EE301 Signals and Systems — Spring 2015
Exam 2 — Tuesday, Mar. 31, 2015

Cover Sheet

Test Duration: 75 minutes.
Coverage: Chaps. 3,4 with emphasis on Chap. 4
Open Book but Closed Notes.
One 8.5 in. x 11 in. crib sheet
Calculators NOT allowed.
All work should be done on the sheets provided.
You can NOT do work on the back of a page unless permission is granted.
No work on the back of a page will be graded unless permission is granted.
You must show all work for each problem to receive full credit.

Problem 1. The sum of (infinite-duration) sinewaves below

$$x(t) = 1 + \pi \cos(\pi t) + \pi \cos(2\pi t)$$

is input to an LTI system with impulse response given by

$$h(t) = \cos(\pi t)\text{rect}(t)$$

Determine and write a closed-form expression for the output $y(t) = x(t) * h(t)$. Show all work, state which Fourier Transform pairs and/or properties you are using, and clearly indicate your final answer.

Additional Space for Problem 1 answer and work.

Problem 2 (a). The Gaussian pulse $x(t) = \frac{1}{6\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma_1^2}}$, with $\sigma_1^2 = 6^2$, is multiplied by a complex-valued sinewave to form $y(t) = e^{j3t}x(t)$. Find the numerical value of $E = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} |Y(\omega)|^2 d\omega$. Show all work, state which Fourier Transform pairs and/or properties you are using, and clearly indicate your final answer.

Problem 2 (b). The Gaussian pulse $x(t) = \frac{1}{6\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma_1^2}}$, with $\sigma_1^2 = 6^2$, is time-shifted to form $y(t) = x(t-3)$. Find the numerical value of $E = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} |Y(\omega)|^2 d\omega$. Show all work, state which Fourier Transform pairs and/or properties you are using, and clearly indicate your final answer.

Problem 2 (c). The Gaussian pulse $x(t) = \frac{1}{6\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma_1^2}}$, with $\sigma_1^2 = 6^2$, is differentiated to form $y(t) = \frac{d}{dt}x(t)$. Find the numerical value of $A = \int_{-\infty}^{\infty} y(t)dt$. Show all work, state which Fourier Transform pairs and/or properties you are using, and clearly indicate your final answer.

Problem 2 (d). The Gaussian pulse $x(t) = \frac{1}{6\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma_1^2}}$, with $\sigma_1^2 = 6^2$, is multiplied by t to form $y(t) = tx(t)$. Find the numerical value of $A = \int_{-\infty}^{\infty} y(t)dt$. Show all work, state which Fourier Transform pairs and/or properties you are using, and clearly indicate your final answer.

Problem 2 (e). The Gaussian pulse $x(t) = \frac{1}{6\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma_1^2}}$, with $\sigma_1^2 = 6^2$, is compressed in time by a factor of 2 to form $y(t) = x(2t)$. Find the numerical value of the energy $E = \int_{-\infty}^{\infty} y^2(t)dt$. Show all work, state which Fourier Transform pairs and/or properties you are using, and clearly indicate your final answer.

Problem 2 (f). The Gaussian pulse $x(t) = \frac{1}{6\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma_1^2}}$, with $\sigma_1^2 = 6^2$, is input to an LTI system with a Gaussian shaped impulse response, $h(t) = \frac{1}{8\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma_2^2}}$, with $\sigma_2^2 = 8^2$. Determine a simple, closed-form expression for the output $y(t) = x(t) * h(t)$. Show all work, state which Fourier Transform pairs and/or properties you are using, and clearly indicate your final answer.

Problem 2 (g). The Gaussian pulse $x(t) = \frac{1}{6\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma_1^2}}$, with $\sigma_1^2 = 6^2$, is input to an LTI system with a Gaussian shaped impulse response, $h(t) = \frac{1}{8\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma_2^2}}$, with $\sigma_2^2 = 8^2$. Determine the numerical value of the area, $A = \int_{-\infty}^{\infty} y(t)dt$, under the output $y(t) = x(t) * h(t)$. Show all work, state which Fourier Transform pairs and/or properties you are using, and clearly indicate your final answer.

Problem 2 (h). The Gaussian pulse $x(t) = \frac{1}{6\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma_1^2}}$, with $\sigma_1^2 = 6^2$, is input to an LTI system with a Gaussian shaped impulse response, $h(t) = \frac{1}{8\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma_2^2}}$, with $\sigma_2^2 = 8^2$. Determine the numerical value of the energy $E = \int_{-\infty}^{\infty} y^2(t)dt$ of the output $y(t) = x(t) * h(t)$. Show all work, state which Fourier Transform pairs and/or properties you are using, and clearly indicate your final answer.

Problem 2 (i). The Gaussian pulse $x(t) = \frac{1}{6\sqrt{2\pi}} e^{-\frac{(t-2)^2}{2\sigma_1^2}}$, with $\sigma_1^2 = 6^2$, is input to an LTI system with a Gaussian shaped impulse response, $h(t) = \frac{1}{8\sqrt{2\pi}} e^{-\frac{(t-3)^2}{2\sigma_2^2}}$, with $\sigma_2^2 = 8^2$. Determine the numerical value of the energy $E = \int_{-\infty}^{\infty} y^2(t)dt$ of the output $y(t) = x(t) * h(t)$. Show all work, state which Fourier Transform pairs and/or properties you are using, and clearly indicate your final answer.

Problem 2 (j).

The signal $z(t) = x(t)y(t)$ is the PRODUCT of the Gaussian pulse $x(t) = \frac{1}{6\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma_1^2}}$, with $\sigma_1^2 = 6^2$, and the Gaussian pulse $y(t) = \frac{1}{8\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma_2^2}}$, with $\sigma_2^2 = 8^2$. Determine a simple expression for the Fourier Transform, $Z(\omega)$, of $z(t) = x(t)y(t)$. Show all work, state which Fourier Transform pairs and/or properties you are using, and clearly indicate your final answer.

Problem 3. The signal

$$x(t) = \frac{1}{t^2+1}\cos(20t) - \left\{\frac{1}{t^2+1} * \frac{1}{\pi t}\right\}\sin(20t)$$

is input to an LTI system with impulse response

$$h(t) = \left\{\frac{\pi}{5}\,\frac{\sin(5t)}{\pi t}\,\frac{\sin(15t)}{\pi t}\right\}$$

Determine the output $y(t) = x(t) * h(t)$. Show all work, state which Fourier Transform pairs and/or properties you are using, and clearly indicate your final answer.

Additional Space for Problem 3 answer and work.