Solution

NAME: **25 February** 2016

EE301 Signals and Systems **Exam 1**

Cover Sheet

Test Duration: 75 minutes.
Coverage: Chaps. 1,2
Open Book but Closed Notes.
One 8.5 in. x 11 in. crib sheet
Calculators NOT allowed.
This test contains **TWO** problems with multiple parts.
All work should be done in the space provided.
You must show ALL work or explain answer for each problem to receive full credit.
WRITE YOUR NAME ON EVERY SHEET.

Prob. No.	**Topic(s)**	**Points**
1.	Continuous Time Signals and System Properties	50
2.	Discrete Time Signals and System Properties	50

If you want to refer to the input signal and output signal for one part of a problem when solving a later part, use that part's letter as a subscript, e.g., you can refer to the input signal and corresponding output signal for part (d) of Prob. 1 as $x_d(t)$ and $y_d(t)$, respectively.

For the relevant parts, you must indicate which of the formulas below you are using to solve that particular problem – just list the formula letter, e.g., "Formula A"

Formula A: $$e^{at}u(t) * e^{bt}u(t) = \frac{1}{a-b}e^{at}u(t) + \frac{1}{b-a}e^{bt}u(t) \quad (1)$$

Formula B: $$\alpha^n u[n] * \beta^n u[n] = \frac{\alpha}{\alpha-\beta}\alpha^n u[n] + \frac{\beta}{\beta-\alpha}\beta^n u[n] \quad (2)$$

Formula C: if $x(t) * h(t) = y(t)$ then: $a\,x(t-t_1) * b\,h(t-t_2) = ab\,y(t-(t_1+t_2))$ (3)

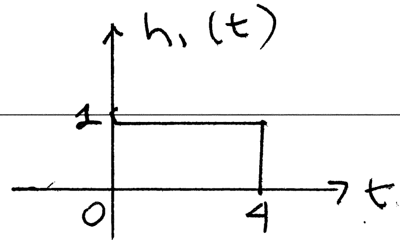
Formula D: if $x[n]*h[n] = y[n]$ then: $a\,x[n-n_1]*b\,h[n-n_2] = ab\,y[n-(n_1+n_2)]$ (4)

Prob. 1. [50 pts] Consider the LTI system characterized by the I/O relationship:

$$\text{System 1:} \quad y(t) = \int_{t-4}^{t} x(\tau)d\tau$$

(a) Determine and plot the impulse response of the system, $h_1(t)$.

$h_1(t) = u(t) - u(t-4)$



(b) Is the system causal? Justify your answer using the impulse response.

Yes: $h_1(t) = 0$ for $t<0$

(c) Is the system stable? Justify your answer using the impulse response.

Yes: $\int_{-\infty}^{\infty} |h(t)|dt = 4 \cdot 1 = 4 < \infty$

(d) **You should solve each part of this problem using known convolution results in conjunction with linearity (homogeneity and superposition) and time-invariance. You do not need to simplify your answers.** Determine and write a closed-form expression for the output ($y(t)$) of System 1 for the input

$$x(t) = 4\,e^{-2t}u(t)$$

$$y(t) = 4\,e^{-2t}u(t) * (u(t) - u(t-4))$$

$$= 4\left(e^{-2t}u(t) * u(t) - e^{-2t}u(t) * u(t-4)\right)$$

$z(t) \triangleq$ Formula A with $a = -2$ and $b = 0$

$a = -2 \quad b = 0$

$$y(t) = 4\,z(t) - 4\,z(t-4)$$

(e) Determine a closed-form expression for the output ($y(t)$) of System 1 for the input

$$x(t) = 4e^{-2t}u(t-4)$$

$$x_e(t) = 4e^{-8}\,e^{-2(t-4)}\,u(t-4) = e^{-8}x_d(t-4)$$

Thus, Formula C

$$y_e(t) = e^{-8}y_d(t-4)$$

(f) Determine and write a closed-form expression for the output $y(t)$ of System 1 for the input

$$x(t) = 4e^{-2t}\{u(t) - u(t-4)\}$$

$$x_f(t) = \underbrace{4e^{-2t}u(t)}_{x_d(t)} - \underbrace{4e^{-8}e^{-2(t-4)}u(t-4)}_{x_e(t)}$$

