ECE 301 Exam 1 Solution SP '09

Prob. 1 (a) System 1 is same as System 2 and they are in series.

Simply prove System 1 is linear and invoke that two linear systems in series is also linear

$$y(t) = \int_{t-3}^{t} \left(a_1 x_1(\tau) + a_2 x_2(\tau)\right) d\tau$$

$$= a_1 \int_{t-3}^{t} x_1(\tau)\, d\tau + a_2 \int_{t-3}^{t} x_2(\tau)\, d\tau$$

$$= a_1 y_1(t) + a_2 y_2(t)$$

(b) System 2 = System 1 $\Rightarrow$ prove System 1 is TI $\Rightarrow$ two TI systems in series is TI

$$x(t-t_0) \longrightarrow \boxed{} \longrightarrow y_{t_0}(t) = \int_{t-3}^{t} x(\tau - t_0)\, d\tau$$

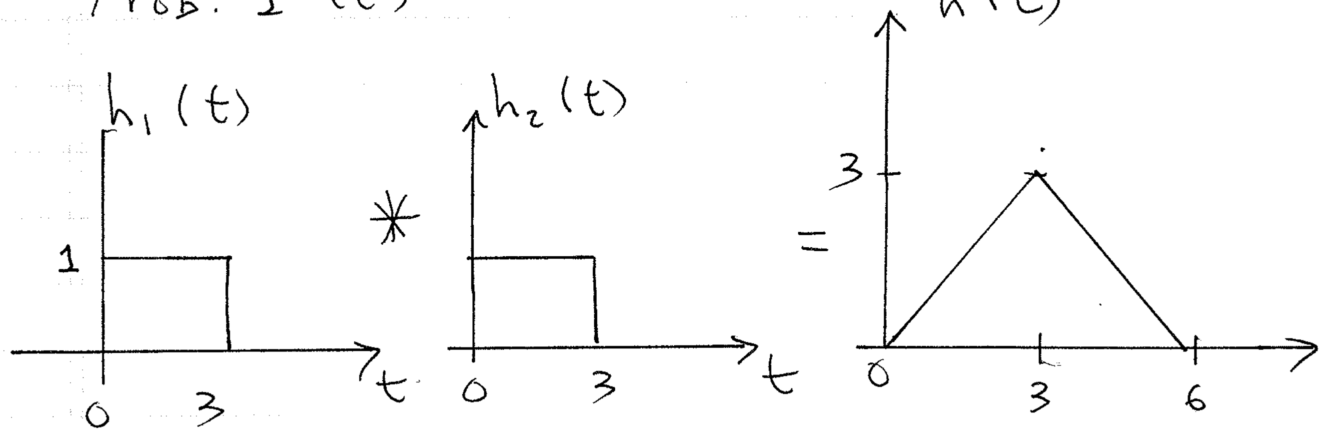
$\lambda = \tau - t_0 \Rightarrow d\lambda = d\tau$

new limits: $\tau \Big|_{t-3}^{t} \Rightarrow \lambda \Big|_{t-3-t_0 = (t-t_0)-3}^{t-t_0}$

$$y_{t_0}(t) = \int_{(t-t_0)-3}^{t-t_0} x(\ )\, d\lambda$$

Since $y(t) = \int_{t-3}^{t} x(\tau)\, d\tau \Rightarrow y(t-t_0) = \int_{t-t_0-3}^{t-t_0} x(\tau)\, d\tau$ $\Bigg\}$ System is TI

Prob. 1 (c)



$$h(t) = h_1(t) * h_2(t)$$

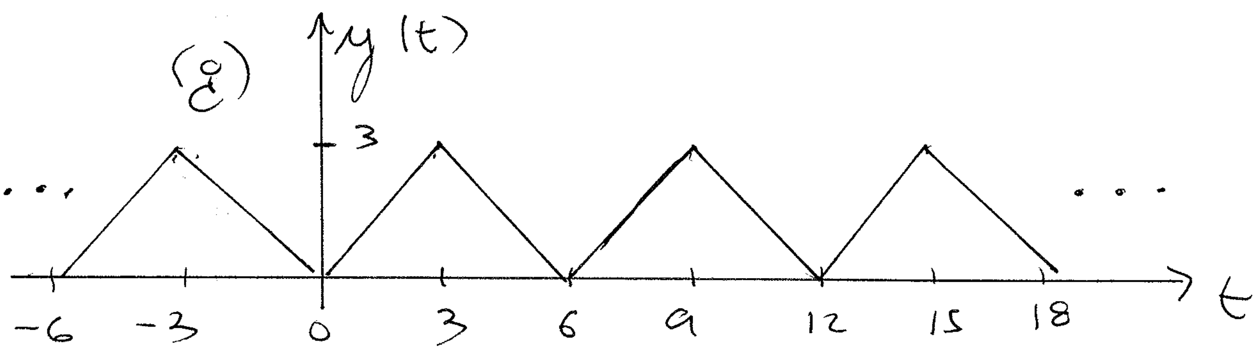
for 2 LTI systems in series

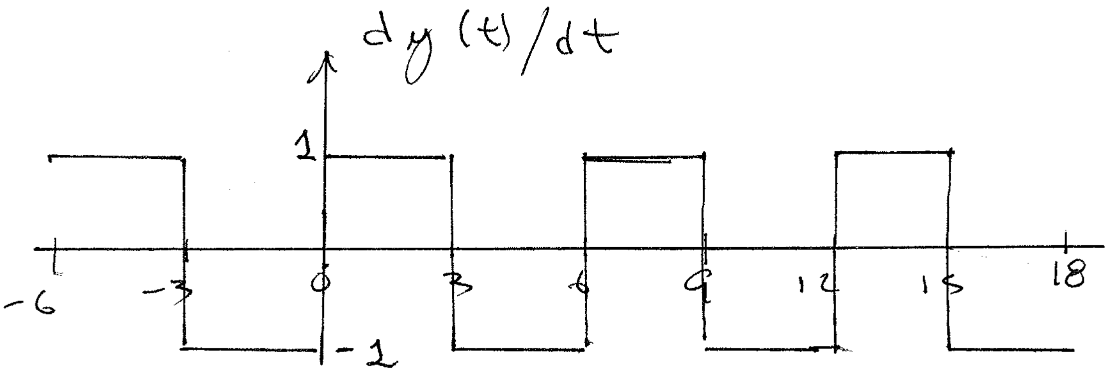
(d) $h(t) = 0$ for $t < 0$ $\Rightarrow$ causal

(e) $\displaystyle\int_{-\infty}^{\infty} |h(t)|\,dt = \underset{<\infty}{9}$ $\Rightarrow$ stable

(f) $a_k = \frac{1}{T} = \frac{1}{6}$

(g)





$T=6$ period

$$b_k = \frac{1}{j\frac{2\pi}{6}k}\left\{\frac{\sin\left(k\pi\frac{3}{6}\right)}{k\pi}e^{-j\frac{k2\pi}{6}\left(\frac{3}{2}\right)} - \frac{\sin\left(k\pi\frac{3}{6}\right)}{k\pi}e^{-j\frac{k2\pi}{6}\left(\frac{-3}{2}\right)}\right\}$$

$$= -6\,\frac{\sin^2\left(k\frac{\pi}{2}\right)}{(k\pi)^2} \quad \text{for } k\neq 0$$

$$b_0 = 9/6 = \frac{3}{2}$$

<u>Problem 2</u>: $x[n] = e^{j\frac{\pi}{3}n} + e^{j\frac{\pi}{2}n}$

period: $\frac{2\pi}{\pi/3} = 6$ period: $\frac{2\pi}{\pi/2} = 4$

since have common divisor of 2,
period is $\frac{6\times 4}{2} = 12$

$y[n] = x[n-1] + 2x[n] + x[n+1]$

<u>System 1</u>: (a) L (b) TI

(c) since linear:

$$y_1[n] = e^{j\frac{\pi}{3}(n-1)} + 2e^{j\frac{\pi}{3}n} + e^{j\frac{\pi}{3}(n+1)}$$

$$= \underbrace{\left\{e^{-j\pi/3} + 2 + e^{j\frac{\pi}{3}}\right\}}_{\alpha} e^{j\frac{\pi}{3}n}$$

$$y_2[n] = e^{j\frac{\pi}{2}n}\underbrace{\left\{e^{-j\frac{\pi}{2}} + 2 + e^{+j\pi/2}\right\}}_{\beta}$$

(c) $y[n] = y_1[n] + y_2[n]$

LTI system 1 only one for which output frequencies same as input frequencies

$$= \alpha e^{j\frac{\pi}{3}n} + \beta e^{j\frac{\pi}{2}n}$$

output freqs. : $\frac{\pi}{3}, \frac{\pi}{2}$ $\Rightarrow$ same as input

System 2 : $y[n] = x[2n]$ (a) L ; (b) not TI

(c) $= e^{j\frac{2\pi}{3}n} + e^{j\frac{2\pi}{2}n}$

output freqs.: $\frac{2\pi}{3}, \pi$

System 3 : $y[n] = x^2[n]$ (a) not L; (b) TI

(c) $y[n] = \left(e^{j\frac{\pi}{3}n} + e^{j\frac{\pi}{2}n}\right)^2$

$$= e^{j\frac{2\pi}{3}n} + 2e^{j\left(\frac{\pi}{3}+\frac{\pi}{2}\right)n} + e^{j\frac{2\pi}{2}n}$$

output freqs.: $\frac{2\pi}{3}, \frac{5\pi}{6}, \pi$

System 4 : $y[n] = (-1)^n x[n]$ (a) L

$= e^{j\pi n} x[n]$ (b) not TI

(c) $y[n] = e^{j\pi n} e^{j\frac{\pi}{3}n} + e^{j\pi n} e^{j\frac{\pi}{2}n}$

output freqs. : $\frac{4\pi}{3} - 2\pi, \frac{3\pi}{2} - 2\pi$

$$= -\frac{2\pi}{3}, -\frac{\pi}{2}$$