**NAME:** **26 February 2015**
**EE301 Signals and Systems** **Exam 1**

# Cover Sheet

Test Duration: 75 minutes.
Coverage: Chaps. 1,2
Open Book but Closed Notes.
One 8.5 in. x 11 in. crib sheet
Calculators NOT allowed.
This test contains **TWO** problems.
All work should be done on the sheets provided.
You must show ALL work or explain answer for each problem to receive full credit.
Plot your answers on the graphs provided.
**WRITE YOUR NAME ON EVERY SHEET.**

| Prob. No. | Topic(s) | Points | No. of Parts |
|---|---|---|---|
| 1. | Continuous Time Signals and System Properties | 50 | 5 parts (a)-(e) |
| 2. | Discrete Time Signals and System Properties | 50 | 4 parts (a)-(d) |

$$\begin{aligned} y_1(t) = \{u(t)-u(t-T_1)\} * t\{u(t)-u(t-T_2)\} &= \frac{t^2}{2}\{u(t)-u(t-T_1)\} \\ &+ \left(T_1 t - \frac{T_1^2}{2}\right)\{u(t-T_1)-u(t-T_2)\} \\ &+ \left(-\frac{t^2}{2} + T_1 t + \frac{T_2^2 - T_1^2}{2}\right)\{u(t-T_2)-u(t-(T_1+T_2))\} \end{aligned} \tag{1}$$

$$\begin{aligned} \{u(t)-u(t-T_1)\} * [-(t-T_2)\{u(t)-u(t-T_2)\}] &= \left(-\frac{t^2}{2} + T_2 t\right)\{u(t)-u(t-T_1)\} \\ &+ \left(-T_1 t + \frac{2T_1T_2 + T_1^2}{2}\right)\{u(t-T_1)-u(t-T_2)\} \\ &+ \left(\frac{t^2}{2} - (T_1+T_2)t + \frac{(T_1+T_2)^2}{2}\right)\{u(t-T_2)-u(t-(T_1+T_2))\} \end{aligned} \tag{2}$$

$$y_2(t) = \{u(t)-u(t-T_1)\} * [-(t-T_2)\{u(t)-u(t-T_2)\}] = y_1\left(-(t-(T_1+T_2)\right) \tag{3}$$

**Prob. 1.** [50 pts] Consider an LTI system characterized by the I/O relationship:

$$y(t) = \int_{t-2}^{t} x(\tau)d\tau - \int_{t-6}^{t-4} x(\tau)d\tau + x(t-7) \tag{4}$$

Note that this can be viewed as three systems in parallel, $y(t) = y_1(t) + y_2(t) + y_3(t)$, where:

$$y_1(t) = \int_{t-2}^{t} x(\tau)d\tau \quad \Rightarrow \text{System 1 impulse response: } h_1(t) \tag{5}$$

$$y_2(t) = -\int_{t-6}^{t-4} x(\tau)d\tau \quad \Rightarrow \text{System 2 impulse response: } h_2(t) \tag{6}$$

$$y_3(t) = x(t-7) \quad \Rightarrow \text{System 3 impulse response: } h_3(t) \tag{7}$$

(a) Determine and plot the impulse response of the overall system, denoted $h(t)$, in the spaced provided on the sheets attached.

(b) Determine and plot the output $y_1(t)$ of System 1 in the space provided on the next few pages when the input to System 1 is the linearly ramping-up input signal below:

$$x(t) = t\{u(t) - u(t-6)\}$$

(c) Determine and plot the output $y_2(t)$ of System 2 in the space provided for the same input $x(t) = t\{u(t) - u(t-6)\}$. You are first required to express $h_2(t)$ in terms of $h_1(t)$ in the space directly below; then express the relationship between $y_2(t)$ and $y_1(t)$.

Relationship between impulse responses of Systems 1 and 2:

(d) Determine and plot the output $y_3(t)$ of System 3 in the space provided on the next few pages for the same input $x(t) = t\{u(t) - u(t-6)\}$. You are first required to write an expression for System 3's impulse response $h_3(t)$ in the space directly below.

Expression for System 3's impulse response:

(e) Using the spaced provided in the sheets attached, plot the output $y(t)$ of the overall system when the input to the system is $x(t) = t\{u(t) - u(t-6)\}$.

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You can show whatever work you want to show on these 2 pages for solving all parts of Problem 1 to do the plots requested on the next few pages. You do NOT have to show a lot of work – you do NOT have to write out large equations. Label you work for each part.

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You can show whatever work you want to show on these 2 pages for solving all parts of Problem 1 to do the plots requested on the next few pages. You do NOT have to show a lot of work – you do NOT have to write out large equations. Label you work for each part.

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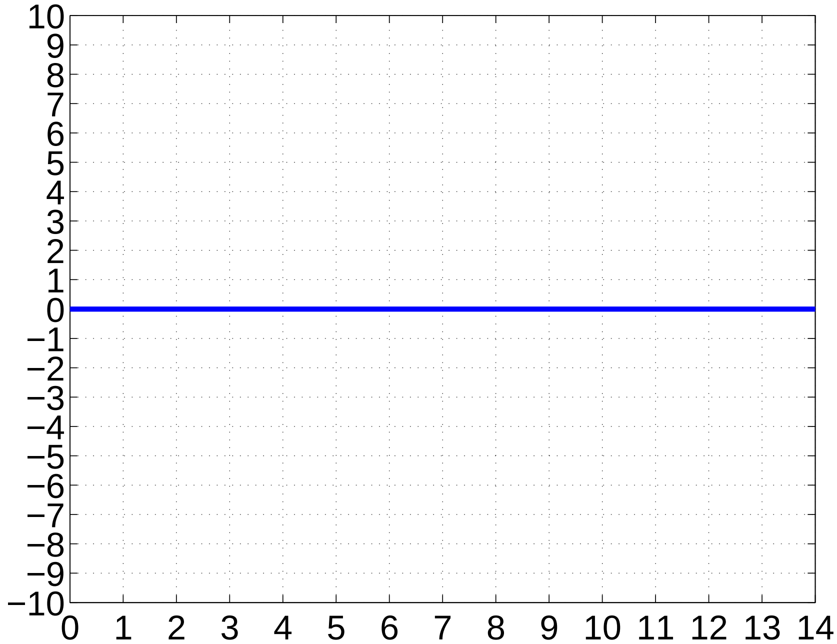
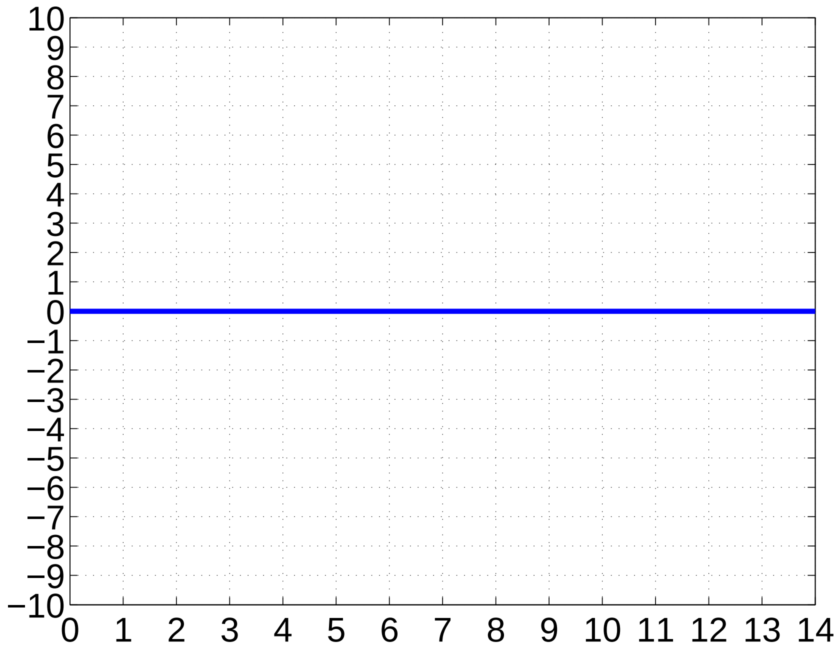
**Plot your answer for $h(t)$ for Problem 1 (a) here.**