Thus:

$$y_f(t) = y_d(t) - y_e(t)$$

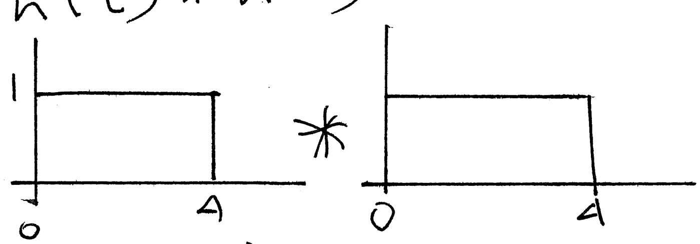
(g) Determine and plot ~~a closed-form expression for~~ the output $(y(t))$ of System 1 for the input

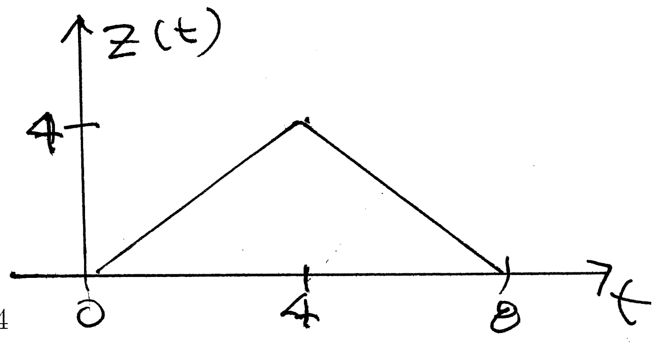
$$x(t) = 3\{u(t-2) - u(t-6)\} \qquad x(t) = h(t-2)$$

$$y(t) = \underbrace{(u(t) - u(t-4))}_{h(t)} * 3(h(t-2))$$

define: $z(t) = h(t) * h(t)$

$$\Rightarrow \boxed{y(t) = 3z(t-2)}$$





(h) Consider a second LTI system described by the following difference equation. Determine the impulse response for System 2, denoted $h_2(t)$.

$$\text{System 2:} \quad y(t) = \int_{-\infty}^{t} e^{-\frac{1}{2}(t-\tau)} x(\tau) d\tau$$

$h(t) = e^{-\frac{1}{2}t} u(t)$

(i) Determine a closed-form expression for the output $y[n]$ when the input is

$$x(t) = 6e^{-3t} u(t-2)$$

$y(t) = e^{-\frac{1}{2}t} u(t) * 6 e^{-3t} u(t-2)$

$= e^{-\frac{1}{2}t} u(t) * 6 e^{-6} e^{-3(t-2)} u(t-2)$

Define $z(t) =$ Formula A with $a = -\frac{1}{2}$ $b = -3$

$y(t) = 6e^{-6} z(t-2)$

(j) Determine and write a closed-form expression for the output of System 2, $y(t)$, for the input

$$x(t) = \{u(t) - u(t-4)\}$$

$$y(t) = e^{-\frac{1}{2}t}u(t) * \left(u(t) - u(t-4)\right)$$

$$= e^{-\frac{1}{2}t}u(t) * u(t) - e^{-\frac{1}{2}t}u(t) * u(t-4)$$

$z(t) =$ Formula A with $a = -\frac{1}{2}$ and $b = 0$

$$y(t) = z(t) - z(t-4)$$

(k) Determine and write a closed-form expression for the output of System 2, $y(t)$, for the input

$$x(t) = 2\{u(t) - u(t-4)\} - 3\{u(t-6) - u(t-10)\}$$

Observe: $x_k(t) = 2x_j(t) - 3x_j(t-6)$

Thus: $y_k(t) = 2y_j(t) - 3y_j(t-6)$

(1) System 1 and System 2 are in series along with a third system (all three in series) which is a differentiator as described below. Determine a closed-form expression for the overall impulse response, denoted $h_0(t)$, for the series combination of all three systems. If you invoke properties of convolution and remember a similar homework problem, this problem should be quick.

