NAME:

EE301 Signals and Systems

16 February 2012

Exam 1

Cover Sheet

Test Duration: 75 minutes.
Coverage: Chaps. 1,2
Open Book but Closed Notes.
One 8.5 in. x 11 in. crib sheet
Calculators NOT allowed.
This test contains **two** problems.
All work should be done on the sheets provided.
You must show work or explain answer for each problem to receive full credit.
Plot your answers on the graphs provided.
WRITE YOUR NAME ON EVERY SHEET.

Prob. No.	Topic(s)	Points
1.	Continuous Time Signals and System Properties	50
2.	Discrete Time Signals and System Properties	50

$$\begin{aligned}\{u(t)-u(t-T_1)\} * t\{u(t)-u(t-T_2)\} &= \frac{t^2}{2}\{u(t)-u(t-T_1)\} \\ &+ \left(T_1 t - \frac{T_1^2}{2}\right)\{u(t-T_1)-u(t-T_2)\} \\ &+ \left(-\frac{t^2}{2} + T_1 t + \frac{T_2^2 - T_1^2}{2}\right)\{u(t-T_2)-u(t-(T_1+T_2))\}\end{aligned} \tag{1}$$

Problem 1. [50 pts] Consider the LTI system characterized by the following I/O relationship:

$$y(t) = \int_{t-1}^{t} x(\tau)d\tau \tag{1}$$

(a) Determine and plot the impulse response of System 1, denoted $h(t)$, in the spaced provided on the sheets attached.

(b) Determine and plot the output $y_1(t)$ in the space provided when the input to the overall system is the ramp-up triangular pulse of duration 2 seconds below.

$$x_1(t) = 2t\{u(t) - u(t-2)\}$$

(c) Determine and plot the output $y_2(t)$ in the space provided when the input to the overall system is the ramp-down triangular pulse of duration 2 seconds below. On the same page as the plot, express $y_2(t)$ in terms of $y_1(t)$. Note: $x_2(t) = x_1(-(t-2))$.

$$x_2(t) = -2(t-2)\{u(t) - u(t-2)\}$$

(d) Determine and plot the output $y_3(t)$ in the space provided when the input to the overall system is the rectangular pulse below:

$$x_3(t) = 4\{u(t) - u(t-2)\}$$

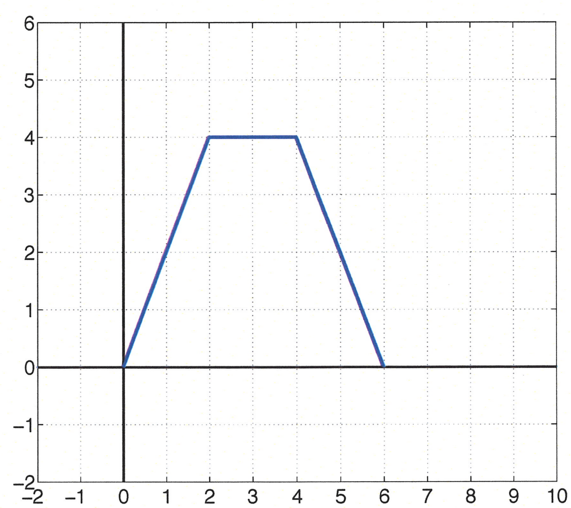
(e) The goal is to determine and plot the output $y(t)$ when the input to the overall system is the signal $x(t)$ below.

(i) Express $x(t)$ in terms of $x_i(t)$, $i = 1, 2, 3$, defined in parts (b), (c), (d), respectively.

(ii) Express $y(t)$ in terms of $y_i(t)$, $i = 1, 2, 3$, your answers to parts (b), (c), (d), respectively.

(iii) Plot $y(t)$ in the space provided on the sheets attached.

Input $x(t)$



Problem 2. [50 points]

(a) For parts (a) and (b), consider causal LTI System 1 characterized by the following difference equation below. Determine and plot (stem plot) the impulse response $h_1[n]$ of System 1 in the space provided on the sheets attached.

$$\text{System 1:} \quad y[n] = \frac{1}{2}y[n-1] + x[n] - \left(\frac{1}{2}\right)^4 x[n-4]$$

(b) Determine the output $y[n]$ of System 1 when the input is the finite-length geometric sequence below. Plot $y[n]$ in terms of a stem plot.

$$x[n] = 4(2)^n \{u[n] - u[n-4]\}$$

(c) For this part and part (d), consider causal LTI System 2 characterized by the difference equation below. Determine and plot the impulse response $h_2[n]$ of System 1 in the space provided on the sheets attached.

$$\text{System 2:} \qquad y[n] = y[n-1] + x[n] - x[n-3]$$

(d) Determine the output $y[n]$ when the input is the finite-length geometric sequence below. Plot $y[n]$ in the space provided on the sheets attached.

$$x[n] = \{u[n] - u[n-5]\}$$

(e) Consider that System 1 and System 2 are put in SERIES such that the output of System 1 is the input to System 2. Determine the impulse response $h[n]$ for the overall series combination and plot it in the space provided on the sheets attached.