**Plot your answer for $y_1(t)$ for Problem 1 (b) here.**



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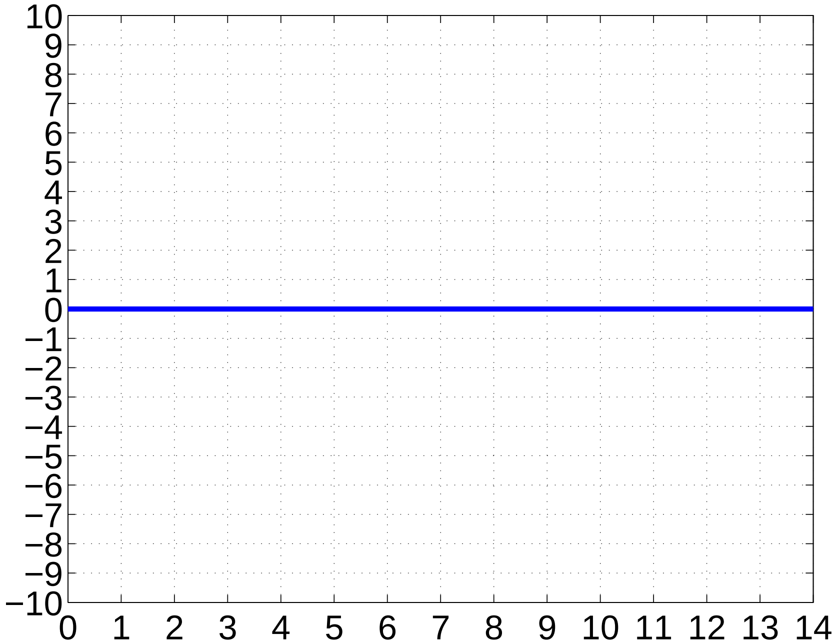
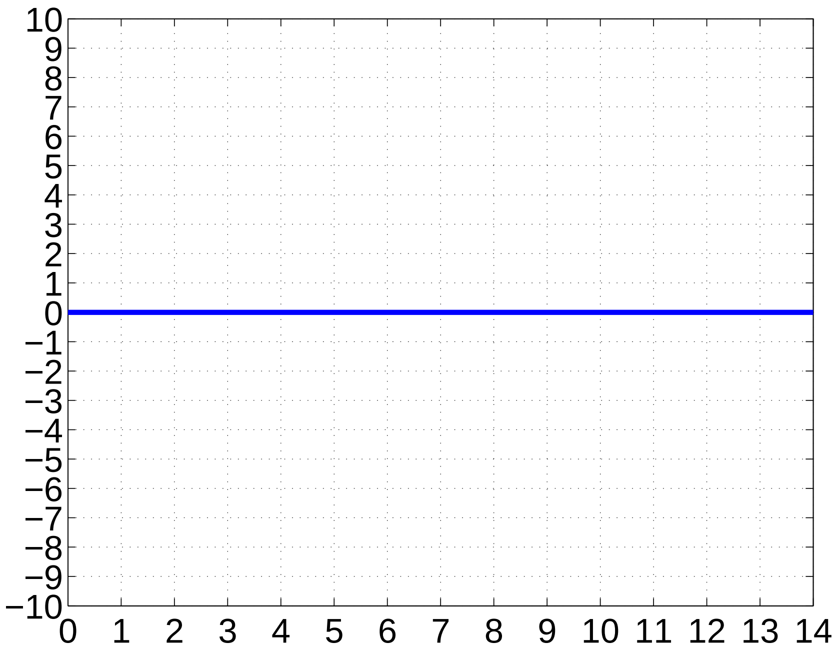
**Plot your answer for $y_2(t)$ for Problem 1 (c) here.**




