SOLUTION

NAME:
EE301 Signals and Systems

23 February 2017
Exam 1

Cover Sheet

Test Duration: 75 minutes.
Coverage: Chaps. 1,2
Open Book but Closed Notes.
One 8.5 in. x 11 in. crib sheet
Calculators NOT allowed.
DO NOT UNSTAPLE THE EXAM!
All work should be done in the space provided.
You must show ALL work or explain answer for each problem to receive full credit.

Prob. No.	Topic(s)	Points
1.	Continuous Time Signals and System Properties	50
2.	Discrete Time Signals and System Properties	50

VIP If you want to refer to the input signal and output signal for one part of a problem when solving a later part, use that part's letter as a subscript, e.g., you can refer to the input signal and corresponding output signal for part (d) of Prob. 1 as $x_d(t)$ and $y_d(t)$, respectively.

VIP: Solving and part, you can just write: z(t)= Formula A with a=-3 and b =-2 BUT don't have to write out Formula A substituting a=-3 and b=-2. Just use z(t) for remainder of your solution.

Formula A: $$e^{at}u(t) * e^{bt}u(t) = \frac{1}{a-b}e^{at}u(t) + \frac{1}{b-a}e^{bt}u(t) \quad (1)$$

Formula B: $$\alpha^n u[n] * \beta^n u[n] = \frac{\alpha}{\alpha-\beta}\alpha^n u[n] + \frac{\beta}{\beta-\alpha}\beta^n u[n] \quad (2)$$

Formula C: if $x(t) * h(t) = y(t)$ then: $a\, x(t-t_1) * b\, h(t-t_2) = ab\, y(t-(t_1+t_2))$ (3)

Formula D: if $x[n] * h[n] = y[n]$ then: $a\, x[n-n_1] * b\, h[n-n_2] = ab\, y[n-(n_1+n_2)]$ (4)

Formula E: if $x(t) * h(t) = y(t)$ then: $x(t-t_0) * h(t) = y(t-t_0)$ and $x(t) * h(t-t_0) = y(t-t_0)$ (5)

Formula F: if $x[n] * h[n] = y[n]$ then: $x[n-n_0] * h[n] = y[n-n_0]$ and $x[n] * h[n-n_0] = y[n-n_0]$ (6)

Prob. 1. [50 pts] Consider the LTI system characterized by the I/O relationship:

$$\text{System 1:} \quad y(t) = \int_{-\infty}^{t} e^{-2(t-\tau)} x(\tau) d\tau$$

(a) Write the impulse response of the system, $h_1(t)$.

$$h_1(t) = e^{-2t} u(t)$$

(b) Is the system causal? Justify your answer using the impulse response.

Yes. $h_1(t) = 0$ for $t < 0$

(c) Is the system stable? Justify your answer using the impulse response.

Yes. $\int_{-\infty}^{\infty} |h(t)| dt = \int_{0}^{\infty} e^{-2t} dt$

$$= \frac{1}{-2} e^{-2t} \Big]_0^{\infty} = -\frac{1}{2}(0 - 1) = \frac{1}{2} < \infty$$

(d) **You should solve each part of this problem using known convolution results in conjunction with linearity (homogeneity and superposition) and time-invariance. You do not need to simplify your answers.** Determine and write a closed-form expression for the output ($y(t)$) of System 1 for the input

$$x(t) = e^{-4t}u(t)$$

$$y(t) = e^{-4t}u(t) * e^{-2t}u(t)$$

$= \text{Formula A with } a = -4 \text{ and } b = -2$

$$= y_d(t)$$

(e) Determine a closed-form expression for the output ($y(t)$) of System 1 for the input

$$x(t) = 3e^{-4t}u(t-2)$$

$$x_e(t) = 3\,e^{-4(t-2)}u(t-2) \underset{\text{times}}{\times} e^{-8}$$

$$= 3e^{-8}\, x_d(t-2)$$

LTI:

$$y_e(t) = 3e^{-8}\, y_d(t-2)$$

(f) Determine and write a closed-form expression for the output $y(t)$ of System 1 for the input

$$x(t) = \{u(t) - u(t-3)\}$$

$$y(t) = \{u(t) - u(t-3)\} * e^{-2t} u(t)$$

$$= \underbrace{e^{-2t} u(t) * u(t)}_{z(t)} - \underbrace{e^{-2t} u(t) * u(t-3)}_{z(t-3)}$$