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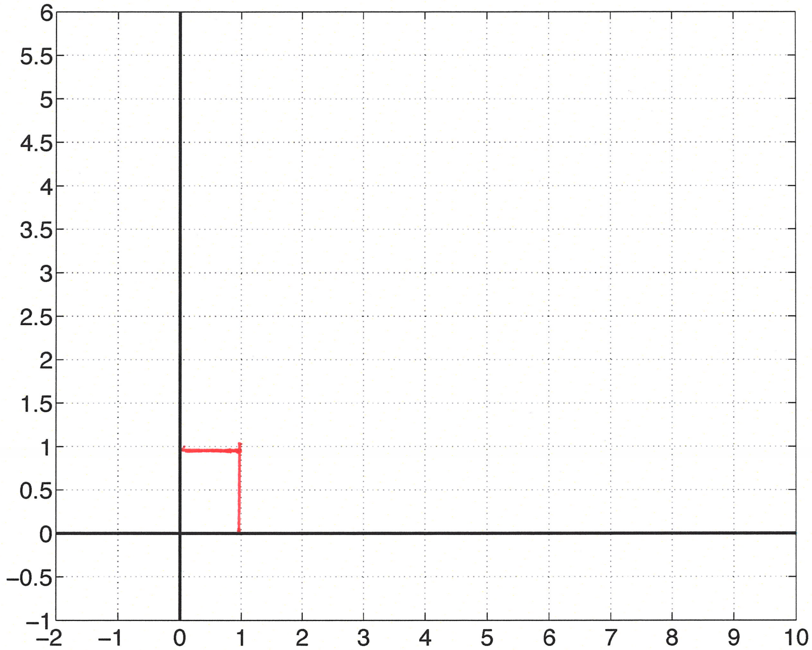
Write your expression for $h(t)$ and plot it, for Problem 1 (a) here.



$h(t) = u(t) - u(t-1)$

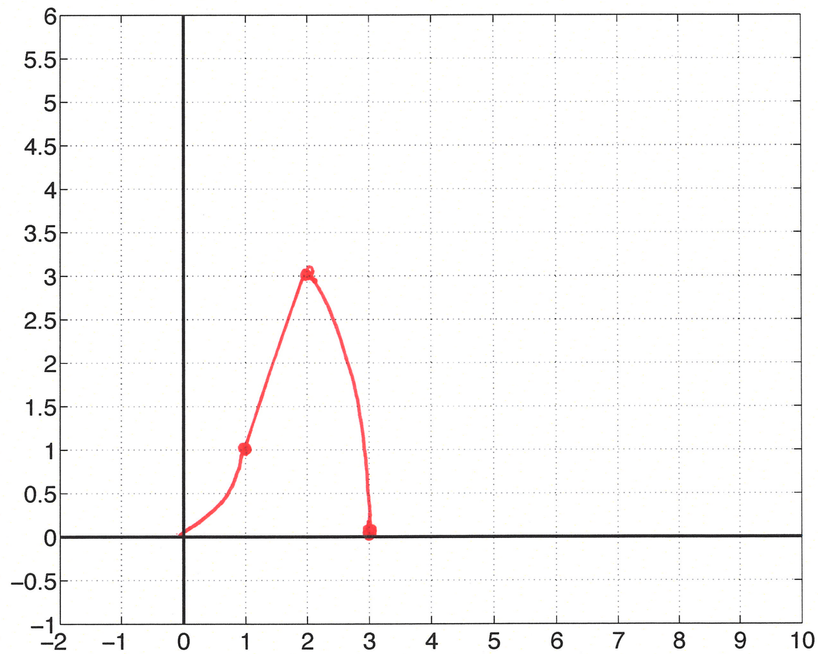
1(b): For each value of t, write the value of $y_1(t)$ in the table below.

t	$y_1(t)$
$t = 0$	0
$t = 1$	1
$t = 2$	3
$t = 3$	0
$t = 4$	0

Mark the correct box with an X for each range for $y_1(t)$.