**Plot your answer for $y_3(t)$ for Problem 1 (d) here.**



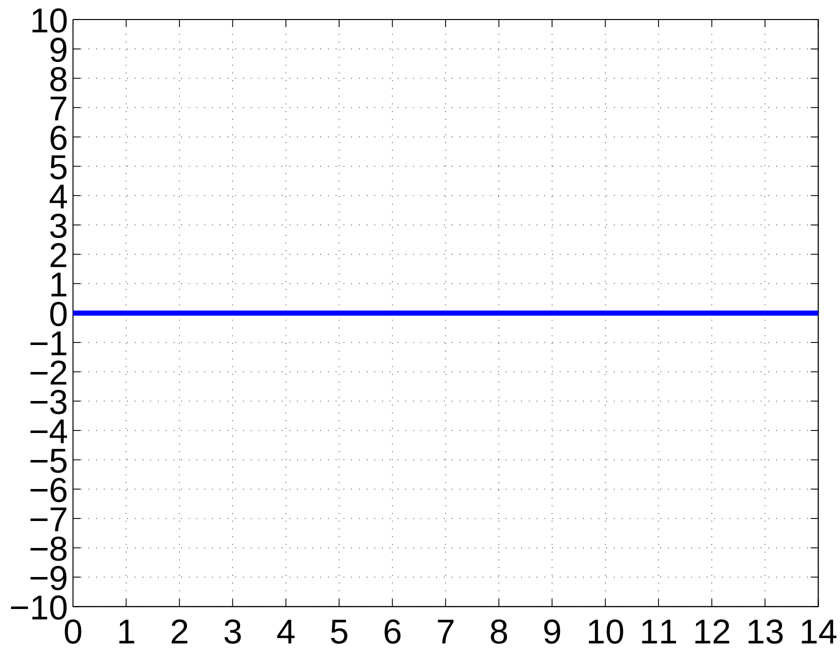
**Part (e). For each range of $t$, put an X in the correct box in the table below.**

| Range for $t$ | Linear pos. slope | Linear neg. slope | Quadratic Concave Up | Quadratic Concave Down |
|---|---|---|---|---|
| $0 < t < 2$ | | | | |
| $2 < t < 4$ | | | | |
| $4 < t < 6$ | | | | |
| $6 < t < 8$ | | | | |
| $8 < t < 10$ | | | | |
| $10 < t < 12$ | | | | |
| $12 < t < 13$ | | | | |

**For each value of $t$, write the value of $y(t)$ in the table below.**

| $t$ | $y(t)$ |
|---|---|
| $t = 0$ | |
| $t = 2$ | |
| $t = 4$ | |
| $t = 6$ | |
| $t = 8$ | |
| $t = 10$ | |
| $t = 12$ | |
| $t = 13$ | |

**Plot $y(t)$ for Prob1, Part (e) below.**



**Problem 2.** [50 points] Four parts. Show work in space provided.

(a) Consider the causal LTI System characterized by the difference equation below. Determine and write a closed-form EXPRESSION for the output $y[n]$ for the input $x[n] = \left(\frac{1}{3}\right)^n u[n]$, which is an infinite length sequence.

$$\text{System 1:} \qquad y[n] = -\frac{1}{2}y[n-1] + x[n]$$

(b) Consider the same system as for part (a), but now determine an expression for the output when the input is $x[n] = 4\left(\frac{1}{3}\right)^n u[n-2]$.

(c) Determine $y[n]$ as the convolution of the two sequences below. You can EITHER do a stem plot for your answer OR write it out in sequence form clearly indicating with an arrow which value corresponds to $n = 0$.

$$x[n] = 2\{u[n] - u[n-4]\} \qquad h[n] = 3\{u[n-2] - u[n-8]\}$$

(d) Determine $y[n]$ as the convolution of the two sequences below. You can EITHER do a stem plot for your answer OR write it out in sequence form clearly indicating with an arrow which value corresponds to $n = 0$.

$$x[n] = 8\left(-\frac{1}{2}\right)^n \{u[n] - u[n-4]\} \qquad h[n] = 16\left(\frac{1}{2}\right)^n \{u[n] - u[n-5]\}$$

Space for doing work for Prob. 2 (d) if needed.