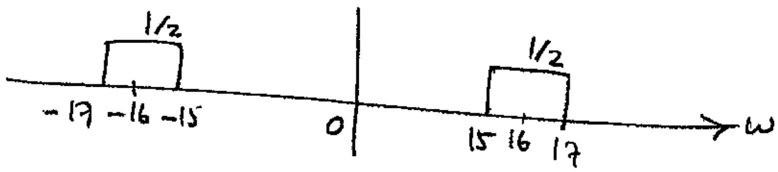
(ii) $|Y_6(\omega)|$



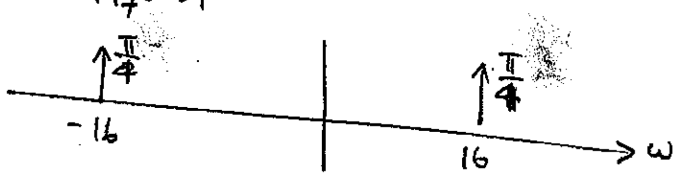
(iii) $y_6(t) = \left\{\frac{\sin(t)}{\pi t}\right\}^2 \cos(8t) \longleftarrow$

(h) $h_7(t) = \dfrac{\sin(t)}{\pi t} \cos(16t)$

(i) $|H_7(\omega)|$



(ii) $|Y_7(\omega)|$

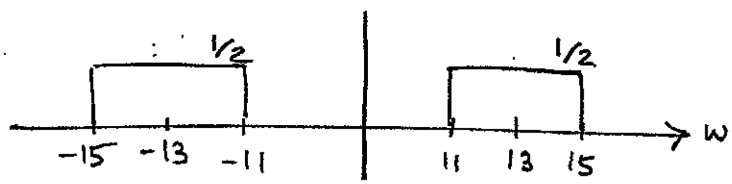


(iii) $y_7(t) = \dfrac{1}{8}\left(e^{-j16t} + e^{j16t}\right)$

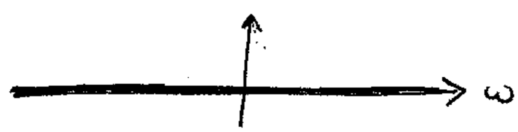
$= \dfrac{1}{4}\cos(16t)$ ←

(i) $h_8(t) = \dfrac{\sin(2t)}{\pi t} \cos(13t)$

(i) $|H_8(\omega)|$



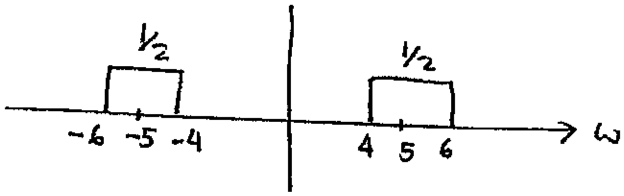
(ii) $|Y_8(\omega)| = 0$



(iii) $y_8(t) = 0$ ←

(j) $h_9(t) = \dfrac{\sin(t)}{\pi t}\cos(5t)$

(i) $|H_9(\omega)|$

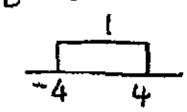


(ii) $|Y_9(\omega)| = 0$

(iii) $y_9(t) = 0 \leftarrow$

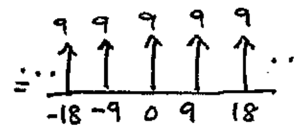
(k) $h_{10}(t) = \left\{\dfrac{\sin(4t)}{\pi t}\right\}\left\{\displaystyle\sum_{n=-\infty}^{\infty}\delta\left(t - n\frac{2\pi}{9}\right)\right\}$

$\dfrac{\sin(4t)}{\pi t} \xleftrightarrow{\mathcal{F}}$

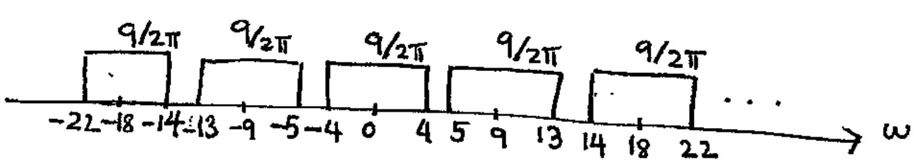


$$\sum_{n=-\infty}^{\infty}\delta\left(t - n\frac{2\pi}{9}\right) \xleftrightarrow{\mathcal{F}} \frac{\cancel{2\pi}}{\cancel{2\pi}/9}\sum_{k=-\infty}^{\infty}\delta\left(\omega - \frac{\cancel{2\pi}}{\cancel{2\pi}/9}k\right)$$

$$= 9\sum_{k=-\infty}^{\infty}\delta(\omega - 9k) :$$

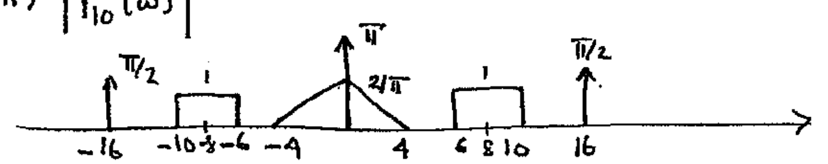


(i) $|H_{10}(\omega)|$



(ii) $|Y_{10}(\omega)|$

$\dfrac{9}{2\pi}\cdot$



$= \dfrac{9}{2\pi}|X(\omega)|$

(iii) $y_{10}(t) = \dfrac{9}{2\pi}x(t) = \dfrac{9}{2\pi}\left(\dfrac{\sin(2t)}{\pi t} + \cos(8t)\right)^2 \leftarrow$