$$\text{System 3:} \quad y(t) = \frac{d}{dt}x(t)$$

$$h_0(t) = (u(t) - u(t-4)) * e^{-\frac{1}{2}t}u(t) * \frac{d}{dt}$$

$$= e^{-\frac{1}{2}t}u(t) * \left\{ (u(t) - u(t-4)) * \frac{d}{dt} \right\}$$

$$= e^{-\frac{1}{2}t}u(t) * \{ \delta(t) - \delta(t-4) \}$$

$$= e^{-\frac{1}{2}t}u(t) - e^{-\frac{1}{2}(t-4)}u(t-4)$$

$u(t)$ is <u>not</u> in exponent :)

Problem 2. [50 points] Show work in space provided.

(a) Consider the causal LTI System characterized by the difference equation below. Write an expression for the impulse response of this system, denoted $h_1[n]$.

$$\text{System 1:} \qquad y[n] = -\frac{3}{4}y[n-1] + x[n]$$

$h[n] = \left(-\frac{3}{4}\right)^n u[n]$

(b) Is the system causal? Justify your answer using the impulse response.

Yes, $h[n] = 0$ for $n < 0$

(c) Is the system stable? Justify your answer using the impulse response.

Yes : $\sum_{n=0}^{\infty} |h[n]| = \sum_{n=0}^{\infty} \left(\frac{3}{4}\right)^n = \frac{1}{1-\frac{3}{4}} < \infty$

(d) Does the system have memory or is it memoryless? Justify your answer using the impulse response.

Has memory since $h[n] \neq K\delta[n]$

(e) Determine and write a closed-form expression for the output $y[n]$ of System 1 for the input

$$x[n] = 4\left(\frac{1}{2}\right)^n u[n]$$

$$y[n] = \left(-\frac{3}{4}\right)^n u[n] * 4\left(\frac{1}{2}\right)^n u[n]$$

$$= 4\, z[n]$$

$z[n]$ = Formula B with $\alpha = \frac{-3}{4}$ and $\beta = \frac{1}{2}$

(f) Determine a closed-form expression for the output $y[n]$ of System 1 when the input is

$$x[n] = 4\left(\frac{1}{2}\right)^n u[n-4]$$

$$x[n] = 4\left(\frac{1}{2}\right)^{n-4} u[n-4] \left(\frac{1}{2}\right)^4$$

$$x_f[n] = 4\left(\frac{1}{2}\right)^4 x_e[n-4]$$

$$y_f[n] = \frac{1}{16}\, y_e[n-4]$$

(g) Determine and write a closed-form expression for the output $y[n]$ of System 1 for the input

$$x[n] = 4\left(\frac{1}{2}\right)^n \{u[n] - u[n-4]\}$$

$$x_g[n] = 4\left(\frac{1}{2}\right)^n u[n] - 4\left(\frac{1}{2}\right)^n u[n-4]$$

$$= x_e[n] - x_f[n]$$

$$y_g[n] = y_e[n] - y_f[n]$$

(h) Determine a closed-form expression for the output $y[n]$ of System 1 when the input is

$$x[n] = 4\{u[n] - u[n-4]\}$$

$$y[n] = 4\{u[n] - u[n-4]\} * \left(\frac{-3}{4}\right)^n u[n]$$

$$= 4\, u[n] * \left(-\frac{3}{4}\right)^n u[n] - 4\, u[n-4] * \left(\frac{-3}{4}\right)^n u[n]$$

$$= 4\, z[n] - 4\, z[n-4]$$

$z[n]$ = Formula B with $\alpha = -\frac{3}{4}$ and $\beta = 1$

(i) Consider a second LTI system described by the following difference equation. Determine the impulse response for System 2, denoted $h_2[n]$. You can write your answer in sequence form, using an arrow to denote the $n = 0$ value.