$z(t)$ = Formula A with $a = -2$ and $b = 0$

$$y_f(t) = z(t) - z(t-3)$$

(g) Determine a closed-form expression for the output $(y(t))$ of System 1 for the input

$$x(t) = 5e^{-4t}u(t-3) - 3\{u(t-3) - u(t-6)\}$$

$$x_g(t) = 5e^{-12} e^{-4(t-3)} u(t-3) - 3x_f(t-3)$$

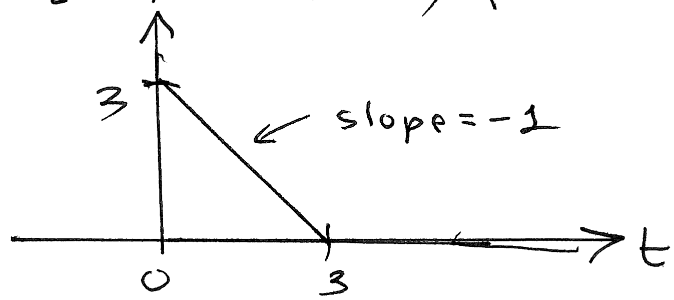
$$= 5e^{-12} x_d(t-3) - 3x_f(t-3)$$

$$= 5e^{-12} y_d(t-3) - 3y_f(t-3)$$

(h) Consider a second LTI system described by the following difference equation. Determine the impulse response for System 2, denoted $h_2(t)$.

