NAME: SOLUTION

EE301 Signals and Systems

13 February 2013

Exam 1

Cover Sheet

Test Duration: ~~75~~ 60 minutes.

Coverage: Chaps. 1,2

Open Book but Closed Notes.

One 8.5 in. x 11 in. crib sheet

Calculators NOT allowed.

This test contains **two** problems.

All work should be done on the sheets provided.

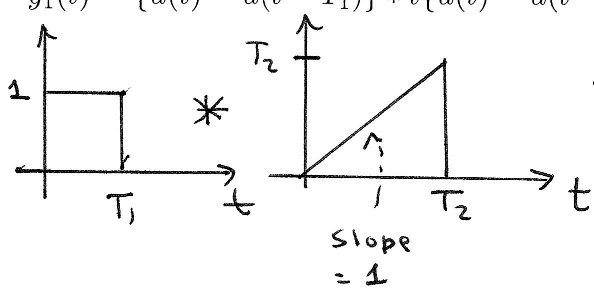
You must show work or explain answer for each problem to receive full credit.

Plot your answers on the graphs provided.

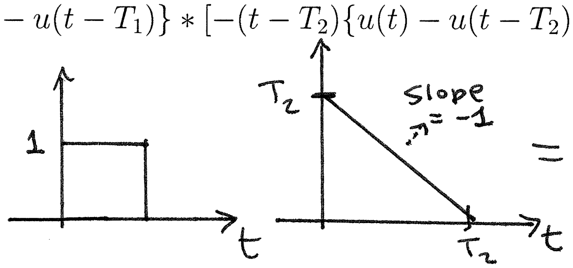
WRITE YOUR NAME ON EVERY SHEET.

Prob. No.	Topic(s)	Points
1.	Continuous Time Signals and System Properties	50
2.	Discrete Time Signals and System Properties	50

$$
\begin{aligned}
y_1(t) = \{u(t)-u(t-T_1)\} * t\{u(t)-u(t-T_2)\} &= \frac{t^2}{2}\{u(t)-u(t-T_1)\} \\
&+ \left(T_1 t - \frac{T_1^2}{2}\right)\{u(t-T_1)-u(t-T_2)\} \\
&+ \left(-\frac{t^2}{2} + T_1 t + \frac{T_2^2 - T_1^2}{2}\right)\{u(t-T_2)-u(t-(T_1+T_2))\}
\end{aligned} \tag{1}
$$



$$
\begin{aligned}
\{u(t)-u(t-T_1)\} * [-(t-T_2)\{u(t)-u(t-T_2)\}] &= \left(-\frac{t^2}{2} + T_2 t\right)\{u(t)-u(t-T_1)\} \\
&+ \left(-T_1 t + \frac{2T_1T_2 + T_1^2}{2}\right)\{u(t-T_1)-u(t-T_2)\} \\
&+ \left(\frac{t^2}{2} - (T_1+T_2)t + \frac{(T_1+T_2)^2}{2}\right)\{u(t-T_2)-u(t-(T_1+T_2))\}
\end{aligned} \tag{2}
$$



$$
y_2(t) = \{u(t)-u(t-T_1)\} * [-(t-T_2)\{u(t)-u(t-T_2)\}] = y_1\left(-(t-(T_1+T_2)\right) \tag{3}
$$

Prob. 1. [50 pts] Consider the LTI system characterized by the I/O relationship:

$$y(t) = \int_{t-1}^{t} x(\tau) d\tau \tag{4}$$

5 pt

(a) Determine and plot the impulse response of this system, denoted $h(t)$, in the spaced provided on the sheets attached.

5 pt

(b) Determine and plot the output $y_1(t)$ in the space provided when the input to the system is the rectangular pulse below:

$$x_1(t) = \{u(t) - u(t-1)\}$$

9 pt

(c) Determine and plot the output $y_2(t)$ in the space provided when the input to the overall system is the ramp-down triangular pulse.

$$x_2(t) = -(t-2)\{u(t) - u(t-2)\}$$

9 pt

(d) Determine and plot the output $y_3(t)$ in the space provided when the input to the system is the ramp-up triangular pulse:

$$x_3(t) = t\{u(t) - u(t-2)\}$$

(e) *GOAL:* determine the output $y(t)$ when the input to the system is $x(t)$ plotted below.