$$\text{System 2:} \quad y[n] = y[n-1] + x[n] - x[n-4]$$

$$h[n] = (1)^n \left(u[n] - u[n-4]\right) = \{\underset{\underset{n=0}{\uparrow}}{1}, 1, 1, 1\}$$

(j) Determine and write a closed-form expression for the output of System 2 ,$y[n]$, for the input below. Write answer in sequence form, using an arrow to denote the $n = 0$ value.

$$x[n] = \{u[n] - u[n-4]\}$$

$$y[n] = \{\underset{\underset{n=0}{\uparrow}}{1}, 1, 1, 1\} * \{\underset{\underset{n=0}{\uparrow}}{1}, 1, 1, 1\}$$

$$= \{\underset{\underset{n=0}{\uparrow}}{1}, 2, 3, 4, 3, 2, 1\}$$

(k) Determine and write a closed-form expression for the output of System 2 ,$y[n]$, for the input below. Write answer in sequence form, using an arrow to denote the $n = 0$ value.

$$x[n] = 2\{u[n] - u[n-4]\} - 3\{u[n-6] - u[n-10]\}$$

$x[n] = 2\,x_j[n] - 3\,x_j[n-6]$

$2\{\underset{\substack{\uparrow \\ n=0}}{1}, 2, 3, 4, 3, 2, 1\}$

$-3\{\underset{\substack{\uparrow \\ n=6}}{1}, 2, 3, 4, 3, 2, 1\}$

$= \{2, 4, 6, 8, 6, 4, \underset{\substack{\uparrow \\ n=6}}{-1}, -6, -9, -12, -9, -6, -3\}$

only point where they overlap

(l) Determine a closed-form expression for the output $y[n]$ of System 2 when the input is

$$x[n] = 2\delta[n] - \delta[n-1]$$

$y[n] = \{\underset{\substack{\uparrow \\ n=0}}{1}, 1, 1, 1\} * \{2, -1\} = h[n] * \{2\delta[n] - \delta[n-1]\}$

$= 2h[n] - h[n-1]$

$2\{\underset{\substack{\uparrow \\ n=0}}{1}, 1, 1, 1\}$

$\{1, 1, 1, 1\}$

$= \{2, 2, 2, 2\}$

$\{-1, -1, -1, -1\}$

$= \{\underset{\substack{\uparrow \\ n=0}}{2}, 1, 1, 1, -1\}$

(m) Determine $y[n]$ as the convolution of the two sequences below. Write your answer in sequence form indicating with an arrow which value corresponds to $n = 0$. You can use next page as well as the space below to show all your work.

$$x[n] = 8\left(\frac{1}{2}\right)^n \{u[n] - u[n-4]\} \qquad h[n] = 16\left(-\frac{1}{2}\right)^n \{u[n] - u[n-4]\}$$

$$\begin{array}{r} 16 \\ 8 \\ \hline 128 \end{array}$$

$x[n] = 8\left\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}\right\} = \{\underset{\uparrow\, n=0}{8}, 4, 2, 1\}$

$h[n] = 16\left\{\underset{\uparrow\, n=0}{1}, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}\right\} = \{16, -8, 4, -2\}$

n	0	1	2	3	4	5	6
$x[0]h[n]$	128	-64	32	-16	0	0	0
$x[1]h[n-1]$	0	64	-32	16	-8	0	0
$x[2]h[n-2]$	0	0	32	-16	8	-4	0
$x[3]h[n-3]$	0	0	0	16	-8	4	-2
$y[n]$	$\underset{\uparrow\, n=0}{128}$	0	32	0	8	0	-2

(additional space for work for part (m) if needed: