**EE301 Signals and Systems** **Spring 2013**
**Exam 2** **Wednesday, Mar. 27, 2013**

# Cover Sheet

Test Duration: 60 minutes.
Coverage: Chaps. 3,4 with emphasis on Chap. 4
Open Book but Closed Notes.
One 8.5 in. x 11 in. crib sheet
Calculators NOT allowed.
All work should be done on the sheets provided.
You must show all work for each problem to receive full credit.
Plot your answers on the graphs provided.

**Yes or No Questions with Explanation.** Circle Yes or No for each question below, and briefly **explain** your answer in the space provided. You must cite at least one Fourier Transform Property in your answer for each part.

Yes No Consider a signal $x(t)$ that is real-valued and non-negative for all time, i.e., $x(t) > 0$ for $-\infty < t < \infty$. Under these conditions, is it possible that the Fourier Transform $X(\omega)$ at $\omega = 0$ can be 0? That is, under these conditions, can $X(0) = 0$?

Yes No Consider the signal $x(t) = \sqrt{2}\,\mathrm{rect}(2t) + \frac{sin(\pi t)}{\pi t} + \sqrt{\pi}\, e^{-t^2}$. Does the Fourier Transform of this signal, $X(\omega)$, have an imaginary part?

Yes No Consider a signal $y(t) = t\, e^{-t^2}$ with Fourier Transform $Y(\omega)$. Is the area under $Y(\omega)$ integrated over all frequencies equal to zero?

Yes No Consider $x(t)$ to be real-valued and having odd symmetry, $x(-t) = -x(t)$. Will the value of the Fourier Transform $X(\omega)$ at $\omega = 0$ always be zero, i.e., $X(0) = 0$ for a signal with odd symmetry?

Yes No Is the energy distribution as a function of frequency for the signal at the output of an LTI filter the same as the energy distribution as a function of frequency for the corresponding input signal?

Yes No Supposed that $X(\omega) = 0$ for $|\omega| > 10$ rads/sec. Will the Fourier Transform of $y(t) = a\,x(t - t_1) + b\,x(t - t_2) + c\,x(t - t_3)$ also be equal to 0 for $|\omega| > 10$ rads/sec, regardless of the values of the time-delays $t_1$, $t_2$, and $t_3$ and the amplitude scalings $a, b$, and $c$?

Yes No Let $y(t)$ denote the output when $x(t)$ is the signal input to an LTI system with impulse response $h(t)$. Supposed that $X(\omega) = 0$ for $|\omega| > 10$ rads/sec. Is it possible to design an LTI system such that $Y(\omega) = 0$ for $|\omega| > 5$ rads/sec? That is, is it possible to lower the bandwidth (the max frequency) of signal through LTI filtering?

Yes No Consider the signal $y(t) = \frac{d}{dt}x(t)$ Is the Fourier Transform $Y(\omega)$ guaranteed to be 0 at at $\omega = 0$, i.e., $Y(0) = 0$, for any signal $x(t)$?

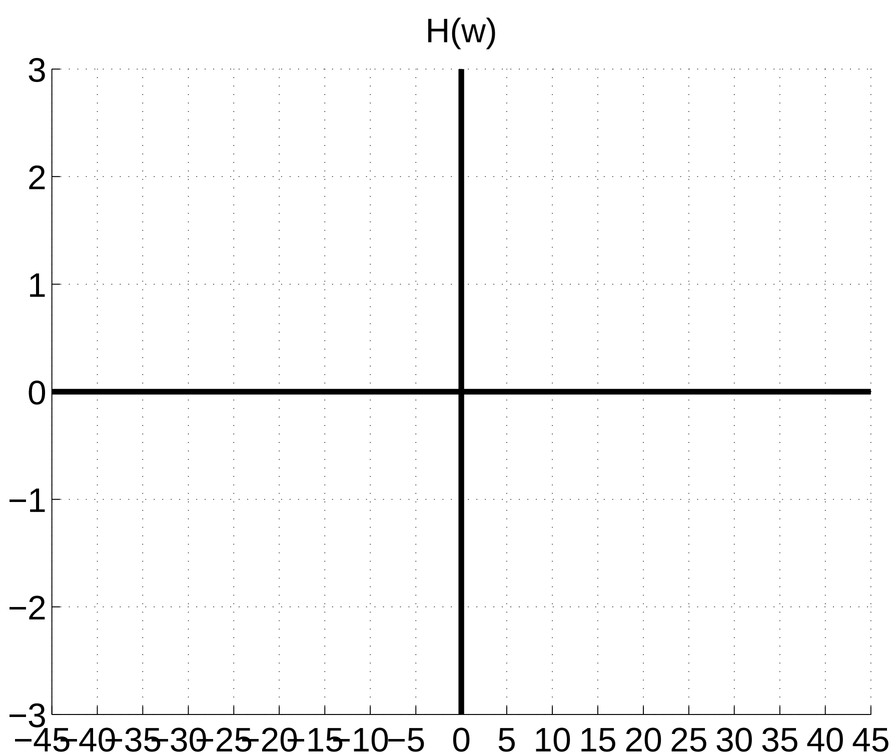
Yes No Is ALL of the energy of a FINITE-duration sinewave totally concentrated at the frequency of the sinewave?