4 pt

(i) Express $x(t)$ in terms of possibly amplitude-scaled and time-shifted versions of $x_i(t)$, $i = 1, 2, 3$, defined in parts (b), (c), and (d). **You can use any of the $x_i(t)$ functions more than once in your expression, and your expression can sum more than just three terms.** For example, (this is NOT correct): $x(t) = \sqrt{2}\, x_1(t-\pi) - \sqrt{\pi}\, x_2(t-\sqrt{2}) + 3\, x_3(t-7) - x_3(t-9) + x_2(t-2\pi) - x_1(t-\sqrt{11})$

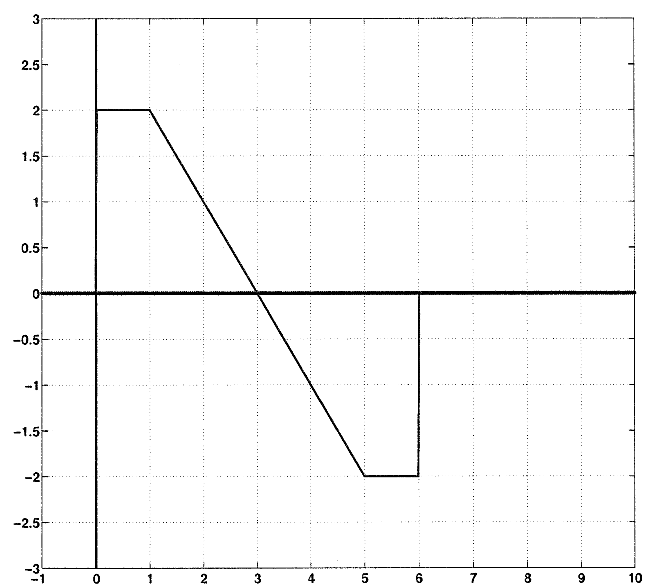
3 pt

(ii) Similarly express $y(t)$ in terms of $y_i(t)$, $i = 1, 2, 3$, answers to parts (b), (c), (d).

(iii) Plot $y(t)$ in the space provided on the sheets attached.

Input $x(t)$

15 pt



Problem 2. [50 points] For parts (a) and (b), show your work and do your plots in the space provided on the sheets attached. Put the answers for the remaining parts on this page.

(a) For parts (a) and (b), consider causal LTI System 1 characterized by the following difference equation below. Determine and plot (stem plot) the impulse response $h_1[n]$ of System 1 in the space provided on the sheets attached.

10 pt

$$\text{System 1:} \quad y[n] = 2y[n-1] + x[n] - 16x[n-4]$$

(b) Determine the output $y[n]$ of System 1 when the input is the finite-length geometric sequence below. Plot $y[n]$ in terms of a stem plot.

15 pt

$$x[n] = 4\left(\frac{1}{2}\right)^n \{u[n] - u[n-4]\}$$

(c) For the REST of this problem, consider System 2 characterized by the equation below.

$$\text{System 2:} \qquad y[n] = x[n] + \cos(\pi n)x[n-1] + n^2 x[n-2]$$

(i) Is System 2 linear? State Yes or No, and briefly explain your answer in words.

5 pt

Answer: Yes, System 2 is linear due to distributive property of multiplication and addition.

(ii) Is System 2 Time Invariant? State Yes or No, and explain your answer in words.

5 pt

Answer: No. Proved in class that the system $y[n]=g[n]\,x[n]$ is not TI. The system $y[n]=g[n]\,x[n-1]$ is not TI for the same reasoning, where $g[n]=\cos(\pi n)$. The system $y[n]=n^2\,x[n-2]$ is also not TI, as this is a case where $g[n]=n^2$

(iii) Let $h[n]$ denote the output of System 2 when the input is $x[n] = \delta[n]$. For any other input, is the output $y[n]$ equal to the convolution of $x[n]$ with the impulse response $h[n]$? State Yes or No, and briefly explain your answer in words.

5 pt

Answer: NO

From part (b), we know System 2 is not TI. So, the system is not LTI. The convolutional relationship between input and output, $y[n]=x[n]*h[n]$, only holds if the system is LTI. This system is Time-Varying.

