

Cover Sheet

Test Duration: 75 minutes.

Coverage: Chaps. 1,2,3, and 4, Emphasis on Chap. 4

Open Book but Closed Notes. NO LOOSE SHEETS OF ANY KIND

Calculators NOT allowed.

This test contains **two** problems, each with multiple parts.

All work should be done in the blue books provided.

You must show all work for each problem to receive full credit.

Do **not** return this test sheet, just return the blue books.

Problem 1. Note: The answer to this problem is needed to solve Problem 2. This is because the signal $x_0(t)$ defined in equation (1) below is what is used in Equation (2) in Problem 2. Consider the signal $x_0(t)$ below which is equal to the sum of two sinc functions.

$$x_0(t) = \frac{\sin(t)}{\pi t} + \frac{\sin(3t)}{\pi t} \quad (1)$$

(a) Plot the magnitude $|X_0(\omega)|$ of the Fourier Transform of $x_0(t)$ as a function of frequency.

(b) Consider squaring the signal $x_0(t)$ above to create the signal $z(t) = x_0^2(t)$:

$$\begin{aligned} z(t) &= x_0^2(t) \\ &= \left\{ \frac{\sin(t)}{\pi t} + \frac{\sin(3t)}{\pi t} \right\}^2 \\ &= \left\{ \frac{\sin(t)}{\pi t} \right\}^2 + 2 \frac{\sin(t)}{\pi t} \frac{\sin(3t)}{\pi t} + \left\{ \frac{\sin(3t)}{\pi t} \right\}^2 \end{aligned}$$

(i) Plot the magnitude $|X_1(\omega)|$ of the Fourier Transform of

$$x_1(t) = \left\{ \frac{\sin(t)}{\pi t} \right\}^2$$

(ii) Plot the magnitude $|X_2(\omega)|$ of the Fourier Transform of

$$x_2(t) = 2 \frac{\sin(t)}{\pi t} \frac{\sin(3t)}{\pi t}$$

(iii) Plot the magnitude $|X_3(\omega)|$ of the Fourier Transform of

$$x_3(t) = \left\{ \frac{\sin(3t)}{\pi t} \right\}^2$$

(iv) Sum the results above to plot the magnitude $|Z(\omega)|$ of the Fourier Transform of

$$z(t) = x_0^2(t) = \left\{ \frac{\sin(t)}{\pi t} + \frac{\sin(3t)}{\pi t} \right\}^2$$

Problem 2. Consider the input signal $x(t)$ below which is the square of a signal equal to the sum of $x_0(t)$ (defined previously in Equation (1) in Problem (1)) plus a sinewave.

$$x(t) = \{x_0(t) + \cos(10t)\}^2 \quad (2)$$

(a) Plot the magnitude $|X(\omega)|$ of the Fourier Transform of this signal as a function of frequency. Show as much detail as possible.

For EACH of the remaining parts of this problem, the signal above is input to an LTI system whose impulse response is given (a different impulse response for each part.) For EACH part, you must do EACH of the following THREE steps. You MUST show your work, explaining how you got your answer concisely but with sufficient detail to receive full credit.

- (i) Plot the magnitude $|H_i(\omega)|$ of the Fourier Transform of the impulse response $h_i(t)$.
- (ii) Plot the magnitude $|Y_i(\omega)|$ of the Fourier Transform of the output signal $y_i(t)$.
- (iii) Determine a simple, closed-form expression for the time-domain output $y_i(t)$.

$$(b) h_1(t) = 2 \frac{\sin(\frac{t}{2})}{\pi t} \cos(6.5t)$$

$$(c) h_2(t) = 2 \frac{\sin(4t)}{\pi t} \cos(18t)$$

$$(d) h_3(t) = 2 \frac{\sin(2t)}{\pi t} \cos(16t)$$

$$(e) h_4(t) = 4\pi \left\{ \frac{\sin(\frac{t}{2})}{\pi t} \frac{\sin(\frac{7t}{2})}{\pi t} \right\} \cos(10t)$$

$$(f) h_5(t) = 2 \frac{\sin(5t)}{\pi t} \cos(11t)$$

$$(g) h_6(t) = \frac{\sin(6t)}{\pi t}$$

$$(h) h_7(t) = 4\pi \left\{ \frac{\sin(\frac{t}{2})}{\pi t} \right\}^2 \cos(8t)$$

$$(i) h_8(t) = t h_7(t) = 4\pi t \left\{ \frac{\sin(\frac{t}{2})}{\pi t} \right\}^2 \cos(8t)$$

$$(j) h_8(t) = \left\{ \frac{\sin(t-\frac{\pi}{2})}{\pi(t-\frac{\pi}{2})} + \frac{\sin(t+\frac{\pi}{2})}{\pi(t+\frac{\pi}{2})} \right\} \cos(10t)$$