Yes No Let $y(t)$ denote the output when a periodic signal $x(t)$ with period $T$ is the signal input to an LTI system with impulse response $h(t)$. Will the output $y(t)$ be periodic with the same period $T$ for any impulse response $h(t)$?

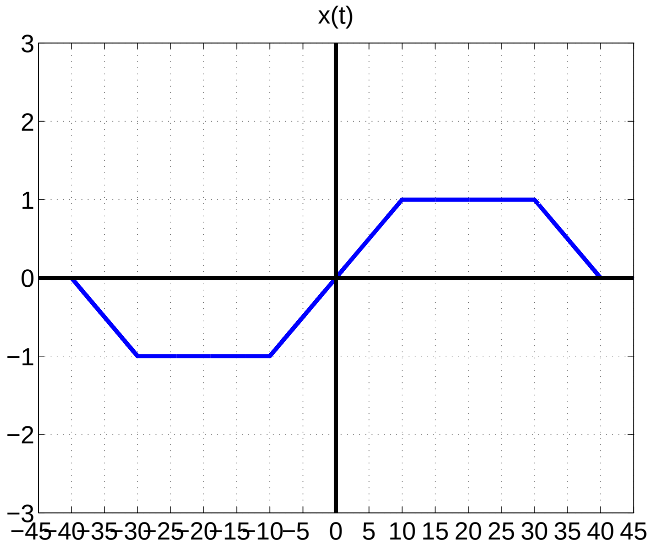
**Problem 2 (a).** Short Workout Problems Using Fourier Transform Properties. Consider an LTI system with the impulse response below. This filter is used in part (c) of Problem 2.

$$h(t) = \left\{ \frac{\pi}{5} \frac{\sin(5t)}{\pi t} \frac{\sin(15t)}{\pi t} \right\} 2j \sin(20t) \tag{1}$$

Determine the frequency response, $H(\omega)$, which is the Fourier Transform of $h(t)$. To this end, first determine the Fourier Transform of what's inside the brackets, and then apply the modulation property of the Fourier Transform. Show work & plot $H(\omega)$ on graph below.



**Problem 2 (b).** Determine the Fourier Transform of the signal $x(t)$ below.




Show all work and write your expression for $X(\omega)$ in the space below. *HINT:* Duality.

**Problem 2 (c).** Consider an LTI system with impulse response

$$h(t) = \left\{ \frac{\pi}{5} \frac{\sin(5t)}{\pi t} \frac{\sin(15t)}{\pi t} \right\} 2j \sin(20t) \tag{2}$$

You already determined the frequency response, $H(\omega)$, which is the Fourier Transform of $h(t)$, in part (a) of this problem. For the LTI system with impulse response above, determine the output $y(t)$ for the input $x(t)$ given below, which is the Fourier Series expansion for a periodic sawtooth waveform with fundamental frequency $\omega_0 = 5$ rads/sec.

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \, e^{jk5t} \qquad\qquad a_k = \frac{j(-1)^k}{k\pi} \quad \text{for } k \neq 0, \quad a_0 = 0$$

Show work and write your expression for $y(t)$ in the space directly below.

**Workout Problem 3**

(a) Determine and plot the magnitude of the Fourier Transform $X(\omega)$ of the signal $x(t)$ defined below. You must indicate which properties and pairs you are using as your arrive at your answer. Draw your plot on the graph provided on the following pages.

$$\begin{aligned} x(t) &= \frac{d}{dt}\left\{\frac{\sin(10t)}{5\pi t}\right\} && (3) \\ &= \frac{1}{5}\frac{d}{dt}\left\{\frac{\sin(10t)}{\pi t}\right\} && (4) \\ & && (5) \end{aligned}$$

(b) Given $x(t)$ defined above, the signal $y(t)$ is created as shown below. Determine the Fourier Transform, $Y(\omega)$, of $y(t)$ and plot the magnitude $|Y(\omega)|$ as a function of frequency. Draw your plot on the graph provided on the following pages.

$$y(t) = x(t)\cos(30t) - \hat{x}(t)\sin(30t) \quad \text{where:} \quad \hat{x}(t) = x(t) * \frac{1}{\pi t}$$

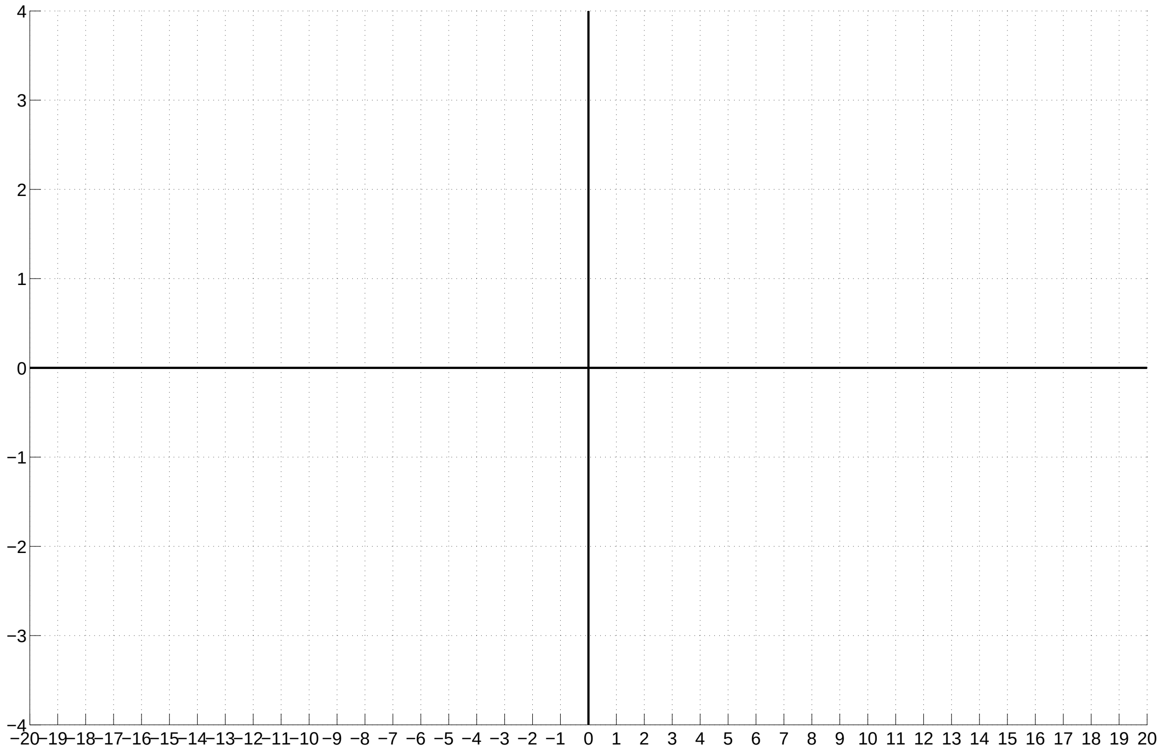
(c) Determine and plot the magnitude of the Fourier Transform $Z(\omega)$ of the signal $z(t)$ defined below. The trig identities $2\cos(\theta)\cos(\phi) = \cos(\theta+\phi)+\cos(\theta-\phi)$ and $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$ should be useful. Draw your plot on the graph provided on the following pages.

$$z(t) = 2y(t)\cos(30t)$$

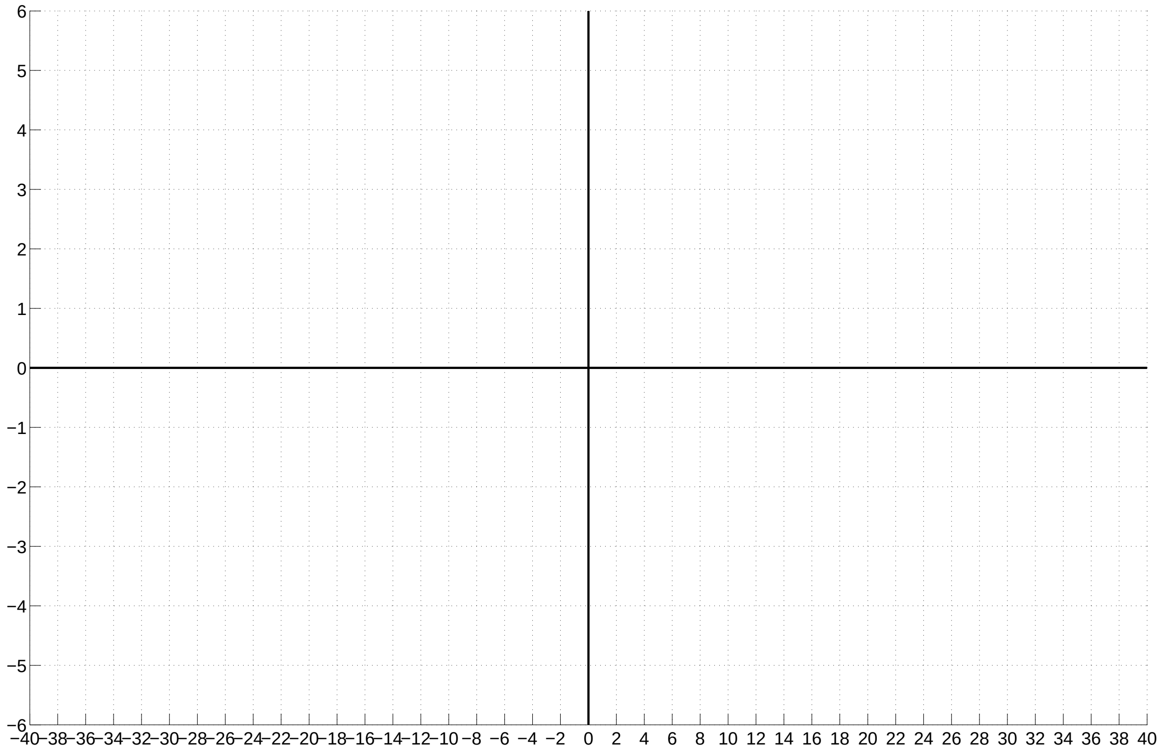
(d) The signal $w(t)$ is the output obtained with $z(t) = 2y(t)\cos(30t)$ from part (c) as the input to the lowpass filter with impulse response defined below. Plot the magnitude of the Fourier Transform $W(\omega)$ of $w(t)$. Draw your plot on the graph provided on the following pages. Does $w(t) = x(t)$?

$$w(t) = z(t) * h(t) \quad \text{where:} \quad h(t) = \frac{\pi}{5}\left\{\frac{\sin(5t)}{\pi t}\,\frac{\sin(15t)}{\pi t}\right\}$$

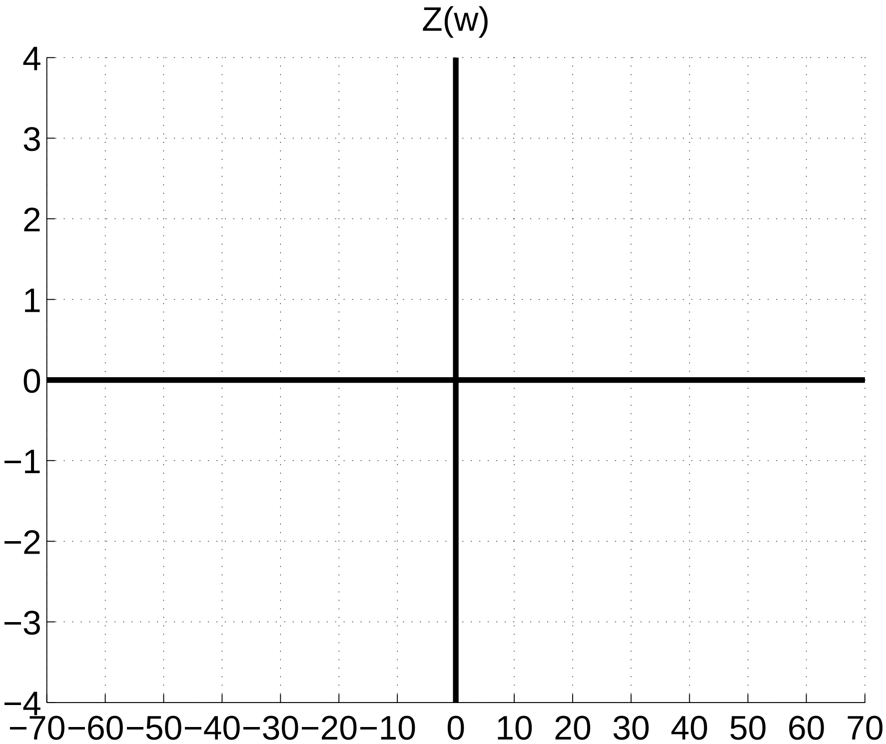
**Plot your answer to Problem 3 (a) here. Show work below.**



**Plot your answer to Problem 3 (b) here. Show work below.**



**Plot your answer to Problem 3 (c) here. Show work below.**



**Plot your answer to Problem 3 (d) here. Show work below.**