Range for t	Linear pos. slope	Linear neg. slope	Quadratic Concave Up	Quadratic Concave Down
$0 < t < 1$			X	
$1 < t < 2$	X			
$2 < t < 3$				X

Plot $y_1(t)$ below.



1(c): Write a simple expression for $y_2(t)$ in terms of $y_1(t)$.

1(c): For each value of t, write the value of $y_2(t)$ in the table below.

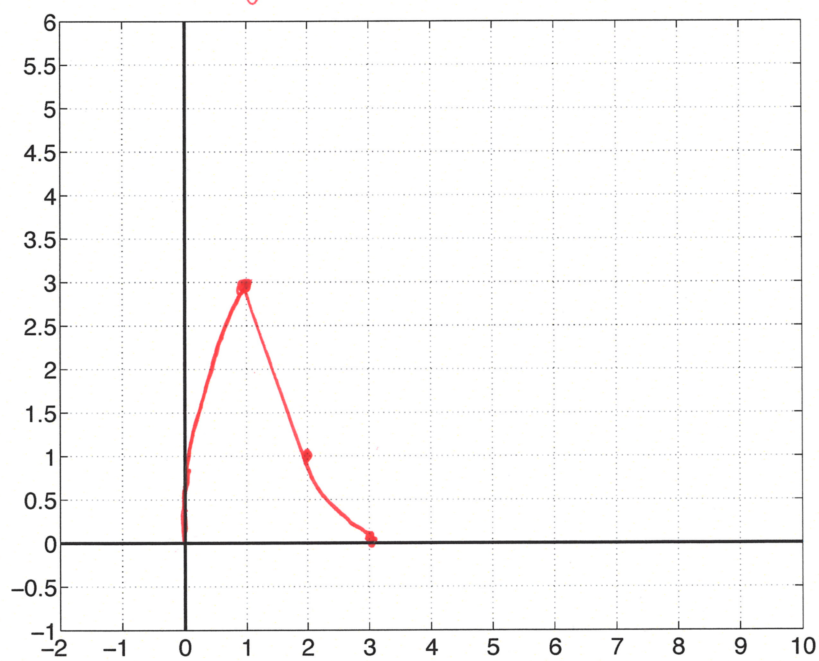
t	$y_2(t)$
$t = 0$	0
$t = 1$	3
$t = 2$	1
$t = 3$	0
$t = 4$	0

Mark the correct box with an for each range for $y_2(t)$.

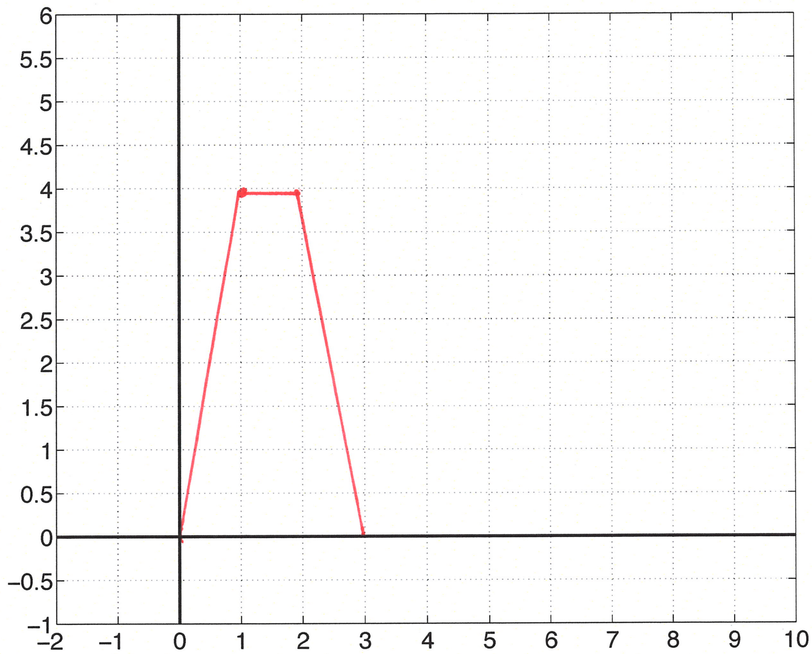
Range for t	Linear pos. slope	Linear neg. slope	Quadratic Concave Up	Quadratic Concave Down
$0 < t < 1$				X
$1 < t < 2$		X		
$2 < t < 3$			X	

Plot $y_2(t)$ below.

$y_2(t) = y_1(-(t-3))$



Plot $y_3(t)$ for Problem 1 (d) here.



