TABLE 4.1 PROPERTIES OF THE FOURIER TRANSFORM

Section	Property	Aperiodic signal	Fourier transform
		$x(t)$ $y(t)$	$X(\omega)$ $Y(\omega)$
4.3.0	Duality	$X(t)$	$2\pi\, x(-\omega)$
4.3.1	Linearity	$ax(t) + by(t)$	$aX(\omega) + bY(\omega)$
4.3.2	Time Shifting	$x(t - t_0)$	$e^{-j\omega t_0}X(\omega)$
4.3.6	Frequency Shifting	$e^{j\omega_0 t}x(t)$	$X(\omega - \omega_0)$
4.3.3	Conjugation	$x^*(t)$	$X^*(-\omega)$
4.3.5	Time Reversal	$x(-t)$	$X(-\omega)$
4.3.5	Time and Frequency Scaling	$x(at)$	$\frac{1}{\|a\|}X\left(\frac{\omega}{a}\right)$
4.4	Convolution	$x(t) * y(t)$	$X(\omega)Y(\omega)$
4.5	Multiplication	$x(t)y(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2\pi} X(\omega) * Y(\omega) =$	$\frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\theta)Y(\omega - \theta)\,d\theta$
4.3.4	Differentiation in Time	$\frac{d}{dt}x(t)$	$j\omega X(\omega)$
4.3.4	Integration	$\int_{-\infty}^{t} x(t)dt$	$\frac{1}{j\omega}X(\omega) + \pi X(0)\delta(\omega)$
4.3.6	Differentiation in Frequency	$tx(t)$	$j\frac{d}{d\omega}X(\omega)$
4.3.3	Conjugate Symmetry for Real Signals	$x(t)$ real	$X(\omega) = X^*(-\omega)$ $\mathcal{Re}\{X(\omega)\} = \mathcal{Re}\{X(-\omega)\}$ $\mathcal{Im}\{X(\omega)\} = -\mathcal{Im}\{X(-\omega)\}$ $\|X(\omega)\| = \|X(-\omega)\|$ $\sphericalangle X(\omega) = -\sphericalangle X(-\omega)$
4.3.3	Symmetry for Real and Even Signals	$x(t)$ real and even	$X(\omega)$ real and even
4.3.3	Symmetry for Real and Odd Signals	$x(t)$ real and odd	$X(\omega)$ purely imaginary and odd
4.3.3	Even-Odd Decomposition for Real Signals	$x_e(t) = \mathcal{Ev}\{x(t)\}$ [$x(t)$ real] $x_o(t) = \mathcal{Od}\{x(t)\}$ [$x(t)$ real]	$\mathcal{Re}\{X(\omega)\}$ $j\mathcal{Im}\{X(\omega)\}$
Initial Value Theorems:		$x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\,d\omega$ $X(0) = \int_{-\infty}^{\infty} x(t)\,dt$	

4.3.7 Parseval's Relation for Aperiodic Signals

$$\int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |X(\omega)|^2 d\omega$$

TABLE 4.2 BASIC FOURIER TRANSFORM PAIRS

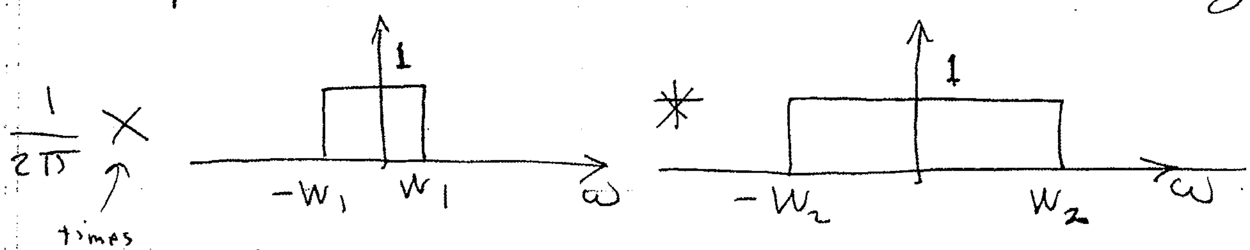
Signal	Fourier transform	Fourier series coefficients (if periodic)
$\sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\omega - k\omega_0)$	a_k
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$	$a_1 = 1$ $a_k = 0$, otherwise
$\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	$a_1 = a_{-1} = \frac{1}{2}$ $a_k = 0$, otherwise
$\sin \omega_0 t$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$	$a_1 = -a_{-1} = \frac{1}{2j}$ $a_k = 0$, otherwise
$x(t) = 1$	$2\pi\,\delta(\omega)$	$a_0 = 1, \quad a_k = 0, \; k \neq 0$ (this is the Fourier series representation for any choice of $T > 0$)
Periodic square wave $x(t) = \begin{cases} 1, & \|t\| < T_1 \\ 0, & T_1 < \|t\| \leq \frac{T}{2} \end{cases}$ and $x(t + T) = x(t)$	$\sum_{k=-\infty}^{+\infty} \frac{2 \sin k\omega_0 T_1}{k} \delta(\omega - k\omega_0)$	$\frac{\omega_0 T_1}{\pi} \operatorname{sinc}\left(\frac{k\omega_0 T_1}{\pi}\right) = \frac{\sin k\omega_0 T_1}{k\pi}$
$\sum_{n=-\infty}^{+\infty} \delta(t - nT)$	$\frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{T}\right)$	$a_k = \frac{1}{T}$ for all k
$x(t) \begin{cases} 1, & \|t\| < T_1 \\ 0, & \|t\| > T_1 \end{cases}$	$\frac{2 \sin \omega T_1}{\omega}$	— $\mathrm{rect}\left(\frac{t}{T}\right) \overset{F}{\longleftrightarrow} \frac{\sin\left(T\frac{\omega}{2}\right)}{\frac{\omega}{2}}$
$\frac{\sin Wt}{\pi t}$	$X(j\omega) = \begin{cases} 1, & \|\omega\| < W \\ 0, & \|\omega\| > W \end{cases}$	—
$\delta(t)$	1	—
$u(t)$	$\frac{1}{j\omega} + \pi\,\delta(\omega)$	—
$\delta(t - t_0)$	$e^{-j\omega t_0}$	—
$e^{-at}u(t)$, $\mathcal{R}e\{a\} > 0$	$\frac{1}{a + j\omega}$	—
$te^{-at}u(t)$, $\mathcal{R}e\{a\} > 0$	$\frac{1}{(a + j\omega)^2}$	—
$\frac{t^{n-1}}{(n-1)!} e^{-at}u(t)$, $\mathcal{R}e\{a\} > 0$	$\frac{1}{(a + j\omega)^n}$	—

Some Fourier Transform Results involving sinc functions:

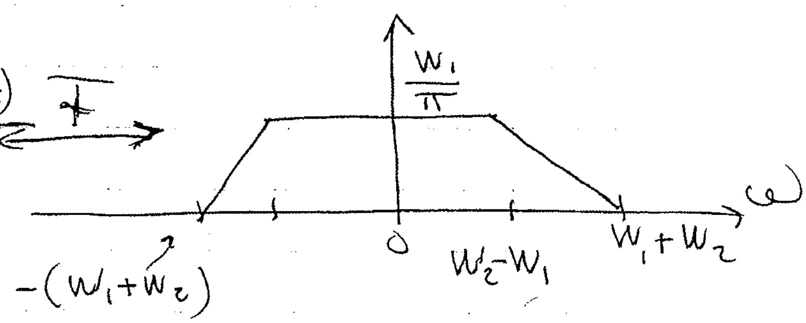
- Two sinc functions <u>multiplied</u> in <u>time</u>:

$$\frac{\sin(W_1 t)}{\pi t}\,\frac{\sin(W_2 t)}{\pi t} \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2\pi}\,\mathcal{F}\left\{\frac{\sin(W_1 t)}{\pi t}\right\} * \mathcal{F}\left\{\frac{\sin(W_2 t)}{\pi t}\right\}$$

multiplication in time $\longrightarrow$ convolution in frequency



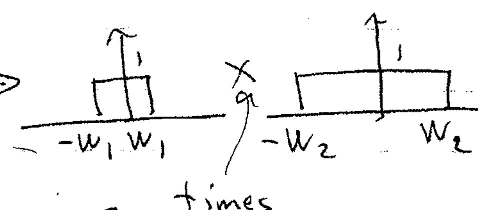
$$\frac{\sin(W_1 t)}{\pi t}\,\frac{\sin(W_2 t)}{\pi t} \overset{\mathcal{F}}{\longleftrightarrow}$$



$(W_1 < W_2)$

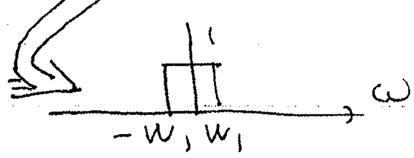
- Two sinc functions <u>convolved</u> in <u>time</u>:

$$\frac{\sin(W_1 t)}{\pi t} * \frac{\sin(W_2 t)}{\pi t} \overset{\mathcal{F}}{\longleftrightarrow}$$



$\boxed{W_1 < W_2}$

convolution in time $\rightarrow$ multiplication in frequency $\Rightarrow$



THUS: $\dfrac{\sin(W_1 t)}{\pi t} * \dfrac{\sin(W_2 t)}{\pi t} = \dfrac{\sin(W_1 t)}{\pi t}$

$W_1 < W_2$

Block Diagrams for Problem 3

$$x_1(t) = \frac{\sin(10t)}{\pi t} \longrightarrow \boxed{h(t) = 20\frac{\pi}{5}\left\{\frac{\sin(5t)}{\pi t}\,\frac{\sin(15t)}{\pi t}\right\}\sin(20t)} \longrightarrow y_1(t)$$

Plot $X_1(\omega)$ for 3(c)

Plot $H(\omega)$ for 3(b)

Write output $y_1(t)$ for 3(d)

$$x(t) = \frac{2\pi}{5}\left\{\frac{\sin(5t)}{\pi t}\right\}^2 \cos(20t) \longrightarrow \boxed{h(t) = 20\frac{\pi}{5}\left\{\frac{\sin(5t)}{\pi t}\,\frac{\sin(15t)}{\pi t}\right\}\sin(20t)} \longrightarrow y(t)$$

Plot $X(\omega)$ for 3(e)

Plot $Y(\omega)$ for 3(f)

For part 3(g): Form $z(t) = 10x(t) + j\,y(t)$ and Plot $Z(\omega)$

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For part 3(g): Form $z(t) = 10x(t) + j\,y(t)$ and Plot $Z(\omega)$