SOLUTION

NAME: EE301 Signals and Systems

26 February 2015 Exam 1

Cover Sheet

Test Duration: 75 minutes.
Coverage: Chaps. 1,2
Open Book but Closed Notes.
One 8.5 in. x 11 in. crib sheet
Calculators NOT allowed.
This test contains **two** problems.
All work should be done on the sheets provided.
You must show work or explain answer for each problem to receive full credit.
Plot your answers on the graphs provided.
WRITE YOUR NAME ON EVERY SHEET.

Prob. No.	Topic(s)	Points
1.	Continuous Time Signals and System Properties	50
2.	Discrete Time Signals and System Properties	50

$$
\begin{aligned}
y_1(t) = \{u(t)-u(t-T_1)\} * t\{u(t)-u(t-T_2)\} &= \frac{t^2}{2}\{u(t)-u(t-T_1)\} \\
&+ \left(T_1 t - \frac{T_1^2}{2}\right)\{u(t-T_1)-u(t-T_2)\} \\
&+ \left(-\frac{t^2}{2} + T_1 t + \frac{T_2^2 - T_1^2}{2}\right)\{u(t-T_2)-u(t-(T_1+T_2))\}
\end{aligned} \quad (1)
$$

$$
\begin{aligned}
u(t)-u(t-T_1)\} * [-(t-T_2)\{u(t)-u(t-T_2)\}] &= \left(-\frac{t^2}{2} + T_2 t\right)\{u(t)-u(t-T_1)\} \\
&+ \left(-T_1 t + \frac{2T_1T_2 + T_1^2}{2}\right)\{u(t-T_1)-u(t-T_2)\} \\
&+ \left(\frac{t^2}{2} - (T_1+T_2)t + \frac{(T_1+T_2)^2}{2}\right)\{u(t-T_2)-u(t-(T_1+T_2))\}
\end{aligned} \quad (2
$$

$$
y_2(t) = \{u(t)-u(t-T_1)\} * [-(t-T_2)\{u(t)-u(t-T_2)\}] = y_1\left(-(t-(T_1+T_2)\right) \quad (3)
$$

Prob. 1. [50 pts] Consider an LTI system characterized by the I/O relationship:

$$y(t) = \int_{t-2}^{t} x(\tau)d\tau - \int_{t-6}^{t-4} x(\tau)d\tau + x(t-7) \tag{4}$$

Note that this can be viewed as three systems in parallel, $y(t) = y_1(t) + y_2(t) + y_3(t)$, where:

$$y_1(t) = \int_{t-2}^{t} x(\tau)d\tau \quad \Rightarrow \text{System 1 impulse response: } h_1(t) \tag{5}$$

$$y_2(t) = -\int_{t-6}^{t-4} x(\tau)d\tau \quad \Rightarrow \text{System 2 impulse response: } h_2(t) \tag{6}$$

$$y_3(t) = x(t-7) \quad \Rightarrow \text{System 3 impulse response: } h_3(t) \tag{7}$$

(a) Determine and plot the impulse response of the overall system, denoted $h(t)$, in the spaced provided on the sheets attached.

(b) Determine and plot the output $y_1(t)$ of System 1 in the space provided on the next few pages when the input to System 1 is the linearly ramping-up input signal below:

$$x(t) = t\{u(t) - u(t-6)\}$$

(c) Determine and plot the output $y_2(t)$ of System 2 in the space provided for the same input $x(t) = t\{u(t) - u(t-6)\}$. You are first required to express $h_2(t)$ in terms of $h_1(t)$ in the space directly below; then express the relationship between $y_2(t)$ and $y_1(t)$.

Relationship between impulse responses of Systems 1 and 2: $h_2(t) = -h_1(t-4)$

(d) Determine and plot the output $y_3(t)$ of System 3 in the space provided on the next few pages for the same input $x(t) = t\{u(t) - u(t-6)\}$. You are first required to write an expression for System 3's impulse response $h_3(t)$ in the space directly below.

Expression for System 3's impulse response: $h_3(t) = \delta(t-7)$

(e) Using the spaced provided in the sheets attached, plot the output $y(t)$ of the overall system when the input to the system is $x(t) = t\{u(t) - u(t-6)\}$.