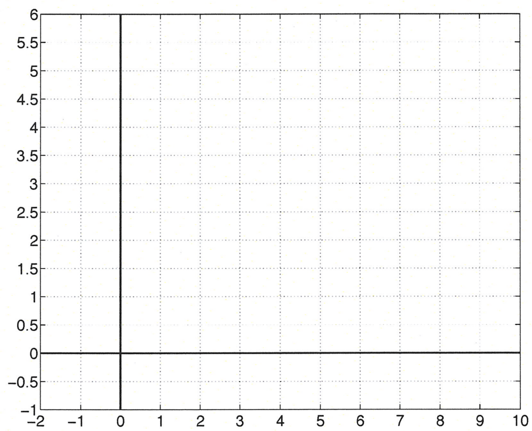
1 (e). Express $x(t)$ in terms of $x_i(t)$, $i = 1, 2, 3$.

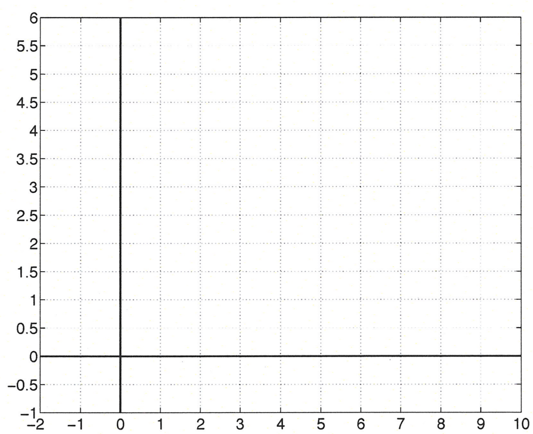
$x(t) = x_1(t) + x_2(t-4) + x_3(t-2)$

1 (e). Express $y(t)$ in terms of $y_i(t)$, $i = 1, 2, 3$.

$y(t) = y_1(t) + y_2(t-4) + y_3(t-2)$

You can use the plots below if they're helpful for answering 1(e).





