NAME: SOLUTION

EE301 Signals and Systems

21 February 2020

Exam 1

# Cover Sheet

Test Duration: 50 minutes.
Coverage: Chaps. 1,2
One 8.5 in. x 11 in. crib sheet
Calculators NOT allowed.
**DO NOT UNSTAPLE THE EXAM!**
All work should be done in the space provided.
You must show ALL work or explain answer for each problem to receive full credit.

| **Prob. No.** | **Topic(s)** | **Points** |
|---|---|---|
| 1. | Continuous Time Signals and System Properties | 60 |
| 2. | Discrete Time Signals and System Properties | 40 |

**VIP** If you want to refer to the input signal and output signal for one part of a problem when solving a later part, use that part's letter as a subscript, e.g., you can refer to the input signal and corresponding output signal for part (d) of Prob. 1 as $x_d(t)$ and $y_d(t)$, respectively.

**VIP:** Solving and part, you can just write: z(t)= Formula A with a=-3 and b =-2 BUT don't have to write out Formula A substituting a=-3 and b=-2. Just use z(t) for remainder of your solution.

Formula A: $$e^{at}u(t) * e^{bt}u(t) = \frac{1}{a-b}e^{at}u(t) + \frac{1}{b-a}e^{bt}u(t) \quad (1)$$

Formula B: $$\alpha^n u[n] * \beta^n u[n] = \frac{\alpha}{\alpha-\beta}\alpha^n u[n] + \frac{\beta}{\beta-\alpha}\beta^n u[n] \quad (2)$$

Formula C: if $x(t) * h(t) = y(t)$ then: $a\, x(t-t_1) * b\, h(t-t_2) = ab\, y(t-(t_1+t_2))$ (3)

Formula D: if $x[n] * h[n] = y[n]$ then: $a\, x[n-n_1] * b\, h[n-n_2] = ab\, y[n-(n_1+n_2)]$ (4)

Formula E: $$e^{at}u(t) * e^{at}u(t) = te^{at}u(t) \quad (5)$$

Formula F: $$\alpha^n u[n] * \alpha^n u[n] = (n+1)\alpha^n u[n] \quad (6)$$

# NAME:

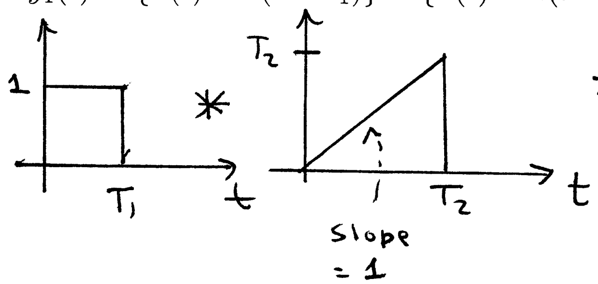
# EE301 Signals and Systems Exam 1

## Cover Sheet

Test Duration: ~~75~~ 60 minutes.
Coverage: Chaps. 1,2
Closed Book but Closed Notes.
One 8.5 in. x 11 in. crib sheet
Calculators NOT allowed.
This test contains **two** problems.
All work should be done on the sheets provided.
You must show work or explain answer for each problem to receive full credit.
Plot your answers on the graphs provided.
**WRITE YOUR NAME ON EVERY SHEET.**

| **Prob. No.** | **Topic(s)** | **Points** |
|---|---|---|
| 1. | Continuous Time Signals and System Properties | |
| 2. | Discrete Time Signals and System Properties | |

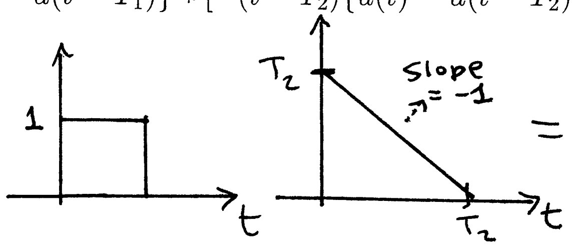
$$y_1(t) = \{u(t) - u(t-T_1)\} * t\{u(t) - u(t-T_2)\} \;=\; \frac{t^2}{2}\{u(t) - u(t-T_1)\} \tag{1}$$




$$+ \left(T_1 t - \frac{T_1^2}{2}\right)\{u(t-T_1) - u(t-T_2)\}$$

$$+ \left(-\frac{t^2}{2} + T_1 t + \frac{T_2^2 - T_1^2}{2}\right)\{u(t-T_2) - u(t-(T_1+T_2))\}$$

$$(t) - u(t-T_1)\} * [-(t-T_2)\{u(t) - u(t-T_2)\}] \;=\; \left(-\frac{t^2}{2} + T_2 t\right)\{u(t) - u(t-T_1)\} \tag{2}$$




$$+ \left(-T_1 t + \frac{2T_1T_2 + T_1^2}{2}\right)\{u(t-T_1) - u(t-T_2)\}$$

$$+ \left(\frac{t^2}{2} - (T_1+T_2)t + \frac{(T_1+T_2)^2}{2}\right)\{u(t-T_2) - u(t-(T_1+T_2))\}$$

$$y_2(t) = \{u(t) - u(t-T_1)\} * [-(t-T_2)\{u(t) - u(t-T_2)\}] = y_1(-(t-(T_1+T_2)) \tag{3}$$

**Prob. 1.** Consider the LTI system characterized by the I/O relationship:

$$\text{System 1:} \quad y(t) = \int_{-\infty}^{t} e^{-3(t-\tau)} x(\tau) d\tau$$

(a) Write the impulse response of the system, $h_1(t)$.

$$h(t) = e^{-3t} u(t)$$

5

(b) Is the system causal? Justify your answer using the impulse response.

Causal ✓ $\quad h(t) = 0$ for $t < 0$

5

(c) Is the system stable? Justify your answer using the impulse response.

$$\int_{-\infty}^{\infty} |h(t)| dt = \int_{0}^{\infty} e^{-3t} dt =$$

5

$$\frac{1}{-3} e^{-3t} \Big]_0^{\infty} = \frac{-1}{3}(0-1) = \frac{1}{3} < \infty$$

Stable ✓

(d) Determine an expression for the output $y(t)$ when the input is a rectangular pulse with amplitude 1 and of duration 3 secs as defined below.

$$x(t) = u(t) - u(t-3)$$

$$y_d(t) = e^{-3t}u(t) * \{u(t) - u(t-3)\}$$

5

$$= z(t) - z(t-3)$$

$z(t) =$ Formula A w/ $a = -3$ $b = 0$

(e) Determine an expression for the output of System 1, $y(t)$, when the input is

$$x(t) = -4\{u(t-1) - u(t-4)\} + 2\{u(t-5) - u(t-8)\}$$

5

$$y_e(t) = -4\, y_d(t-1) + 2\, y_d(t-5)$$

(f) **Next:** Consider LTI System 2 characterized by the I/O relationship below. Suppose that System 2 is in SERIES with System 1. Determine the overall output $y(t)$ of these two systems in series when the input is $x(t) = e^{-t}u(t)$. Show all work.

$$\text{System 2:} \qquad y(t) = \int_{-\infty}^{t} e^{-5(t-\tau)} x(\tau) d\tau$$

$$\text{Input to series connection of Systems 1 and 2:} \quad x(t) = e^{-t}u(t)$$

Simplify your answer as much as possible. You should use Formula A for this problem BUT you will have to do the substitution and NOT simply cite Formula A.

$$y_f(t) = \underbrace{e^{-t}u(t) * e^{-3t}u(t)}_{\text{Formula A w/ } a=-1,\ b=-3} * e^{-5t}u(t)$$