(iv) If the input to System 2 is a sinewave $x[n] = e^{jw_on}$, will the output also be a sinewave but with a different amplitude and frequency? Yes or No? Explain.

5 pt

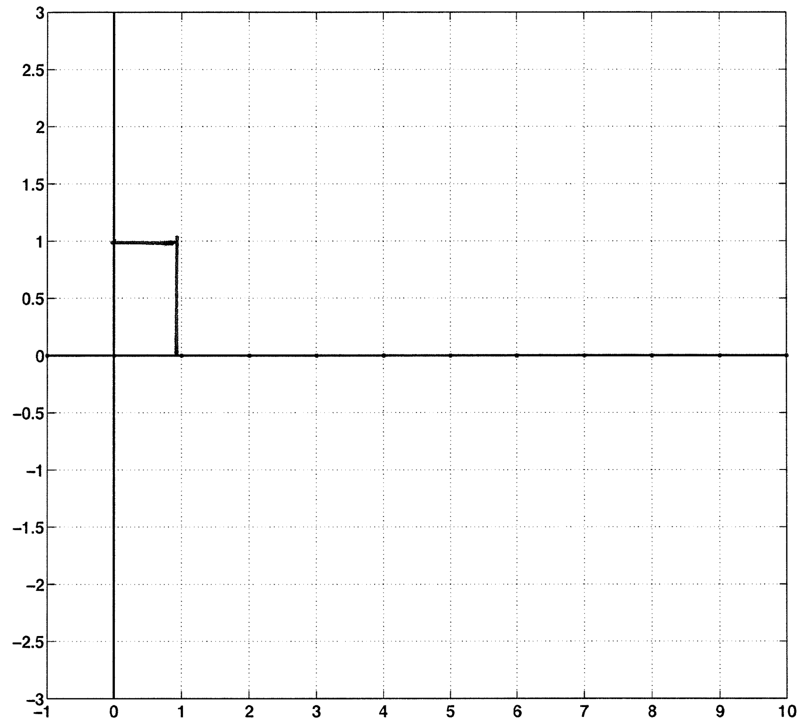
Answer: NO. Due to third term: $n^2\,x[n-2]$

$$y[n] = e^{j\omega_0 n} + \frac{1}{2}\left(e^{j\pi n} + \frac{1}{2}e^{-j\pi n}\right)e^{j\omega_0(n-1)} + \underbrace{n^2\,e^{+j\omega_0(n-2)}}_{\text{not a sinewave}}$$

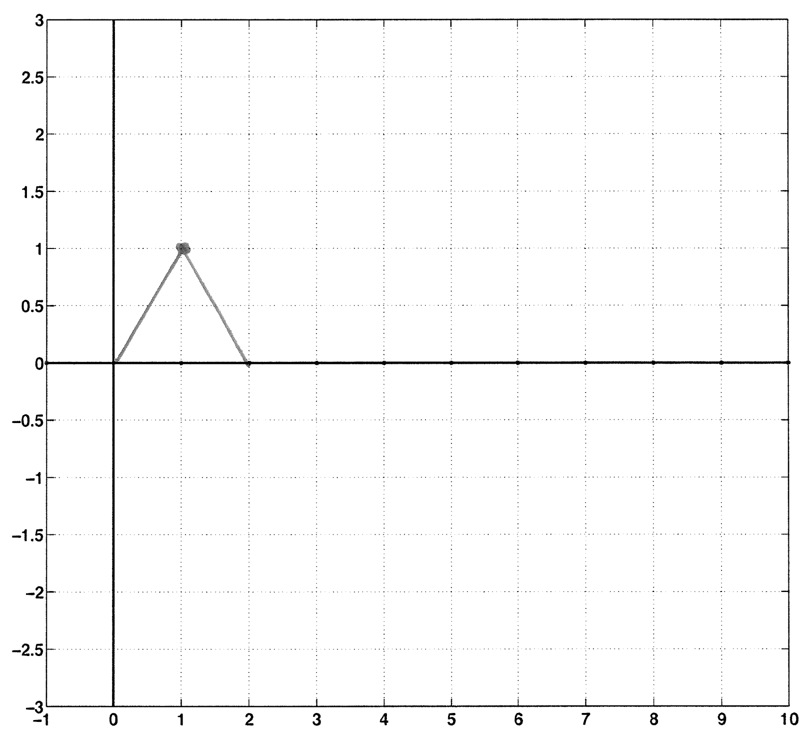
(v) Is System 2 stable? State Yes or No, and briefly explain your answer in words.