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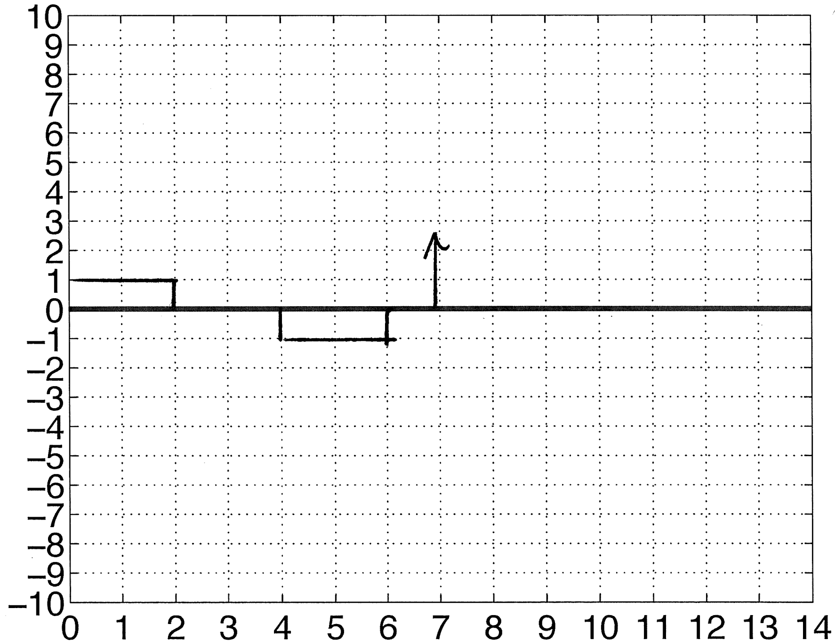
Plot your answer for $h(t)$ for Problem 1 (a) here.

$h(t) = h_1(t) - h_1(t-4) + \delta(t-7)$

$h_1(t) =$
$u(t) - u(t-2)$

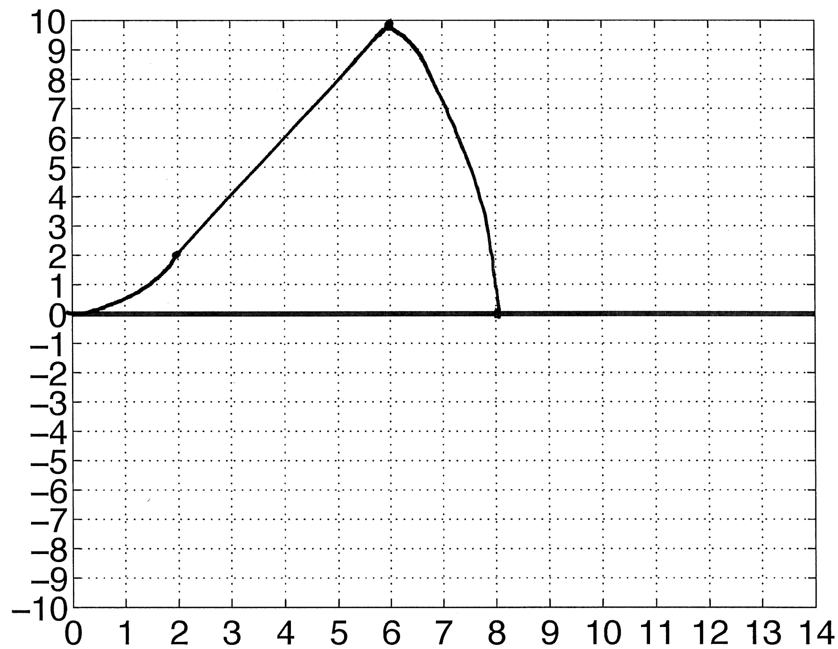


Plot your answer for $y_1(t)$ for Problem 1 (b) here.

$T_1 = 2$

$T_2 = 6$

ramp-up triangle



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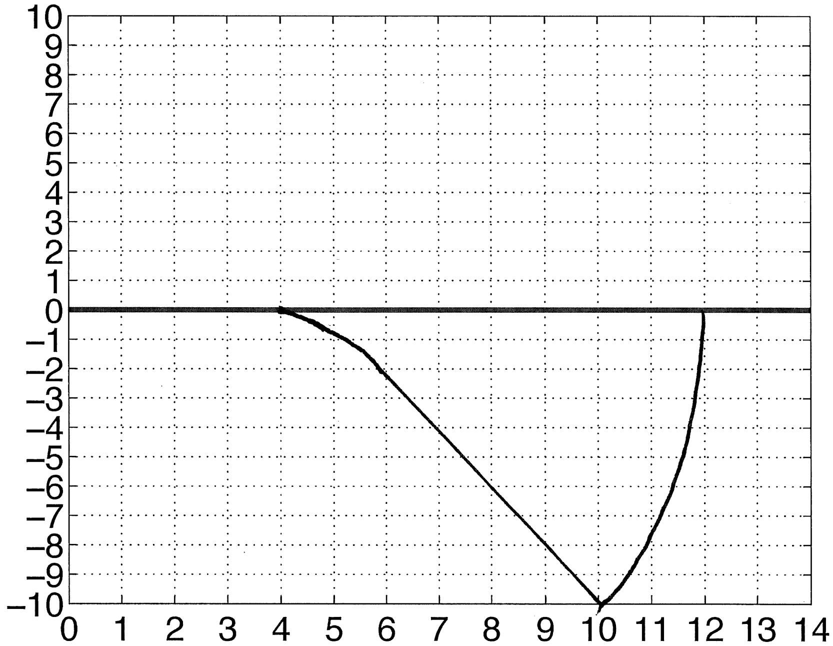
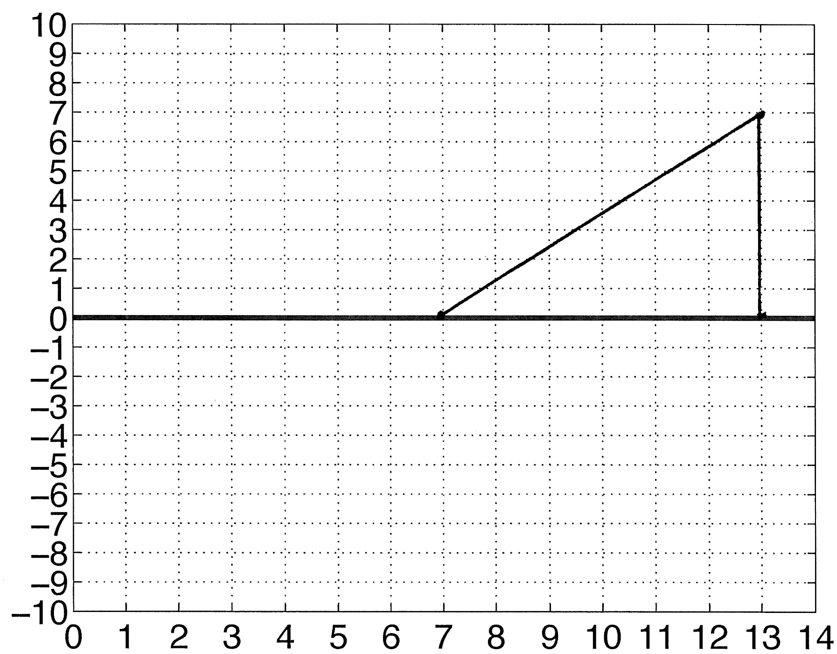
Plot your answer for $y_2(t)$ for Problem 1 (c) here.



Plot your answer for $y_3(t)$ for Problem 1 (d) here.



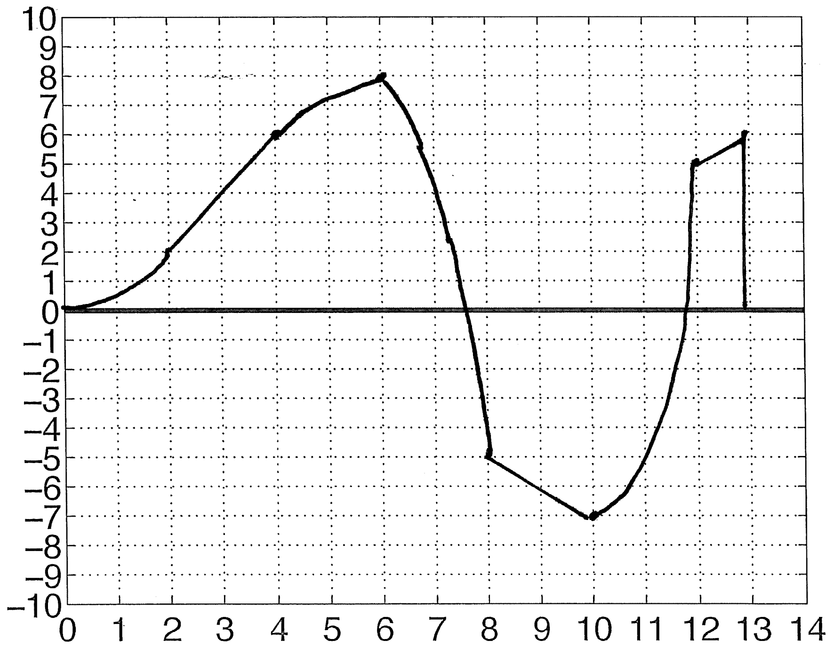
Part (e). For each range of t, put an X in the correct box in the table below.

Range for t	Linear pos. slope	Linear neg. slope	Quadratic Concave Up	Quadratic Concave Down
$0 < t < 2$			X	
$2 < t < 4$	X			
$4 < t < 6$				X
$6 < t < 8$				X
$8 < t < 10$		X		
$10 < t < 12$			X	
$12 < t < 13$	X			

For each value of t, write the value of $y(t)$ in the table below.

t	$y(t)$
$t = 0$	0
$t = 2$	2
$t = 4$	6
$t = 6$	8
$t = 8$	-5
$t = 10$	-7
$t = 12$	5
$t = 13$	6

Plot $y(t)$ for Prob1, Part (e) below.



Problem 2. [50 points] Put the answers for the remaining parts on this page.

(a) Consider the causal LTI System characterized by the difference equation below. Determine and write a closed-form EXPRESSION for the output $y[n]$ for the input $x[n] = \left(\frac{1}{3}\right)^n u[n]$, which is an infinite length sequence.

$$\text{System 1:} \quad y[n] = -\frac{1}{2}y[n-1] + x[n]$$

Proved in class: $h[n] = \left(-\frac{1}{2}\right)^n u[n]$

$y[n] = \left(\frac{1}{3}\right)^n u[n] * \left(-\frac{1}{2}\right)^n u[n]$ $\qquad \alpha = \frac{1}{3} \quad \beta = -\frac{1}{2}$

$$\alpha^n u[n] * \beta^n u[n] = \frac{\beta}{\beta - \alpha} \beta^n u[n] - \frac{\alpha}{\beta - \alpha} \alpha^n u[n]$$

$$= \frac{-1/2}{-\frac{1}{2} - \frac{1}{3}} \left(-\frac{1}{2}\right)^n u[n] - \frac{\frac{1}{3}}{-\frac{1}{2} - \frac{1}{3}} \left(\frac{1}{3}\right)^n u[n]$$

$$= \frac{3}{5}\left(-\frac{1}{2}\right)^n u[n] + \frac{2}{5}\left(\frac{1}{3}\right)^n u[n]$$

(b) Consider the same system as for part (a), but now determine an expression for the output when the input is $x[n] = 4\left(\frac{1}{3}\right)^n u[n-2]$.

$$x_{new}[n] = 4\left(\frac{1}{3}\right)^2 \left(\frac{1}{3}\right)^{n-2} u[n-2]$$

$$= \frac{4}{9} x[n-2]$$

Linearity and Time-Invariance dictates:

$$y_{new}[n] = \frac{4}{9} y[n-2]$$

$$= \frac{4}{9} \cdot \frac{3}{5}\left(-\frac{1}{2}\right)^{n-2} u[n-2] + \frac{4}{9}\,\frac{2}{5}\left(\frac{1}{3}\right)^{n-2} u[n-2]$$

(c)

Determine $y[n]$ as the convolution of the two sequences below. You can EITHER do a stem plot for your answer OR write it out in sequence form clearly indicating with an arrow which value corresponds to $n = 0$.

$$x[n] = 2\underbrace{\{u[n] - u[n-4]\}}_{\text{length } 4} \qquad h[n] = 3\underbrace{\{u[n-2] - u[n-8]\}}_{\text{length } 6}$$

conv. of 2 rectangles: length is $6 + 4 - 1 = 9$

has trapezoidal shape with peak value = 4

If $h[n]$ had started at $n=0$, the answer would be

$$6\{\underset{\substack{\uparrow \\ n=0}}{1}, 2, 3, 4, 4, 4, 3, 2, 1\}$$

Since $h[n]$ starts at 2, we just have to slide the result above to the right by 2 invoking time-invariance

$$y[n] = \{\underset{\substack{\uparrow \\ n=0}}{0}, 0, 6, 12, 18, 24, 24, 24, 18, 12, 6\}$$

(d) Determine $y[n]$ as the convolution of the two sequences below. You can EITHER do a stem plot for your answer OR write it out in sequence form clearly indicating with an arrow which value corresponds to $n = 0$.

$$x[n] = 8\left(-\frac{1}{2}\right)^n \{u[n] - u[n-4]\} \qquad h[n] = 16\left(\frac{1}{2}\right)^n \{u[n] - u[n-5]\}$$

These are finite length sequences, so use Table Method:

$$x[n] = \{\underset{\uparrow}{8}, -4, 2, -1\} \quad \text{length } 4$$

$$h[n] = \{\underset{\uparrow}{16}, 8, 4, 2, 1\} \quad \text{length } 5$$

conv. length = $4 + 5 - 1 = 8$

First, compute:

$$8\,h[n] = \{128, 64, 32, 16, 8\}$$

$$-4\,h[n] = \{-64, -32, -16, -8, -4\}$$

$$2\,h[n] = \{32, 16, 8, 4, 2\}$$

n	0	1	2	3	4	5	6	7
$8\,h[n]$	128	64	32	16	8	0	0	0
$-4\,h[n-1]$	0	-64	-32	-16	-8	-4	0	0
$2\,h[n-2]$	0	0	32	16	8	4	2	0
$-h[n-3]$	0	0	0	-16	-8	-4	-2	-1
$y[n]$	128	0	32	0	0	-4	0	-1