$$\text{System 2:} \qquad y(t) = -\int_{t-3}^{t} (t - \tau - 3)x(\tau)d\tau$$

$$h_2(t) = -(t-3)\{u(t) - u(t-3)\}$$

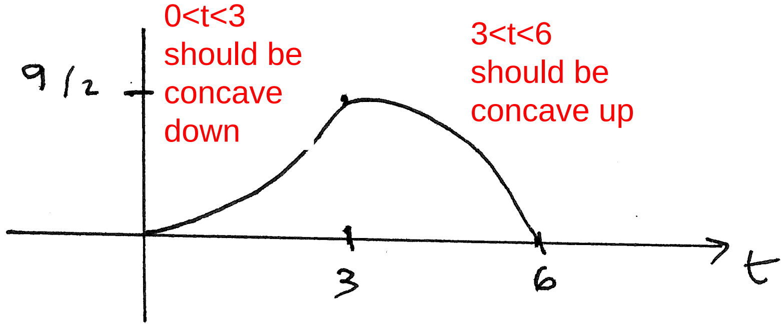


(i) Determine and plot the output $y(t)$ when the input is

$$x(t) = u(t) - u(t-3)$$

$$y(t) = \{u(t) - u(t-3)\} * -(t-3)\{u(t) - u(t-3)\}$$

$T_1 = T_2 = 3$ no linear part

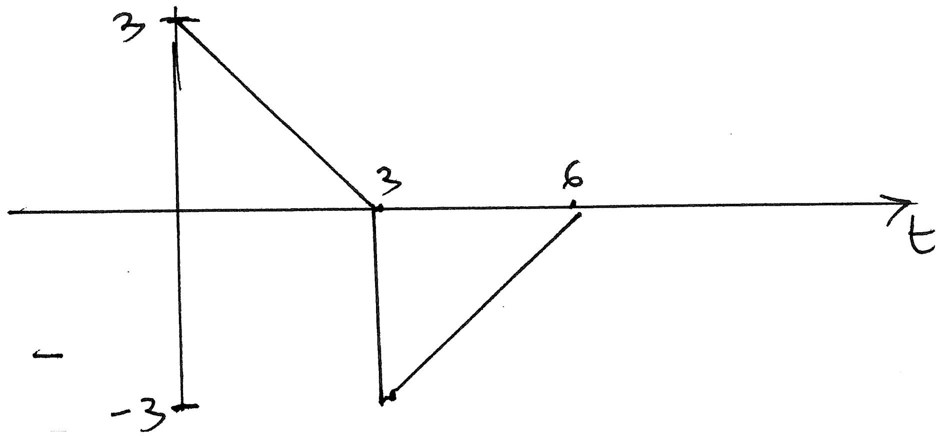


(j) Determine and plot for the output of System 2, $y(t)$, for the input

$$x(t) = \delta(t) - \delta(t-3)$$

$$y_i(t) = \left(\delta(t) - \delta(t-3)\right) * h_2(t)$$

$$= h_2(t) - h_2(t-3)$$



(k) Determine and write an expression for the output of System 2, $y(t)$, for the input below, in terms of your answer to part (i).

$$x(t) = 4\{u(t) - u(t-3)\} - 5\{u(t-6) - u(t-9)\}$$

$$x_k(t) = 4\,x_i(t) - 5\,x_i(t-6)$$

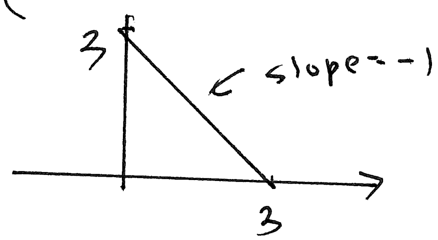
$$y_k(t) = 4\,y_i(t) - 5\,y_i(t-6)$$

(1) System 1 and System 2 are in series along with a third system (all three in series) which is a differentiator as described below. Determine a closed-form expression for the overall impulse response, denoted $h_0(t)$, for the series combination of all three systems. You should invoke the properties of convolution and use Formula A in conjunction with linearity and time-invariance in Formula C.

$$\text{System 3:} \qquad y(t) = \frac{d}{dt}x(t)$$

$$h_0(t) = h_1(t) * h_2(t) * \frac{d}{dt}$$

$$= e^{-2t}u(t) * \left\{ -(t-3)\{u(t) - u(t-3)\} \right\} * \frac{d}{dt}$$



$$= e^{-2t}u(t) * \left\{ -\{u(t) - u(t-3)\} + 3\delta(t) \right\}$$

$$= -y_f(t) + 3e^{-2t}u(t)$$

Problem 2. [50 points] Problem 2 is separate from Problem 1; the first DT system is enumerated as System 1.

(a) Consider the causal LTI System characterized by the difference equation below. Write an expression for the impulse response of this system, denoted $h_1[n]$.

$$\text{System 1:} \quad y[n] = -\frac{2}{3}y[n-1] + x[n] + \frac{8}{27}x[n-3]$$

$y[n] = a\,y[n-1] + x[n] - a^3 x[n-3] \qquad a = -\frac{2}{3}$

$h_1[n] = \left(-\frac{2}{3}\right)^n \{u[n] - u[n-3]\}$

(b) Is the system causal? Justify your answer using the impulse response.

Yes. $h_1[n] = 0$ for $n < 0$

(c) Is the system stable? Justify your answer using the impulse response.

Yes. $\sum_{n=-\infty}^{\infty} |h_1[n]| = \sum_{n=0}^{2} \left(\frac{2}{3}\right)^n = 1 + \frac{2}{3} + \frac{4}{9} < \infty$

(d) Does the system have memory or is it memoryless? Justify your answer using the impulse response.