5 pt

Answer: NO. Due to third term: $n^2\,x[n-2]$ The coefficient of $x[n-2]$ grows without bound as n approaches infinity



Plot your answer for $h(t)$ for Problem 1 (a) here.



Plot your answer for $y_1(t)$ for Problem 1 (b) here.

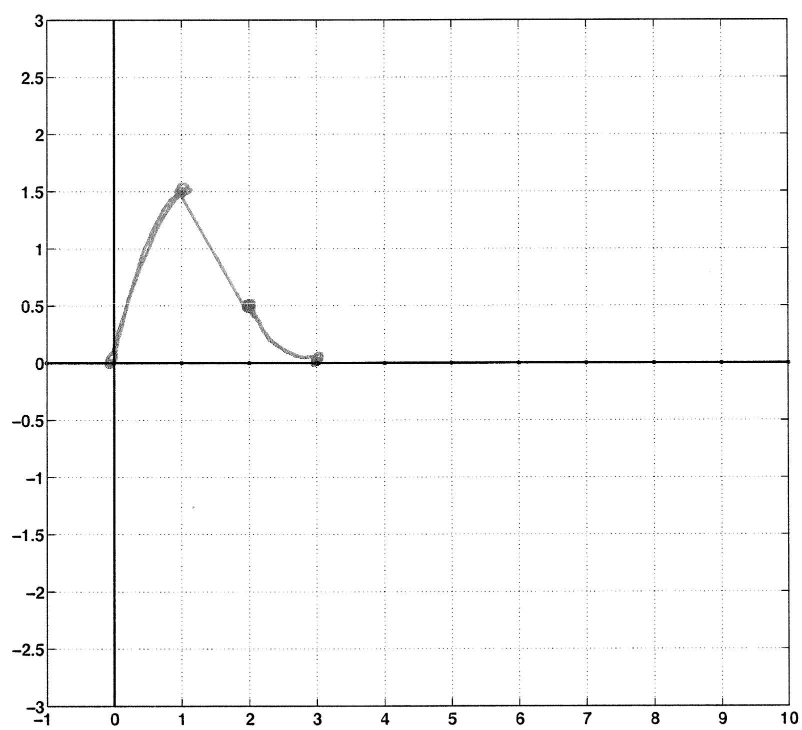
1(c): For each value of t, write the value of $y_2(t)$ in the table below.

t	$y_2(t)$
$t=0$	0
$t=1$	1.5
$t=2$	0.5
$t=3$	0

Mark the correct box with an X for each range for $y_2(t)$.

Range for t	Linear pos. slope	Linear neg. slope	Quadratic Concave Up	Quadratic Concave Down
$0<t<1$				X
$1<t<2$		X		
$2<t<3$			X	

Plot $y_2(t)$ below.



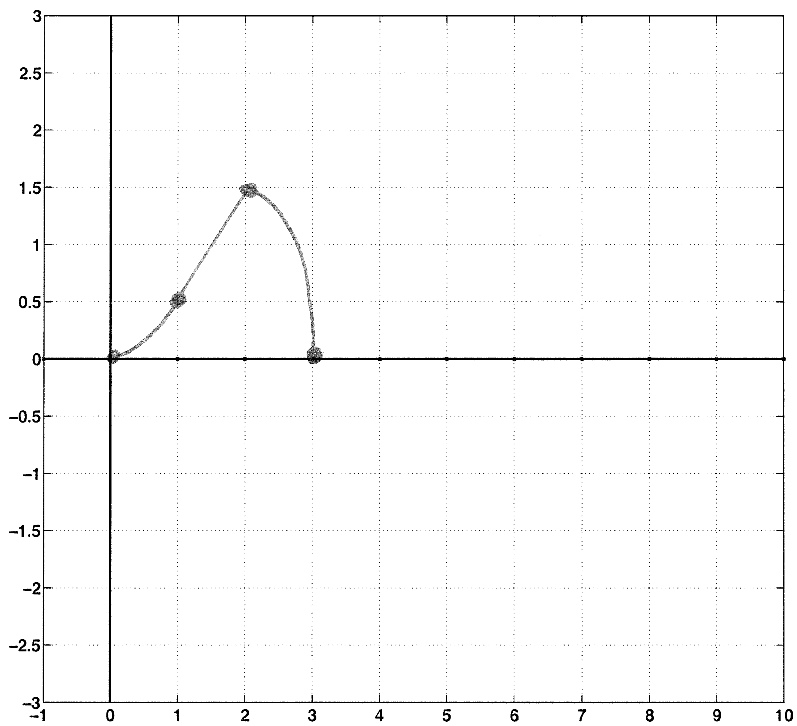
1(d): For each value of t, write the value of $y_3(t)$ in the table below.

t	$y_3(t)$
$t = 0$	0
$t = 1$	0.5
$t = 2$	1.5
$t = 3$	0

Mark the correct box with an X for each range for $y_3(t)$.

Range for t	Linear pos. slope	Linear neg. slope	Quadratic Concave Up	Quadratic Concave Down
$0 < t < 1$			X	
$1 < t < 2$	X			
$2 < t < 3$				X

Plot $y_3(t)$ below.



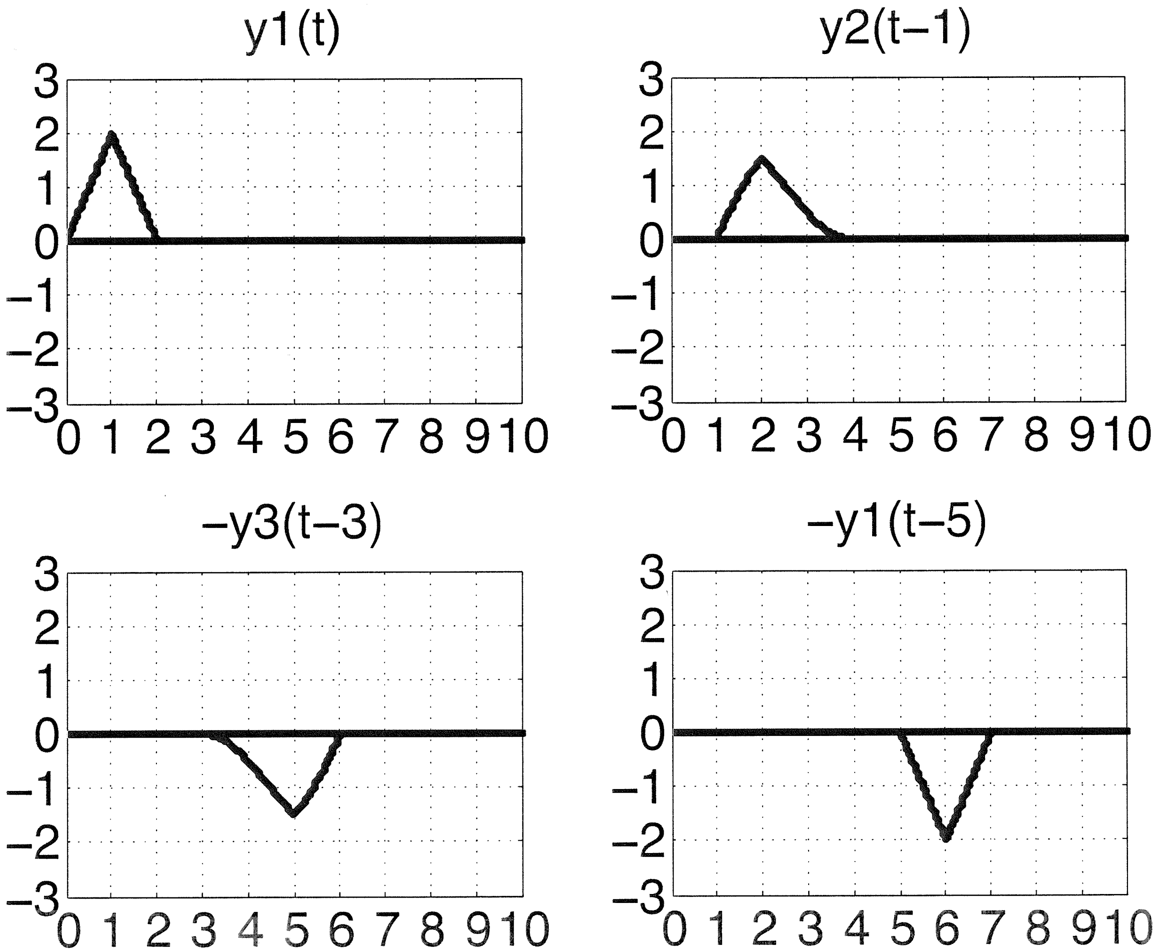
1 (e). Express $x(t)$ in terms of $x_i(t)$, $i = 1, 2, 3$.

