EE301 Signals and Systems **Exam 2**
Exam 2 **Thursday, April 2, 2009**

Cover Sheet

Test Duration: 75 minutes.
Coverage: Chaps. 1,2,3, and 4, Heavy emphasis on Chap. 4
Open Book but Closed Notes. One two-sided handwritten sheet.
Calculators NOT allowed.
This test contains **two** problems, each with multiple parts.
All work should be done in the blue books provided.
You must show all work for each problem to receive full credit.
Do **not** return this test sheet, just return the blue books.

Problem 1.

(a) Determine and plot the magnitude of the Fourier Transform $X_1(\omega)$ of the signal $x_1(t)$ defined below. You must indicate which properties and pairs you are using as your arrive at your answer. You must show all steps to receive full credit.

$$x_1(t) = \frac{1}{2}\left\{\frac{\sin(10(t-\frac{\pi}{20}))}{\pi(t-\frac{\pi}{20})} \; + \; \frac{\sin(10(t+\frac{\pi}{20}))}{\pi(t+\frac{\pi}{20})}\right\}$$

(b) Determine and plot the magnitude of the Fourier Transform $X_2(\omega)$ of the signal $x_2(t)$ defined below. Same directions as for part (a). Show all work.

$$x_2(t) = \frac{d}{dt}\left\{\frac{\sin(10t)}{\pi t}\right\}$$

(c) Given $x_1(t)$ and $x_2(t)$ defined above, the signal $x(t)$ is created as shown below. Determine the Fourier Transform, $X(\omega)$, of $x(t)$ and plot the magnitude $|X(\omega)|$ as a function of frequency, showing as much detail as possible.

$$x(t) = 2x_1(t)\cos(30t) + 2x_2(t)\cos(60t)$$

For EACH of parts (d) thru (g) of this problem, the signal $x(t)$ from part (c) above is input to an LTI system whose impulse response is given. For EACH part, you must do EACH of the following THREE steps. You MUST show all your work.

(i) Plot the magnitude $|H_i(\omega)|$ of the Fourier Transform of the impulse response $h_i(t)$.

(ii) Plot the magnitude $|Y_i(\omega)|$ of the Fourier Transform of the output signal $y_i(t)$.

(iii) Determine a simple, closed-form expression for the time-domain output $y_i(t)$.

(d) $h_1(t) = 2\dfrac{\sin(5t)}{\pi t}\;\cos(45t)$

(e) $h_2(t) = \dfrac{2\pi}{5}\left\{\dfrac{\sin(5t)}{\pi t}\;\dfrac{\sin(15t)}{\pi t}\right\}\cos(60t)$

(f) $h_3(t) = \dfrac{\sin(20t)}{\pi t}$

(g) $h_4(t) = \dfrac{2\pi}{5}\left\{\dfrac{\sin(5t)}{\pi t}\;\dfrac{\sin(15t)}{\pi t}\right\}\cos(30t)$

(h) Determine and plot the magnitude of the Fourier Transform $Z(\omega)$ of the signal $z(t)$ defined below, where $x(t) = 2x_1(t)\cos(30t) + 2x_2(t)\cos(60t)$ as defined in part (c). The trig identity $2\cos(\theta)\cos(\phi) = \cos(\theta+\phi) + \cos(\theta-\phi)$ should be useful.

$$z(t) = 2x(t)\cos(30t)$$

(i) The signal $w(t)$ is the output obtained with $z(t) = 2x(t)\cos(30t)$ from part (h) as the input to the lowpass filter with impulse response defined below. Plot the magnitude of the Fourier Transform $W(\omega)$ of $w(t)$. Is $w(t) = x_1(t)$?

$$h(t) = \frac{\pi}{5}\left\{\frac{\sin(5t)}{\pi t}\ \frac{\sin(15t)}{\pi t}\right\}$$

Problem 2. The output signal $y_2(t)$ is obtained by passing the signal $x_2(t) = \dfrac{d}{dt}\left\{\dfrac{\sin(10t)}{\pi t}\right\}$ (from part (b) of Problem 1) through a filter with impulse response given below.

$$h(t) = 2\frac{\sin(5t)}{\pi t}\ e^{j5t}$$

(a) Plot the frequency response of this system, that is, plot the magnitude of the Fourier Transform $H(\omega)$ of $h(t)$.

(b) Plot the magnitude of the Fourier Transform $Y_2(\omega)$ of the output $y_2(t)$.

(c) Compute the energy of the signal at the output:

$$E_y = \int_{-\infty}^{\infty} |y_2(t)|^2 dt$$