has memory since $h_1[n] \neq k\delta[n]$

(e) Determine and write a closed-form expression for the output $y[n]$ of System 1 for the input

$$x[n] = 3\left(\frac{1}{3}\right)^n u[n]$$

$$y_e[n] = 3\left\{ \left(-\frac{2}{3}\right)^n u[n] * \left(\frac{1}{3}\right)^n u[n] \right\}$$

$$= 3\left\{ \left(-\frac{2}{3}\right)^3 \left(-\frac{2}{3}\right)^{n-3} u[n-3] * \left(\frac{1}{3}\right)^n u[n] \right\}$$

$$= 3\left(z[n] + \frac{8}{27} z[n-3] \right)$$

where: $z[n]$ = Formula B with: $\alpha = -2/3$, $\beta = 1/3$

Rewriting:

$$y_e[n] = 3\left(z[n] + \frac{8}{27} z[n-3] \right)$$

(f) Determine a closed-form expression for the output $y[n]$ of System 1 when the input is

$$x[n] = 3\left(\frac{1}{3}\right)^n u[n-3]$$

$$x_f[n] = \left(\frac{1}{3}\right)^3 3 \left(\frac{1}{3}\right)^{n-3} u[n-3]$$

$$= \frac{1}{27} x_e[n]$$

$$y_f[n] = \frac{1}{27} y_e[n-3]$$

$$= \frac{1}{27} y_e[n-3]$$

(g) Determine and write a closed-form expression for the output $y[n]$ of System 1 for the input

$$x[n] = 12\left(\frac{1}{3}\right)^n \{u[n] - u[n-3]\}$$

$$x_g[n] = 4\, x_e[n] - 4\, x_f[n]$$

$$y_g[n] = 4\{ y_e[n] - y_f[n] \}$$

(h) Determine a closed-form expression for the output $y[n]$ of System 1 when the input is

$$x[n] = 4\{u[n] - u[n-4]\}$$

$$y_h[n] = 4 \times \left\{ (u[n] - u[n-4]) * \left(-\frac{2}{3}\right)^n (u[n] - u[n-3]) \right\}$$

$= \quad u[n] * \left(-\frac{2}{3}\right)^n u[n] = z[n] =$ Formula B with $\alpha = 1$ and $\beta = -\frac{2}{3}$

$4 \times \quad -u[n] * \left(-\frac{2}{3}\right)^3 \left(-\frac{2}{3}\right)^{n-3} u[n-3] = -\left(-\frac{2}{3}\right)^3 z[n-3]$

$- u[n-4] * \left(-\frac{2}{3}\right)^n u[n] = -z[n-4]$

$+ u[n-4] * \left(-\frac{2}{3}\right)^{n-3} u[n-3] \left(-\frac{2}{3}\right)^3 = \left(-\frac{2}{3}\right)^3 z[n-7]$

$y_h[n] =$

$$4 \left\{ z[n] + \frac{8}{27} z[n-3] - z[n-4] - \frac{8}{27} z[n-7] \right\}$$

(i) Consider a second LTI system described by the following difference equation. Determine the impulse response for System 2, denoted $h_2[n]$. You can write your answer in sequence form, using an arrow to denote the $n = 0$ value.

$$\text{System 2:} \quad y[n] = -y[n-1] + x[n] + x[n-5]$$

$$h_2[n] = (-1)^n \{u[n] - u[n-5]\}$$

$$= \{\underset{\substack{\uparrow \\ n=0}}{1}, -1, 1, -1, 1\}$$

$$y[n] = a\,y[n-1] + x[n] - a^5 x[n-5] \qquad a = -1$$

(j) Determine and write a closed-form expression for the output of System 2 ,$y[n]$, for the input below. Write answer in sequence form, using an arrow to denote the $n = 0$ value.