$$x(t) = 2x_1(t) + x_2(t-1) - x_3(t-3) - 2x_1(t-5)$$

1 (e). Express $y(t)$ in terms of $y_i(t)$, $i = 1, 2, 3$.

$$y(t) = 2y_1(t) + y_2(t-1) - y_3(t-3) - 2y_1(t-5)$$

You can use the plots below if they're helpful for answering 1(e).



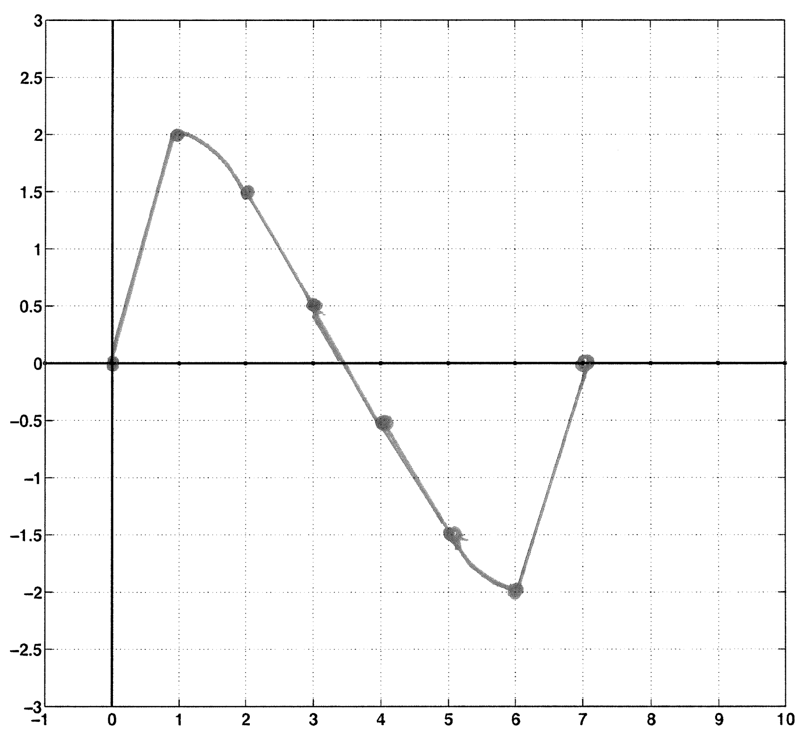
Part (e). For each range of t, put an X in the correct box in the table below.

Range for t	Linear pos. slope	Linear neg. slope	Quadratic Concave Up	Quadratic Concave Down
$0 < t < 1$	X			
$1 < t < 2$				X
$2 < t < 3$		X		
$3 < t < 4$		X		
$4 < t < 5$		X		
$5 < t < 6$			X	
$6 < t < 7$	X			

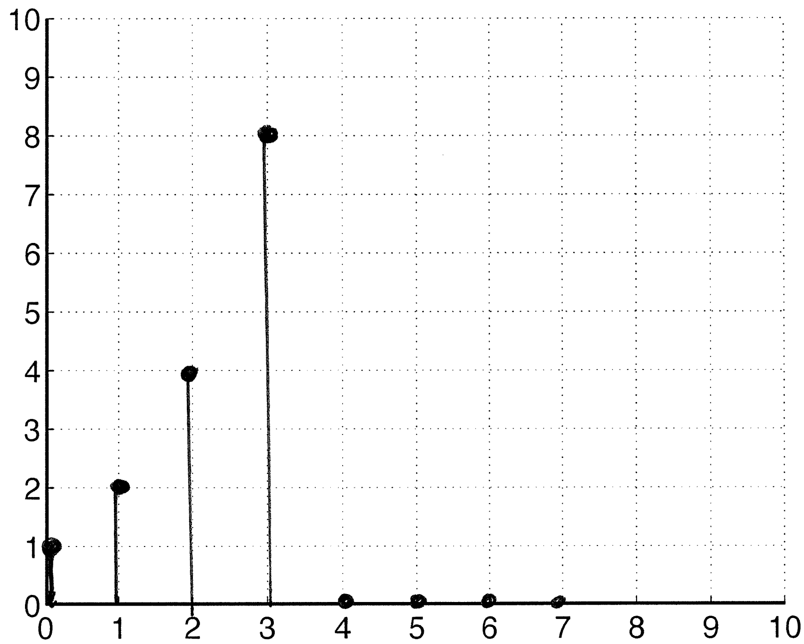
For each value of t, write the value of $y(t)$ in the table below.

t	$y(t)$
$t = 0$	0
$t = 1$	2
$t = 2$	1.5
$t = 3$	0.5
$t = 4$	-0.5
$t = 5$	-1.5
$t = 6$	-2
$t = 7$	0

Plot $y(t)$ for Prob1, Part (e) below.



Plot your answer $h_1[n]$ to Problem 2, part (a) on this page.



Special case: $y[n] = a\,y[n-1] + x[n] - a^D x[n-D]$

where $a = 2$ and $D = 4$

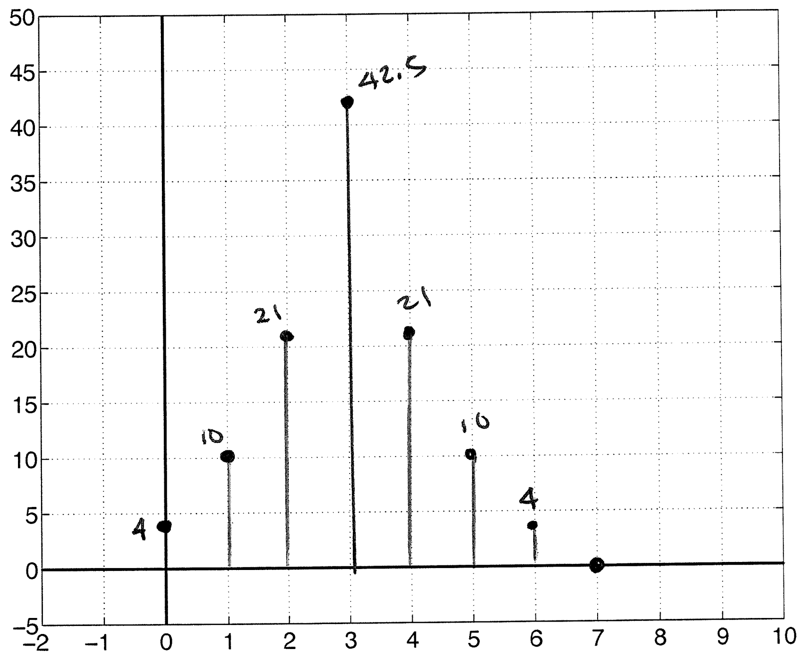
$h[n] = 2^n \{u[n] - u[n-4]\}$

$= \{1, 2, 4, 8\}$

$\uparrow$

$n=0$

Show your work and plot your answer to Problem 2, part (b) on this page.



$$y[n] = x[n] * h[n]$$

$$= \{4, 2, 1, \tfrac{1}{2}\} * \{1, 2, 4, 8\}$$

$\uparrow$ $n=0$ $\quad$ $\uparrow$ $n=0$

length $= 4 + 4 - 1$

$= 7$

	n	0	1	2	3	4	5	6	7
$x[0] =$	4	4	8	16	32	0	0	0	0
$x[1] =$	2	0	2	4	8	16	0	0	0
$x[2] =$	1	0	0	1	2	4	8	0	0
$x[3] =$	1/2	0	0	0	1/2	1	2	4	0
$y[n] =$		4	10	21	42.5	21	10	4	0

the 1st column above just shows you what h[n] is being multiplied by