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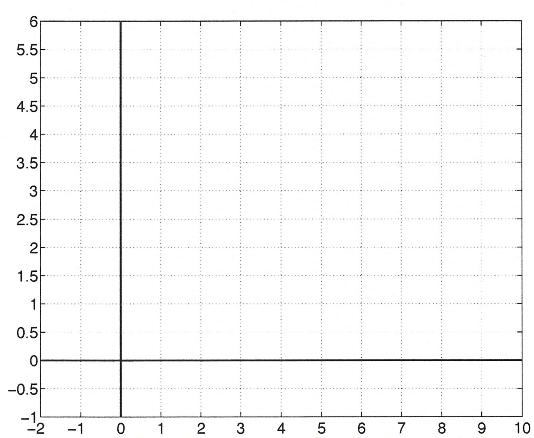
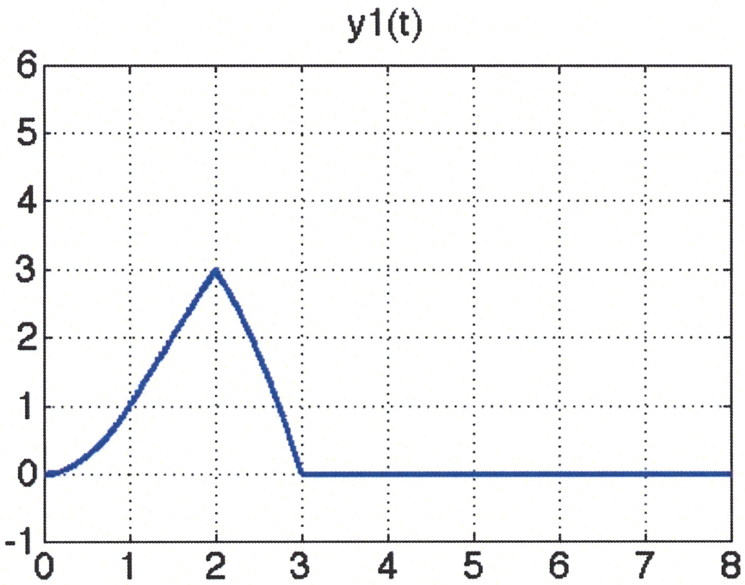
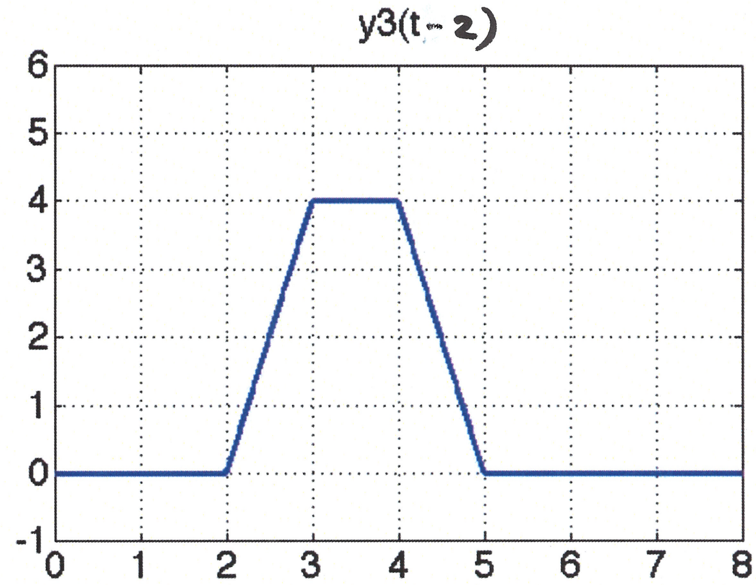
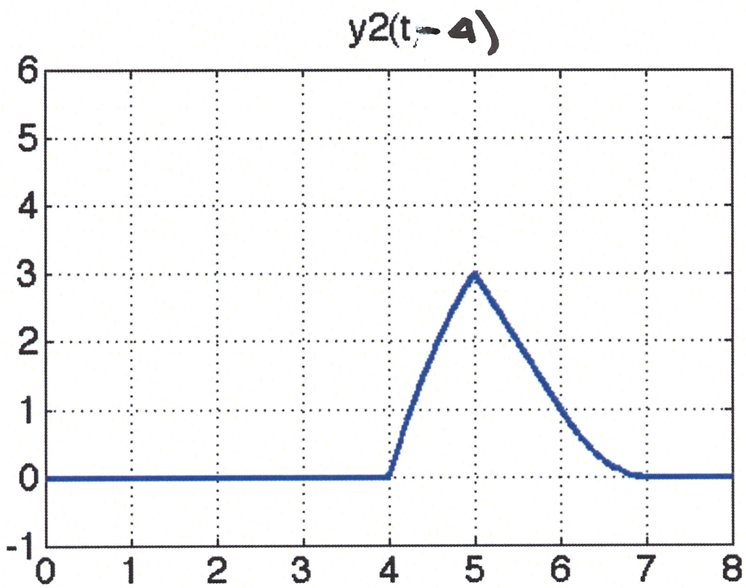
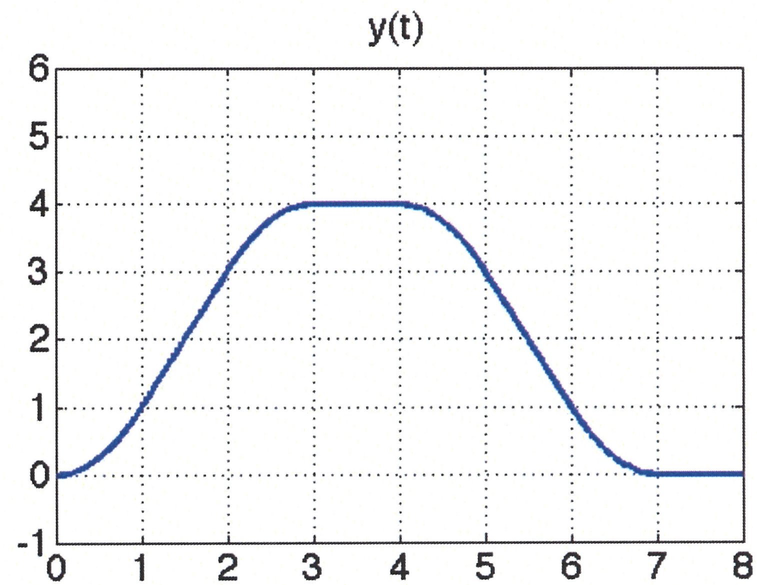



y1(t)
6 5 4 3 2 1 0 -1
0 1 2 3 4 5 6 7 8


y3(t- 2)
6 5 4 3 2 1 0 -1
0 1 2 3 4 5 6 7 8


y2(t- 4)
6 5 4 3 2 1 0 -1
0 1 2 3 4 5 6 7 8


y(t)
6 5 4 3 2 1 0 -1
0 1 2 3 4 5 6 7 8

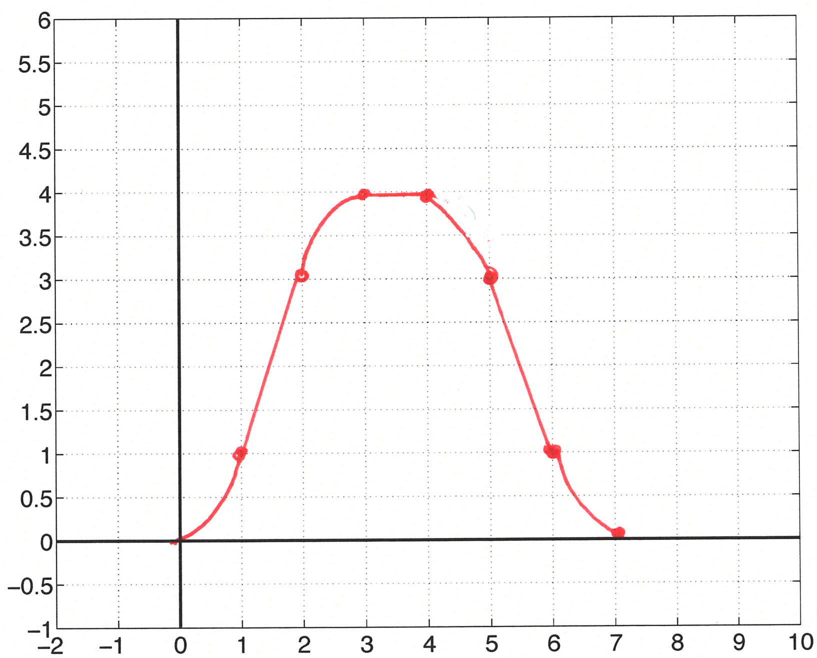
Part (e). For each range of t, put an X in the correct box in the table below.

Range for t	Linear pos. slope	Linear neg. slope	Quadratic Concave Up	Quadratic Concave Down
$0 < t < 1$			X	
$1 < t < 2$	X			
$2 < t < 3$				X
$3 < t < 4$	ze	ro		
$4 < t < 5$				X
$5 < t < 6$		X		
$6 < t < 7$			X	

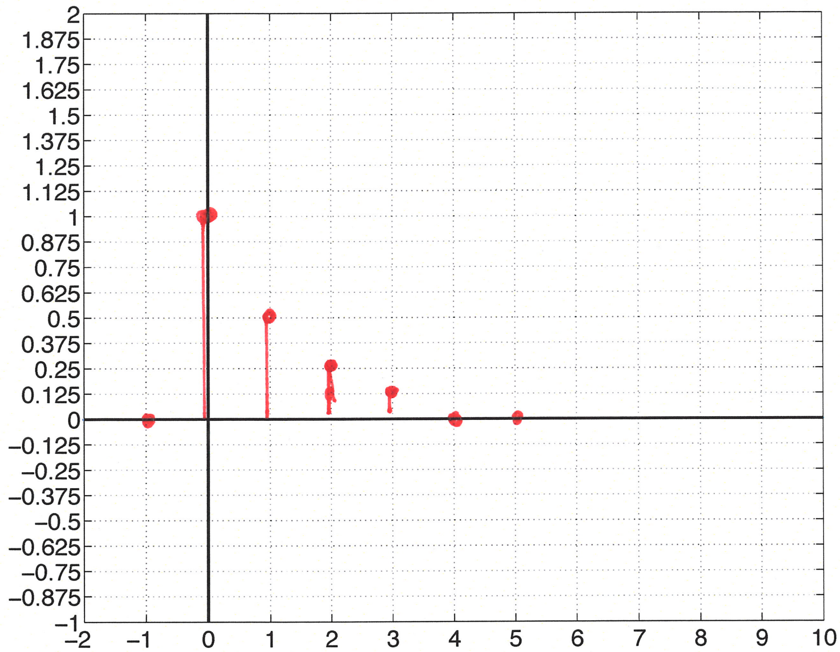
For each value of t, write the value of $y(t)$ in the table below.

t	$y(t)$
$t = 0$	
$t = 1$	
$t = 2$	
$t = 3$	
$t = 4$	
$t = 5$	
$t = 6$	
$t = 7$	

Plot $y(t)$ for Prob1, Part (e) below.



Plot your answer $h_1[n]$ to Problem 2, part (a) on this page.



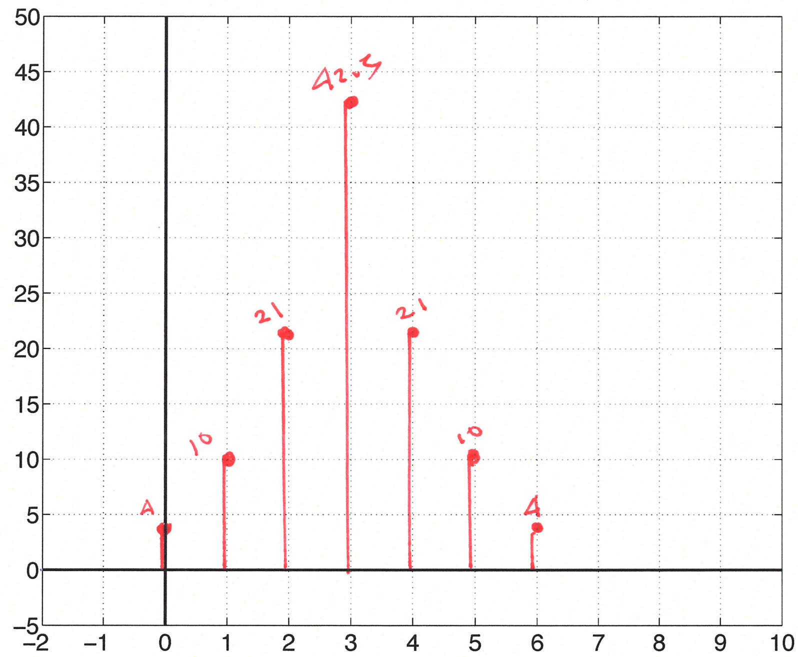
(a) $h_1[n] = \left(\frac{1}{2}\right)^n \{u[n] - u[n-4]\}$

Example of

$y[n] = a\,y[n-1] + x[n] - a^D x[n-D]$

with $a = \frac{1}{2}$ and $D = 4$

Show your work and plot your answer to Problem 2, part (b) on this page.



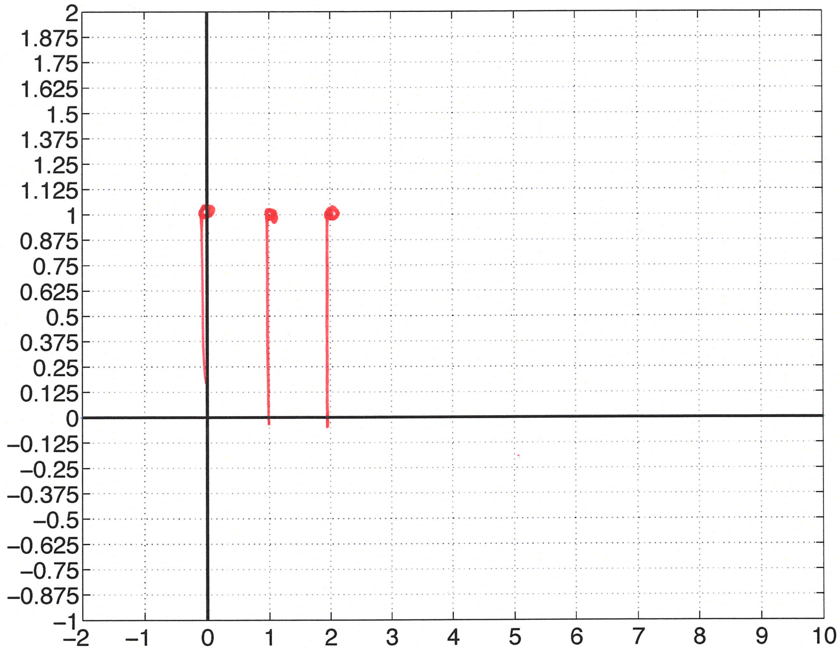
$x[n] = 4 \times \{1, 2, 4, 8\}$

$= \{4, 8, 16, 32\}$

$h[n] = \{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}\}$

$y[n] = 4h[n]$	4	2	1	0.5	0	0	0
$+ 8h[n-1]$	0	8	4	2	1	0	0
$+ 16h[n-2]$	0	0	16	8	4	2	0
$+ 32h[n-3]$	0	0	0	32	16	8	4
	4	10	21	42.5	21	10	4
$n =$	0	1	2	3	4	5	6

Show your work and plot your answer $h_2[n]$ to Problem 2, part (c) on this page.



$a = 1$ $\quad h[n] = u[n] - u[n-3]$

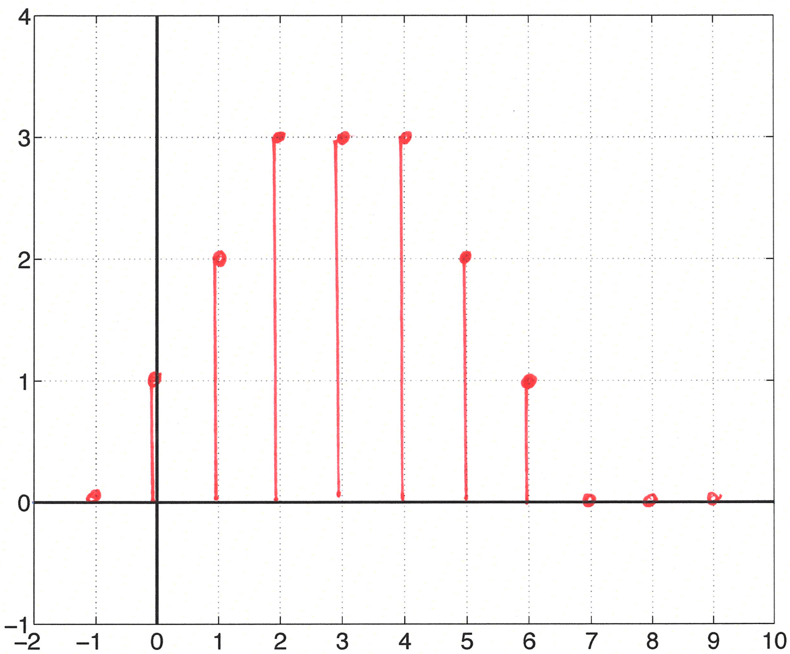
example of

$y[n] = a\, y[n-1] + x[n] - a^D x[n-D]$

with $a = 1$ and $D = 3$

$a^n = 1^n = 1$ for all n

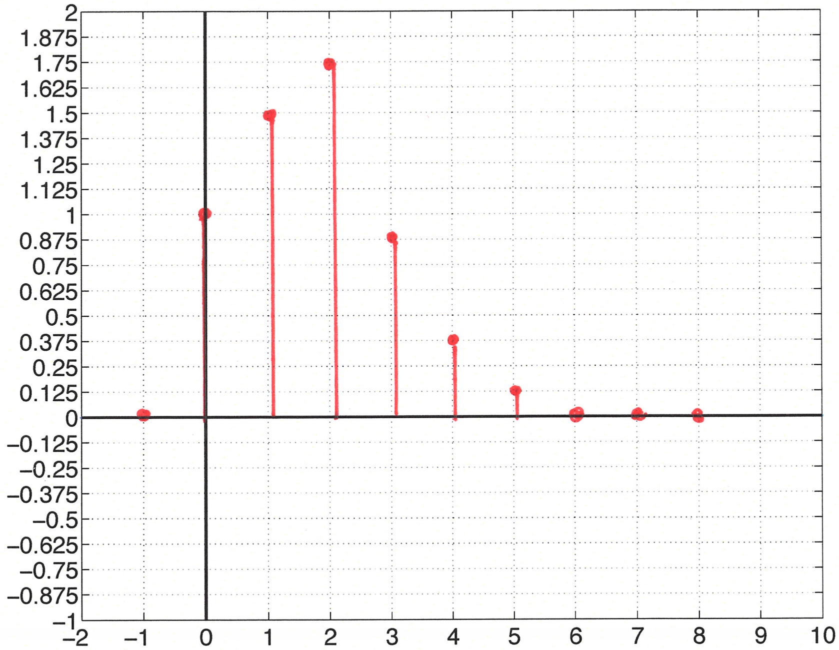
Show your work and plot your answer $y[n]$ to Problem 2, part (d) on this page.



$$\{u[n] - u[n-3]\} * \{u[n] - u[n-5]\}$$

$N_1 = 3$ $\qquad\qquad$ $N_2 = 5$

Show your work and plot your answer $h[n]$ to Problem 2, part (e) on this page.



$$h[n] = h_1[n] * h_2[n]$$

$$\left(\frac{1}{2}\right)^n \{u[n] - u[n-4]\} * \{u[n] - u[n-3]\}$$

$h_1[n]$	1	1/2	1/4	1/8	0	0
$h_1[n-1]$	0	1	1/2	1/4	1/8	0
$h_2[n-2]$	0	0	1	1/2	1/4	1/8
	1	$\frac{3}{2}$	$1\frac{3}{4}$	$\frac{7}{8}$	$\frac{3}{8}$	$\frac{1}{8}$