$$x[n] = \{u[n] - u[n-3]\}$$

$$y_j[n] = \underbrace{\left(u[n] - u[n-3]\right)}_{\{\underset{\substack{\uparrow \\ n=0}}{1}, 1, 1\}} * \{\underset{\substack{\uparrow \\ n=0}}{1}, -1, 1, -1, 1\}$$

length: $3 + 5 - 1 = 7 \} \quad n = 0, 1, \ldots, 6$

Table Method:

n	0	1	2	3	4	5	6
	1	-1	1	-1	1	0	0
	0	1	-1	1	-1	1	0
	0	0	1	-1	1	-1	1
sum columns	1	0	1	-1	1	0	1

$$y_j[n] = \{\underset{\substack{\uparrow \\ n=0}}{1}, 0, 1, -1, 1, 0, 1\}$$

(k) Determine and write a closed-form expression for the output of System 2, $y[n]$, for the input below. Write answer in sequence form, using an arrow to denote the $n = 0$ value. So, for this problem, you are required to list the numbers that comprise the output signal in sequence form.

$$x[n] = -2\{u[n] - u[n-3]\} + 3\{u[n-5] - u[n-8]\}$$

$$x_k[n] = -2\, x_j[n] + 3\, x_j[n-5]$$

$$y_k[n] = -2\, y_j[n] + 3\, y_j[n-5]$$

$$\underset{\underset{n=0}{\uparrow}}{-2},\ 0,\ -2,\ 2,\ -2,\ \underset{\underset{n=5}{\uparrow}}{0},\ -2$$

$$3,\ 0,\ 3,\ -3,\ 3,\ 0,\ 3$$

$$y_k[n] = \{\underset{\underset{n=0}{\uparrow}}{-2},\ 0,\ -2,\ 2,\ -2,\ 3,\ -2,\ 3,\ -3,\ 3,\ 0,\ 3\}$$

(l) Determine a closed-form expression for the output $y[n]$ of System 2 when the input is

$$x[n] = 3\left(\frac{1}{2}\right)^n u[n]$$

$$y_l[n] = 3 \times \left(\frac{1}{2}\right)^n u[n] * (-1)^n \{u[n] - u[n-5]\}$$

$$= 3 \times \Big\{ \underbrace{\left(\frac{1}{2}\right)^n u[n] * (-1)^n u[n]}_{z[n] \,=\, \text{Formula B with } \alpha = 1/2 \text{ and } \beta = -1} - \left(\frac{1}{2}\right)^n u[n] * \underset{-1}{\wedge} (-1)^{n-5} u[n-5] \Big\}$$

$$y_l[n] = 3\{z[n] + z[n-5]\}$$

(m) Determine the output, $y[n]$, when $x[n]$ below

$$x[n] = (n+1)\{u[n] - u[n-4]\} = \{1, 2, 3, 4\}$$

input to two LTI systems in SERIES, with respective impulse responses indicated below. Write your answer in sequence form indicating with an arrow which value corresponds to $n = 0$. You can use next page as well as the space below to show all your work.

$$h_1[n] = 8\left(-\frac{1}{2}\right)^n \{u[n] - u[n-4]\} \qquad h_2[n] = \delta[n] - \delta[n-1]$$

$y[n] = x[n] * h_1[n] * h_2[n]$

$= h_1[n] * \left(x[n] * h_2[n]\right)$

$x[n] * h_2[n] = \{\underset{\uparrow}{1}, -1\} * \{\underset{\uparrow}{1}, 2, 3, 4\} = z[n]$

Table Method

n	0	1	2	3	4
	1	2	3	4	0
	0	−1	−2	−3	−4
$z[n]$	1	1	1	1	−4

$y[n] = z[n] * \{\underset{\substack{\uparrow \\ n=0}}{8}, -4, 2, \underset{\substack{\uparrow \\ n=3}}{-1}\}$

length
$5 + 4 - 1 = 8$
$n = 0, 1, \ldots, 7$

n	0	1	2	3	4	5	6	7
$8z[n]$	8	8	8	8	−32	0	0	0
$-4z[n-1]$	0	−4	−4	−4	−4	16	0	0
$2z[n-2]$	0	0	2	2	2	2	−8	0
$-z[n-3]$	0	0	0	−1	−1	−1	−1	4
$y[n]$	8	4	6	5	−35	17	−9	4

$\uparrow$
$n=0$

(additional space for work for part (m) if needed: