Sol'n to Exam 1 EE 301 SP 2004

Sol'n to Prob. 1 : 1

(a) Linear.

$$y(t) = \int_{t-1}^{t+1} (a_1 x_1(\tau) + a_2 x_2(\tau)) \, d\tau$$

$$= a_1 y_1(t) + a_2 y_2(t)$$

$$= a_1 \int_{t-1}^{t+1} x_1(\tau) \, d\tau + a_2 \int_{t-1}^{t+1} x_2(\tau) \, d\tau$$

(b) Time-Invariant

$$y(t-t_0) = \int_{t-t_0-1}^{t-t_0+1} x(\tau) \, d\tau$$

$$z(t) = \int_{t-1}^{t+1} x(\tau - t_0) \, d\tau \qquad \lambda = \tau - t_0$$

$$= \int_{t-t_0-1}^{t-t_0+1} x(\lambda) \, d\lambda = y(t-t_0)$$

(c) Noncausal Depends on $x(t)$ thru $x(t+1)$

(d) Stability: As long as input is bounded, area under 2 second duration will be finite

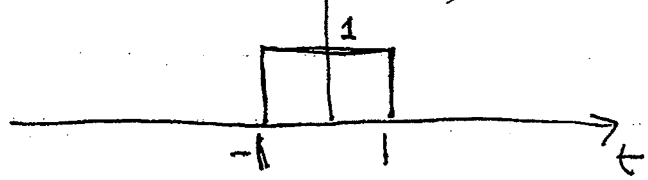
Sol'n. to Prob. 1 (cont.)

(e)

$$h(t) = \int_{t-1}^{t+1} \delta(\tau)\, d\tau$$

$$= \begin{cases} 1, & t+1>0 \text{ and } t-1<0 \Rightarrow -1<t<1 \\ 0, & \text{otherwise} \end{cases}$$

$h(t) = u(t+1) - u(t-1)$



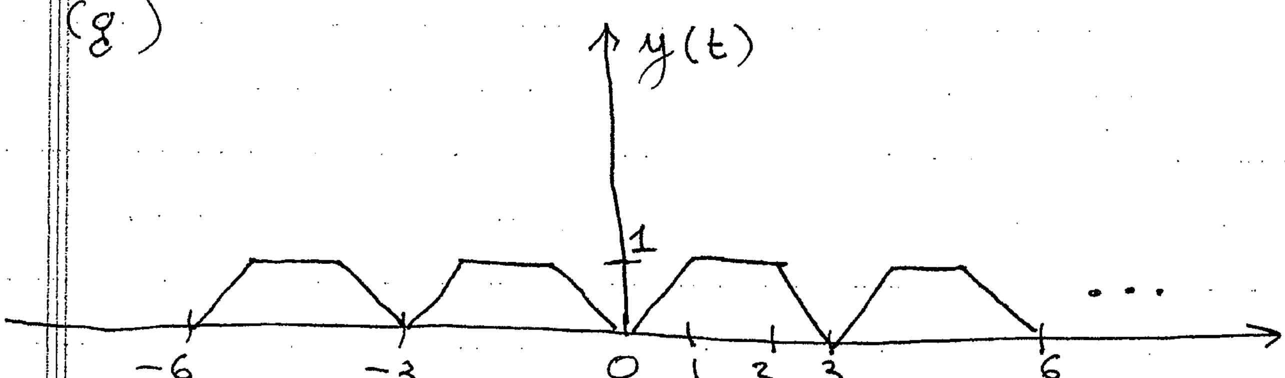
Back to part (d): $\int_{-\infty}^{\infty} |h(t)|\, dt = 2 < \infty \Rightarrow$ stable!

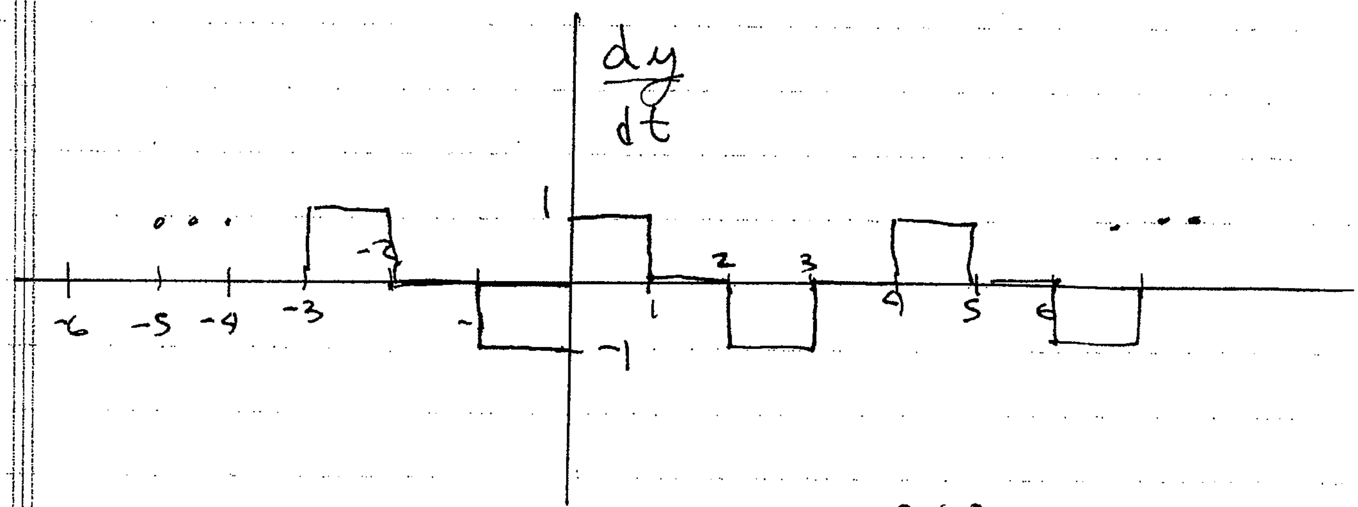
(f)

$$a_k = \frac{\sin\left(k\pi \frac{1}{3}\right)}{k\pi} e^{-j \frac{2\pi k}{3} \frac{3}{2}}$$

$$= (-1)^k \frac{\sin\left(k\frac{\pi}{3}\right)}{k\pi}$$

(g)





$$b_k = \frac{1}{j\,2\pi \frac{k}{3}} \left\{ \frac{\sin\left(k\frac{\pi}{3}\right)}{k\pi} e^{-j\,2\pi\frac{k}{3}\left(\frac{1}{2}\right)} - \frac{\sin\left(k\frac{\pi}{3}\right)}{k\pi} e^{+j\,2\pi\frac{k}{3}\left(\frac{1}{2}\right)} \right\}$$

$$= -3\,\frac{\sin^2\left(k\frac{\pi}{3}\right)}{(k\pi)^2} = -3\,\frac{\sin^2\left(k\frac{\pi}{3}\right)}{(k\pi)^2}$$

$$a_0 = \frac{\text{area under one period}}{\text{period}} = 2/3$$

(h) $$w(t) = x(2t) = \sum_{k=-\infty}^{\infty} a_k\, e^{+j\,2\pi\frac{k}{3}(2t)}$$

$$= \sum_{k=-\infty}^{\infty} a_k\, e^{j\,2\pi\frac{k}{(3/2)}t}$$

same F.S. coefficients $\Rightarrow$ new period = $\frac{3}{2}$ seconds

Sol'n to Prob. 2

(a) Linear

$$y[n] = \sum_{k=-1}^{1} x[n-k] = \sum_{k=n-1}^{n+1} x[k]$$

$$y[n] = \sum_{k=n-1}^{n+1} \{a_1 x_1[k] + a_2 x_2[k]\}$$

$$= a_1 y_1[k] + a_2 y_2[k]$$

(b) Time-Invariant

$$y[n-n_0] = \sum_{k=n-n_0-1}^{n-n_0+1} x(k)$$

$$z[n] = \sum_{k=n-1}^{n+1} x[k-n_0] \qquad l = k-n_0$$

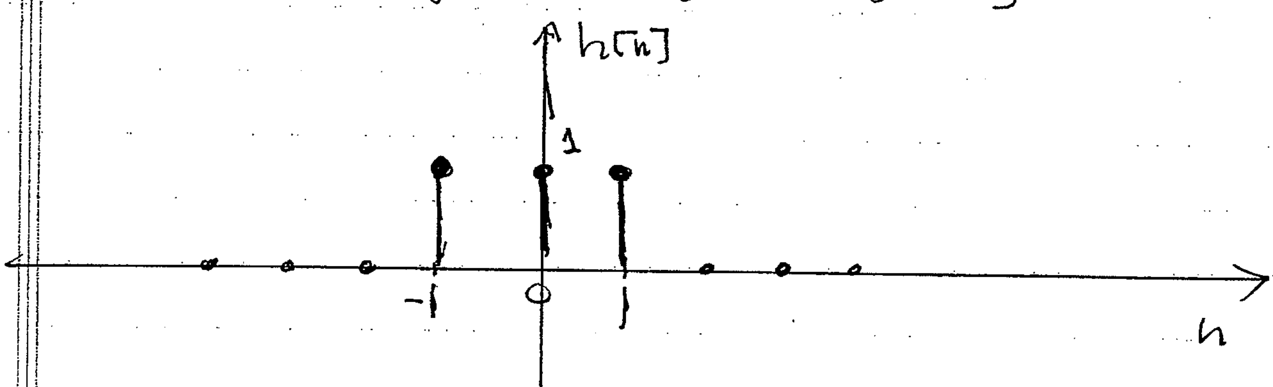
$$= \sum_{l=n-n_0-1}^{n-n_0+1} x[l] = y[n-n_0]$$

(c) Noncausal: $y[n]$ depends on $x[n+1]$

Sol'n to Prob. 2 (cont.)

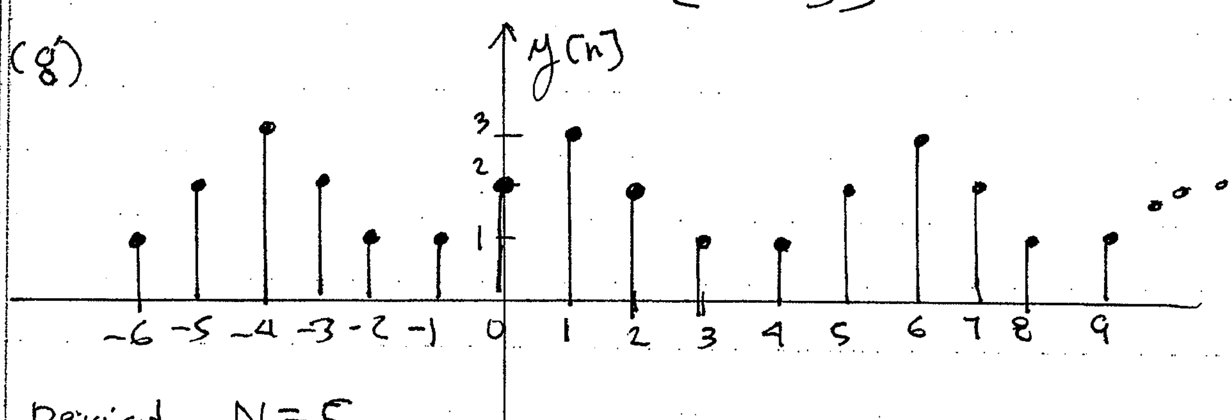
(d) System is stable.
As long as $|x[n]| < \infty$ for all n,
$|y[n]| < \infty$

(e)
$$h[n] = \sum_{k=n-1}^{n+1} \delta[k]$$
$$= \delta[n-1] + \delta[n] + \delta[n+1]$$



(f)
$$a_k = \frac{1}{5} \frac{\sin\left[k\pi \frac{3}{5}\right]}{\sin\left[k\pi \frac{1}{5}\right]} e^{-j\frac{2\pi k}{5}(1)}$$

(g)



period $N=5$

Three different ways to get F.S. coefficients for $y[n]$ denoted b_k

Method 1: Shift to left by one yields real and even-symmetry

$$b'_k = \frac{1}{5}\sum_{n=-2}^{2} y'[n]\, e^{-j\frac{2\pi k}{5}n} \qquad \text{where } y'[n] = y[n+1]$$

$$= \frac{1}{5}\begin{cases} 3 & (n=0) \\ +2e^{j\frac{2\pi k}{5}} + 2e^{-j\frac{2\pi k}{5}} & (n=-1 \text{ and } n=1) \\ + e^{j\frac{4\pi k}{5}} + e^{-j\frac{4\pi k}{5}} & (n=-2 \text{ and } n=2) \end{cases}$$

$$= \frac{1}{5}\left\{ 3 + 4\cos\left(\frac{2\pi k}{5}\right) + 2\cos\left(\frac{4\pi k}{5}\right)\right\}$$

Since $y[n] = y'[n-1]$

$$b_k = \frac{1}{5}\left\{3 + 4\cos\left(2\pi\frac{k}{5}\right) + 2\cos\left(4\pi\frac{k}{5}\right)\right\} e^{-j\frac{2\pi k}{5}(1)}$$

Method 2: Find frequency response

when $x[n] = e^{j\omega_0 n}$

$$y[n] = e^{j\omega_0(n-1)} + e^{j\omega_0 n} + e^{j\omega_0(n+1)}$$

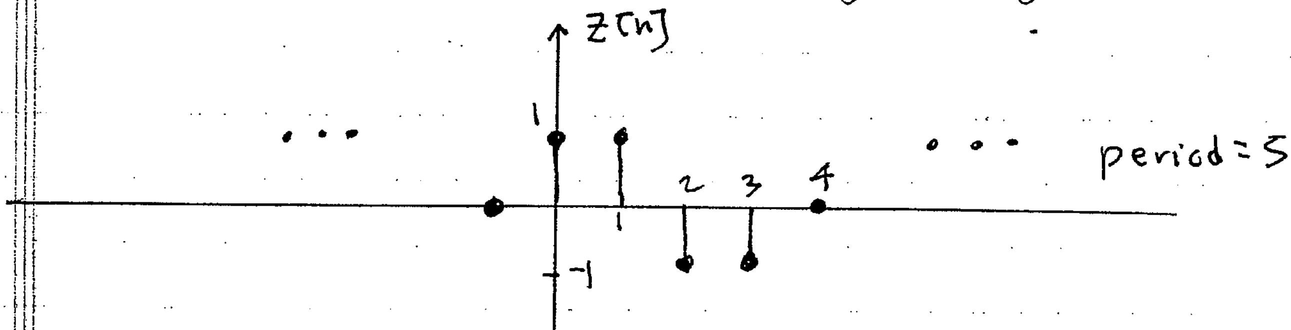
$$= e^{j\omega_0 n}\left\{e^{-j\omega_0} + 1 + e^{+j\omega_0}\right\}$$

Thus:

$$y[n] = \sum_{k=-2}^{2} a_k \left\{ e^{-j\frac{2\pi k}{5}} + 1 + e^{j\frac{2\pi k}{5}} \right\} e^{j\frac{2\pi k}{5} n}$$

$$b_k = \frac{1}{5} \frac{\sin[k\pi \frac{3}{5}]}{\sin[k\pi \frac{1}{5}]} e^{-j\frac{2\pi k}{5}} \left\{ 1 + 2\cos\left(\frac{2\pi k}{5}\right) \right\}$$

<u>Method 3:</u> Create $z[n] = y[n] - y[n-1]$



Can't use formula from text since "turned on" for even no. over period

Think: $\delta[n] + \delta[n-1]$ repeated every $N=5$

$$e_k = \frac{1}{5} + \frac{1}{5} e^{-j\frac{2\pi k}{5}(1)}$$

$$b_k = \frac{1}{1 - e^{-j\frac{2\pi k}{5}}} \left\{ \frac{1}{5} + \frac{1}{5} e^{-j\frac{2\pi k}{5}} - \left(\frac{1}{5} + \frac{1}{5} e^{-j\frac{2\pi k}{5}} \right) e^{-j\frac{2\pi k (2)}{5}} \right\}$$

Sol'n to Prob. 2 (h)

$$w[n] = x[n] - x[n-1]$$

From Table 3.2

$$c_k = b_k \left\{ 1 - e^{-j 2\pi \frac{k}{5}} \right\}$$