Exam 2 Solution Spring 2009

Problem 1 $x_1(t) = \frac{1}{2}\left(x_0(t-t_0) + x_0(t+t_0)\right)$

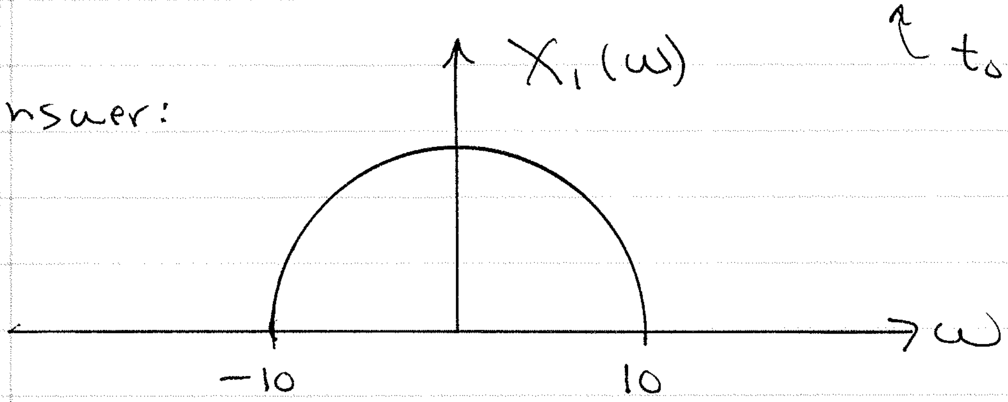
example done in class before exam

$$X_1(\omega) = X_0(\omega)\frac{1}{2}\left\{e^{-j\omega t_0} + e^{+j\omega t_0}\right\}$$

$$X_1(\omega) = X_0(\omega)\cos(\omega t_0)$$

$t_0 = \frac{\pi}{20}$

(a) answer:



$$X_0(\omega) = \text{rect}\left(\frac{\omega}{20}\right)$$

$$= \mathcal{F}\left\{\frac{\sin(10t)}{\pi t}\right\}$$

$$\cos\left(\omega\frac{\pi}{20}\right) = \begin{cases} 1, & \omega = 0 \\ 0, & \omega = 10 \\ 0, & \omega = -10 \end{cases}$$

since $\cos\left(\frac{\pi}{2}\right) = 0$

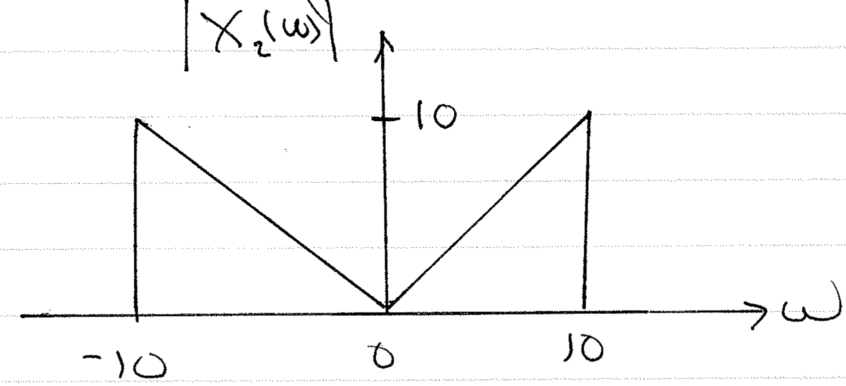
(b) $x_2(t)$

$$= \frac{d}{dt}\left\{\frac{\sin(10t)}{\pi t}\right\} \xleftrightarrow{\mathcal{F}} j\omega\ \text{rect}\left(\frac{\omega}{20}\right)$$

$$= X_2(\omega)$$

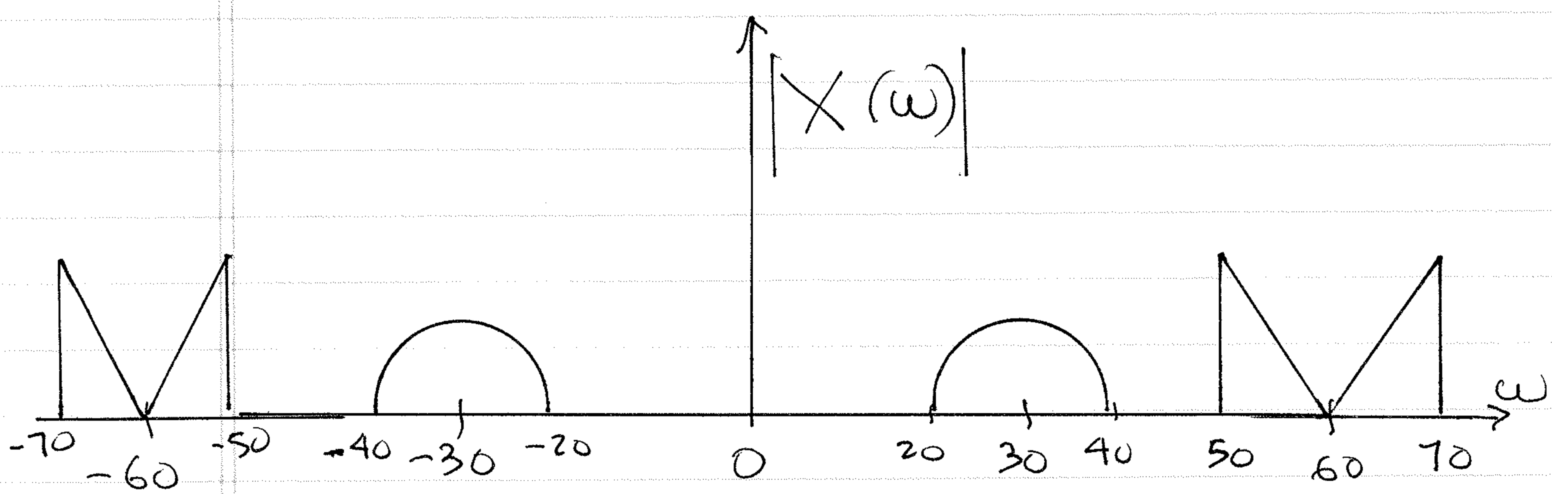
$$|X_2(\omega)| = \begin{cases} |\omega|, & |\omega| < 10 \\ 0, & |\omega| > 10 \end{cases}$$

(b)
answer:



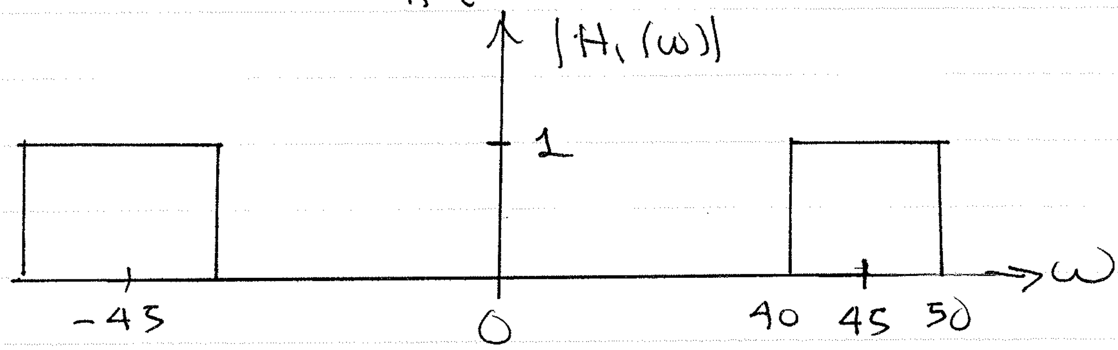
(c) $x(t) = 2x_1(t)\cos(30t) + x_2(t)\cos(60t)2$

$$X(\omega) = X_1(\omega+30) + X_1(\omega-30) + X_2(\omega+60) + X_2(\omega-60)$$



We will refer to this plot for parts (d) thru (g)

(d) $h_1(t) = \frac{\sin(5t)}{\pi t} \, 2\cos(45t)$

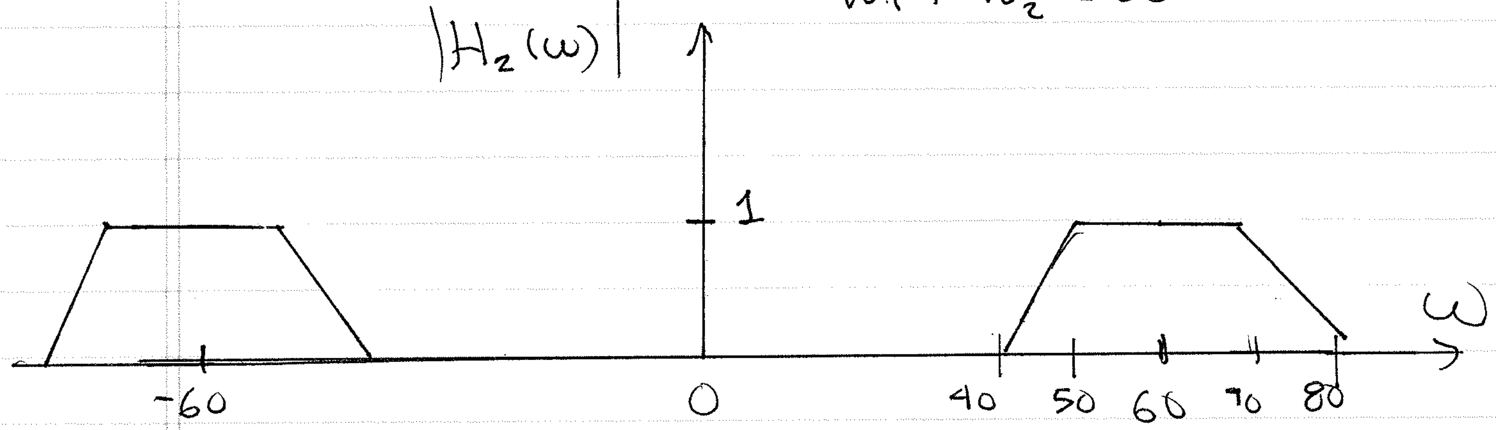


$$Y_1(\omega) = H_1(\omega) X(\omega) = 0$$

$$y_1(t) = 0$$

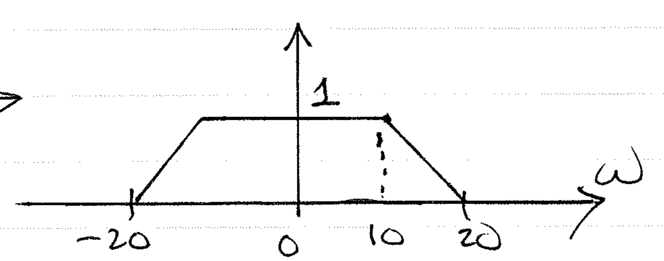
(e) $h_2(t) = \frac{2\pi}{5} \left\{ \frac{\sin(5t)}{\pi t} \frac{\sin(15t)}{\pi t} \right\} \cos(60t)$

$W_1 = 5 \quad W_2 = 15 \Big\} \begin{array}{l} W_2 - W_1 = 10 \\ W_1 + W_2 = 20 \end{array}$



Since:

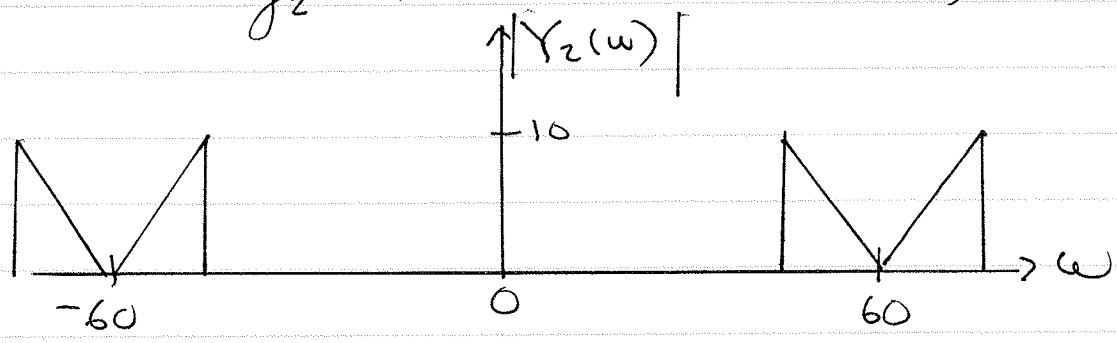
$$\frac{\pi}{5} \frac{\sin(5t)}{\pi t} \frac{\sin(15t)}{\pi t} \xleftrightarrow{\mathcal{F}}$$



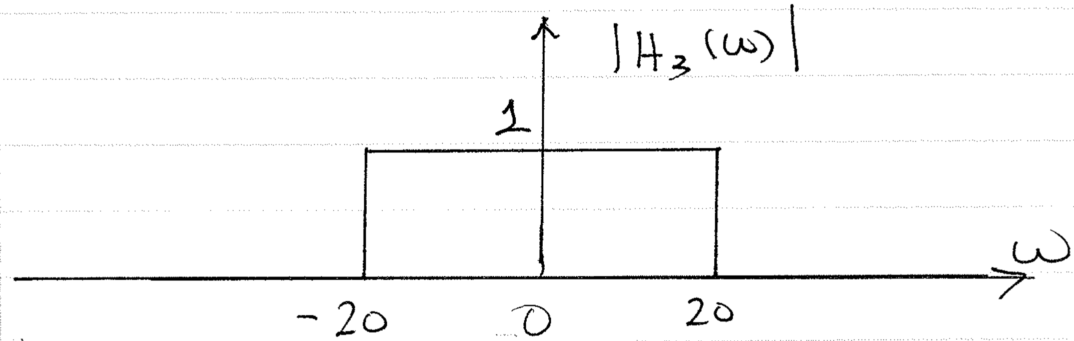
The Flat part of $H_2(\omega)$ passes $2\,x_2(t)\cos(60t)$

The linear roll-off part of $H_2(\omega)$ is where $X(\omega)=0$

THUS: $y_2(t) = 2\,x_2(t)\cos(60t)$



(f) $h_3(t) = \dfrac{\sin(20t)}{\pi t}$



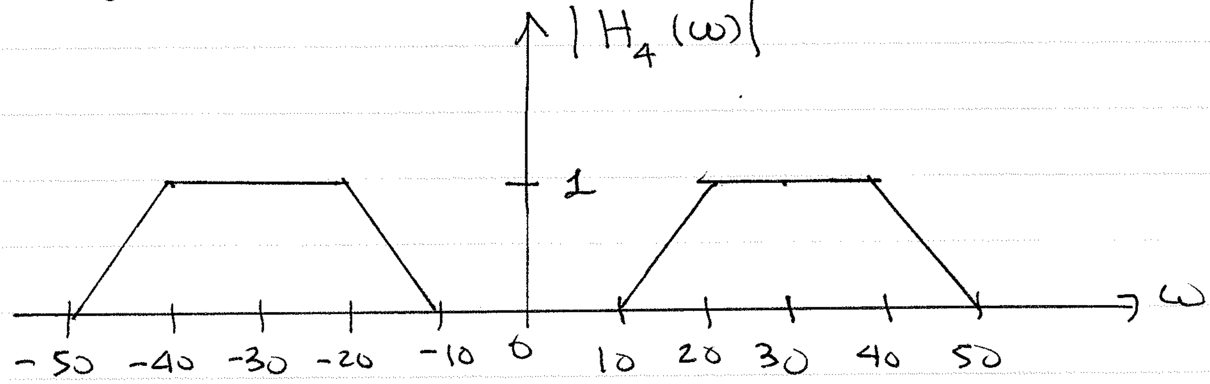
$X(\omega) = 0$ for $|\omega| < 20$

THUS: $Y_3(\omega) = 0$

$y_3(t) = 0$

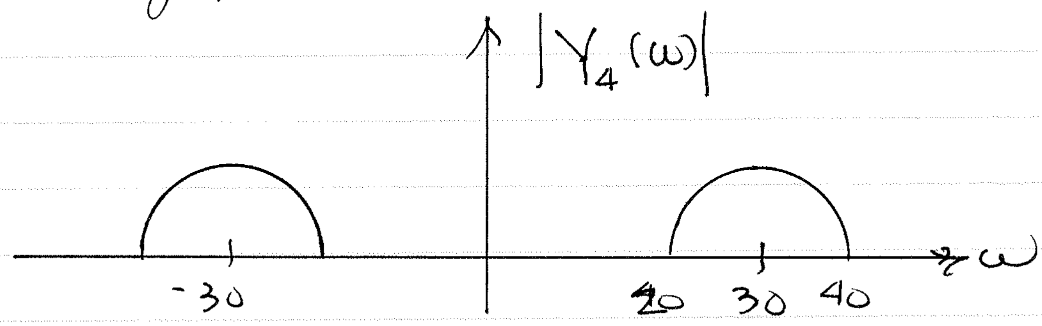
(g) $h_4(t) = \frac{\pi}{5}\left\{\frac{\sin(5t)}{\pi t}\frac{\sin(15t)}{\pi t}\right\} 2\cos(30t)$

Comparing with $h_2(t)$, the only change is being centered at $\omega = \pm 30$ rather than ± 60



Flat part passes $2\,x_1(t)\cos(30t)$

$$y_4(t) = 2\,x_1(t)\cos(30t)$$



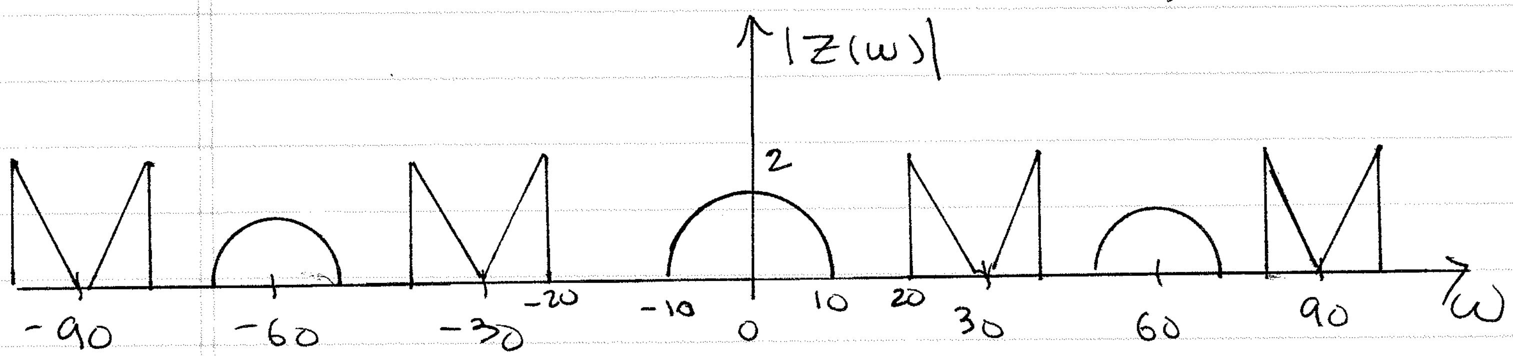
(h)

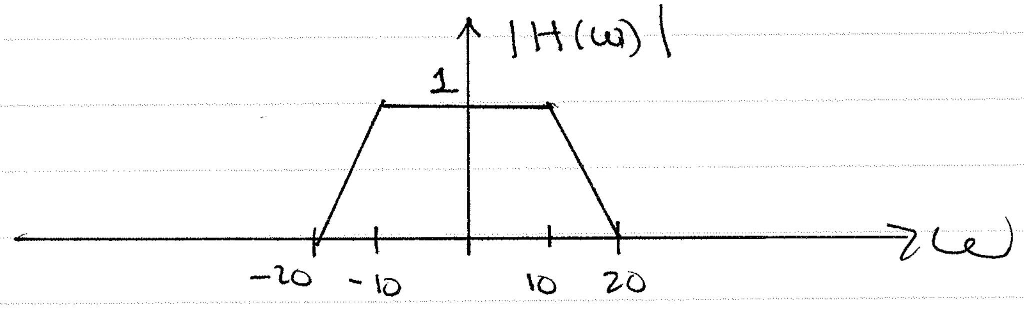
$$z(t) = 2\,x(t)\cos(30t)$$

$$= 2x_1(t)\,2\cos(30t)\cos(30t) + 2x_2(t)\,2\cos(30t)\cos(60t)$$

$$= 2\,x_1(t) + 2\,x_1(t)\cos(60t) + 2\,x_2(t)\cos(30t) + 2\,x_2(t)\cos(90t)$$

$$Z(\omega) = 2X_1(\omega) + X_1(\omega+60) + X_1(\omega-60)$$
$$+ X_2(\omega+30) + X_2(\omega-30)$$
$$+ X_2(\omega+90) + X_2(\omega-90)$$





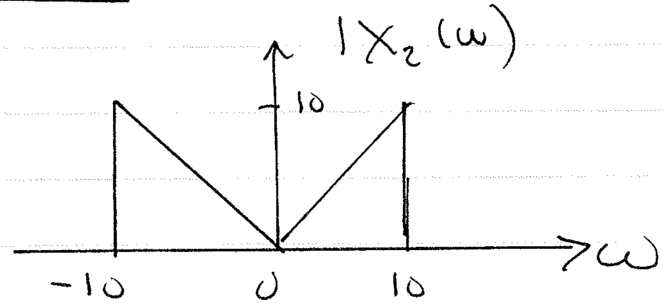
THUS: $w(t) = z(t) * h(t)$

$$\boxed{w(t) = 2x_1(t)}$$

Since $h(t) = \frac{\pi}{5} \frac{\sin(5t)}{\pi t} \frac{\sin(15t)}{\pi t}$

was already used in parts (e) and (g)

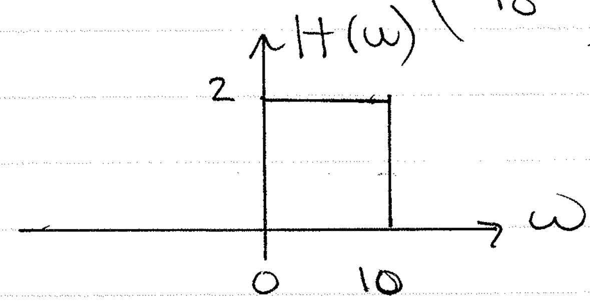
Problem 2 from Prob. 1 (a)



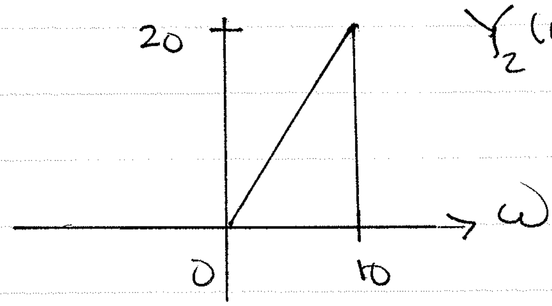
Now: $h(t) = 2\,\dfrac{\sin(5t)}{\pi t}\,e^{j5t}$

$$\overset{\mathcal{F}}{\longleftrightarrow} \; 2\,\text{rect}\left(\frac{\omega-5}{10}\right) = H(\omega)$$

(a)



(b)



$Y_2(\omega) = X_2(\omega)\,H(\omega)$

(c) $E_y = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} |Y_2(\omega)|^2\, d\omega$

$$= \frac{1}{2\pi}\int_0^{10} (2\omega)^2\, d\omega = \frac{4}{2\pi}\,\frac{\omega^3}{3}\Bigg|_0^{10}$$

$$= \frac{2}{\pi}\,(1000-0) = \frac{2000}{\pi}$$