15

$$= \frac{1}{-1+3} e^{-t}u(t) + \frac{1}{-3+1} e^{-3t}u(t)$$

$$\underbrace{\frac{1}{2} e^{-t}u(t) * e^{-5t}u(t)} - \underbrace{\frac{1}{2} e^{-3t}u(t) * e^{-5t}u(t)}$$

$$\frac{1}{2}\left\{\frac{1}{-\ +5} e^{-t}u(t) + \frac{1}{-5+\ } e^{-5t}u(t)\right\}$$

1 1

$$-\frac{1}{2}\left\{\frac{1}{-3+5} e^{-3t}u(t) + \frac{1}{-5+3} e^{-5t}u(t)\right\}$$

$$\left.\begin{array}{l} \frac{1}{8} e^{-t}u(t) - \frac{1}{4} e^{-3t}u(t) \\ \left(-\frac{1}{8} + \frac{1}{4}\right) e^{-5t}u(t) \end{array}\right\} y(t)$$

8 8

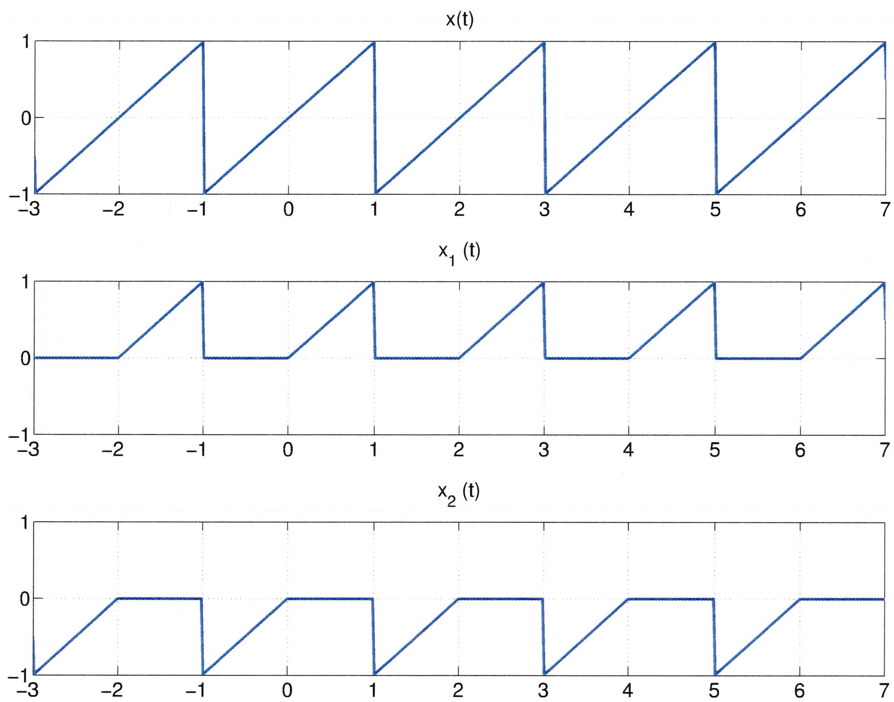
(g) Consider an LTI system with a rectangular impulse response $h(t) = u(t) - u(t-1)$ for which the input signal is the periodic sawtooth $x(t)$ below. In order to find the output $y(t)$ through convolution, decompose the input signal as the sum $x(t) = x_1(t) + x_2(t)$. You MUST plot your answers on graphs provided on the next page.

(i) Determine the output $y_1(t)$ when the input is $x_1(t)$. Plot a couple periods.

(ii) Determine the output $y_2(t)$ when the input is $x_2(t)$. Plot a couple periods.

(iii) Combine your answers to plot the output $y(t)$ when the input is $x(t) = x_1(t) + x_2(t)$.

20



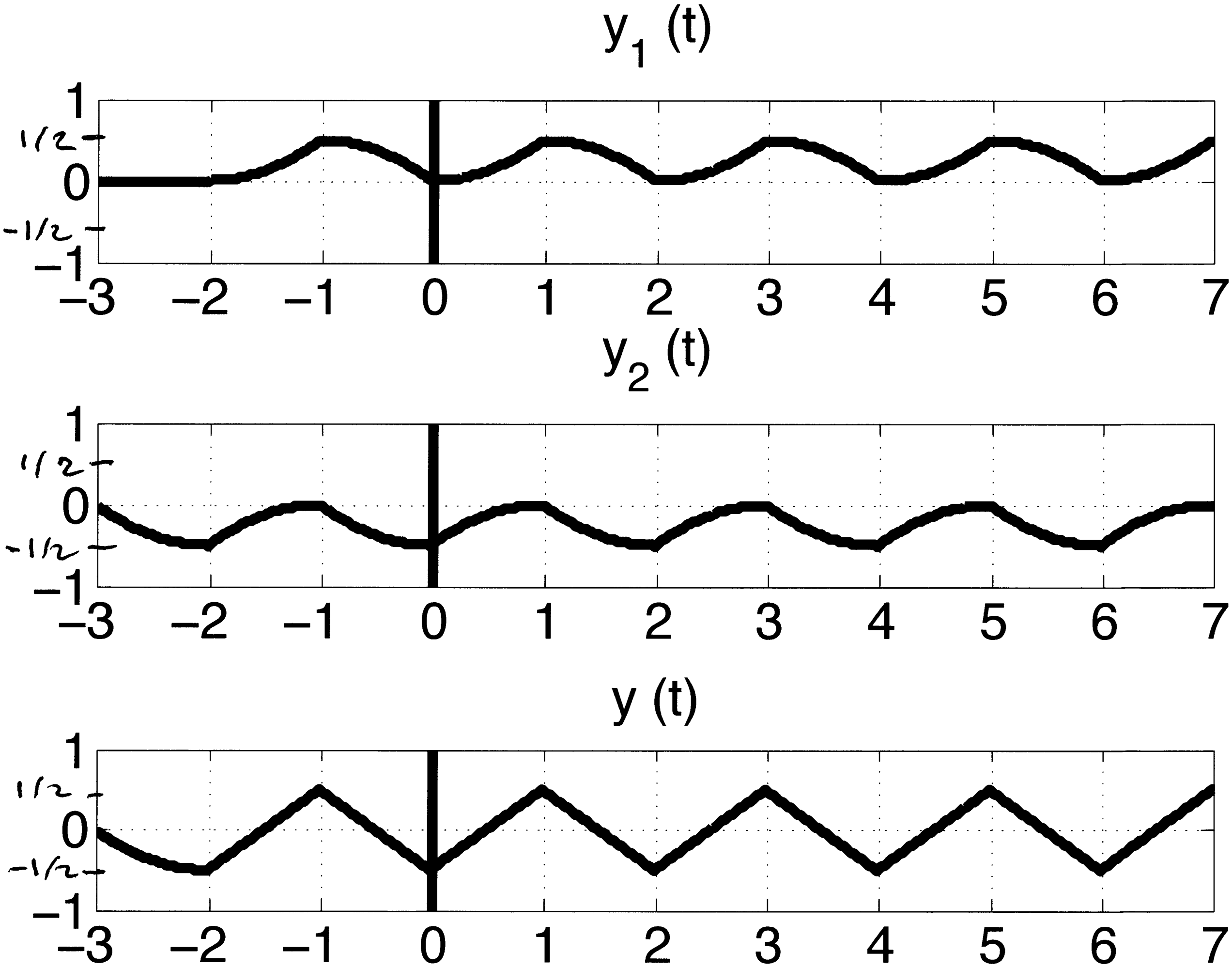

$$y_1(t) = \sum_{k=-\infty}^{\infty} z_1(t - k2)$$

$z_1(t) =$ Formula 1 w/ $T_1 = T_2 = 1$

$$\tilde{y}_2(t) = \sum_{k=-\infty}^{\infty} z_2(t - k2)$$

$z_2(t) =$ Formula 2 w/ $T_1 = T_2 = 1$

$$y(t) = y_1(t) - \tilde{y}_2(t-1)$$



y_1 (t)
1
1/2
0
-1/2
-1
-3 -2 -1 0 1 2 3 4 5 6 7
y_2 (t)
1
1/2
0
-1/2
-1
-3 -2 -1 0 1 2 3 4 5 6 7
y (t)
1
1/2
0
-1/2
-1
-3 -2 -1 0 1 2 3 4 5 6 7

Extra space provided for your work for Prob. 1, part (g)

In $0 < t < 1$: $y(t) =$

$$\frac{t^2}{2} - \left\{ \frac{(t+1)^2}{2} - 2(t+1) + \frac{4}{2} \right\}$$

$$\frac{t^2}{2} - \left\{ \frac{t^2}{2} + t + \frac{1}{2} - 2t - 2 + 2 \right\}$$

$$\frac{t^2}{2} - \left\{ \frac{t^2}{2} - t + \frac{1}{2} \right\}$$

$= t - \frac{1}{2}$ for $0 < t < 1$

**Problem 2.** [50 points] Problem 2 is separate from Problem 1; the first DT system is enumerated as System 1.

(a) Consider the causal LTI System characterized by the difference equation below. Write an expression for the impulse response of this system, denoted $h_1[n]$.

$$\text{System 1:} \quad y[n] = y[n-1] + x[n] - x[n-4]$$

$$h[n] = u[n] - u[n-4]$$
$$= \{\underset{\substack{\uparrow \\ n=0}}{1}, 1, 1, 1\}$$

5

(b) Is the system causal? Justify your answer using the impulse response.

$h[n] = 0$ for $n < 0$

Causal ✓

5

(c) Is the system stable? Justify your answer using the impulse response.

$$\sum_{n=-\infty}^{\infty} |h[n]| = 4 < \infty$$

Stable ✓

5

(d) Does the system have memory or is it memoryless? Justify your answer using the impulse response.

5

$h[n] \neq K\delta[n] \Rightarrow$ System has Memory

(e) Determine the output $y[n]$ of System 1 for the input below.

$$x[n] = \{u[n] - u[n-4]\}$$

$\{1, 1, 1, 1\} * \{1, 1, 1, 1\}$

5

$= \{1, 2, 3, 4, 3, 2, 1\}$

↑

$n=0$

(f) Determine the output, $y[n]$, when $x[n]$ is the input signal below

$$x[n] = 2^n\{u[n] - u[n-4]\} = \{1, 2, 4, 8\}$$

input to two LTI systems in SERIES, with respective impulse responses below. Write answer in sequence form; indicate with arrow the value corresponding to $n = 0$.

$$h_1[n] = (4-n)\{u[n] - u[n-4]\} = \{4, 3, 2, 1\} \qquad h_2[n] = \delta[n] - \delta[n-1] = \{1, -1\}$$

$h_1[n] * h_2[n]$ :

15

| | | | | |
|---|---|---|---|---|
| 4 | 3 | 2 | 1 | 0 |
| 0 | -4 | -3 | -2 | -1 |

$\{\underset{\uparrow}{4}, -1, -1, -1, -1\}$

$n=0$

$x[n] *$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 4 | -1 | -1 | -1 | -1 | 0 | 0 | 0 |
| 0 | 8 | -2 | -2 | -2 | -2 | 0 | 0 |
| 0 | 0 | 16 | -4 | -4 | -4 | -4 | 0 |
| 0 | 0 | 0 | 32 | -8 | -8 | -8 | -8 |

$\{\underset{\uparrow}{4}, 7, 13, 25, -15, -14, -12, -8\}$

$n=0$