EE301 Signals and Systems **In-Class Exam**
Exam 1 **Thursday, Feb. 25, 2010**

Cover Sheet

Test Duration: 75 minutes.
Coverage: Chaps. 1,2,3
Open Book but Closed Notes.
One 8.5 in. x 11 in. crib sheet
Calculators NOT allowed.
This test contains **two** problems.
All work should be done in the blue books provided.
You must show all work for each problem to receive full credit.
Do **not** return this test sheet, just return the blue books.

Prob. No.	**Topic(s)**	**Points**
1.	Continuous Time Signals and System Properties and CT Fourier Series	55
2.	Discrete Time Signals and System Properties	45

Problem 1. [55 points] Consider two LTI systems connected **in parallel**, where each of the two systems in parallel are respectively characterized by the following input-output relationships:

$$\begin{aligned} &\text{System 1:} \quad y_1(t) = 2\int_{t-1}^{t} x(\tau)d\tau \\ &\text{System 2:} \quad y_2(t) = \int_{t-2}^{t-1} x(\tau)d\tau \end{aligned}$$

As is the case with systems in parallel, the three systems have $x(t)$ as a common input and their resective outputs are summed to yield the overall output $y(t) = y_1(t) + y_2(t)$.

(a) Determine and plot the impulse response of the overall system.

(b) Apply a test to the overall impulse response (answer to (a)) to determine if the system is causal or not.

(c) Apply a test to the overall impulse response (answer to (a)) to determine if the system is stable or not.

(d) Determine and plot the output $y(t)$ when the input to this system is the rectangular pulse: $x(t) = \text{rect}(t - 0.5) = u(t) - u(t - 1)$. Your plot needs to explicitly indicate what the value of $y(t)$ is at $t = 1$, $t = 2$, and $t = 3$ seconds.

(e) Determine and plot the output $y(t)$ when the input to this system is the rectangular pulse: $x(t) = 2\text{rect}(t - 2.5) = 2\{u(t - 2) - u(t - 3)\}$.

(f) Consider that the input to this system is the periodic signal $x(t) = \sum_{k=-\infty}^{\infty} (-1)^k \delta(t-k3)$. What is the period of this signal? Determine the Fourier Series coefficients, denoted a_k, $-\infty < k < \infty$, for $x(t)$. Express your answer for a_k as a closed-form function of k.

(g) (i) Determine and plot several periods of the output, $y(t)$, of the overall system above to the periodic input signal $x(t)$ in part (f).

(ii) Determine the Fourier Series coefficients, denoted b_k, $-\infty < k < \infty$, for $y(t)$. Express your answer for b_k as a closed-form function of k that works for all k.

(iii) Determine the numerical value of $\sum_{k=-\infty}^{\infty} |b_k|^2$

Problem 2. [45 points] An signal $x[n]$ is a sum of two DT sinewaves with frequencies $\pi/8$ and $3\pi/4$, respectively.

$$x[n] = e^{j\frac{\pi}{8}n} + e^{j\frac{3\pi}{4}n} \tag{1}$$

Is this signal periodic? if so, what is the period? Consider this signal as the input to each of the four systems described below.

$$\text{System 1:} \quad y[n] = |x[n]|^2 \tag{2}$$

$$\text{System 2:} \quad y[n] = x[4n] \tag{3}$$

$$\text{System 3:} \quad y[n] = -x[n-1] + 2x[n] - x[n+1] \tag{4}$$

$$\text{System 4:} \quad y[n] = (\ j)^n x[n] \tag{5}$$

$$\tag{6}$$

For EACH of the four systems above, you must answer EACH of the following THREE questions:

(a) Is the system linear? Yes or No: don't need to substantiate your answer.

(b) Is the system time-invariant? Yes or No: don't need to substantiate your answer.

(c) Determine the output $y[n]$ of the system given the input in Equation 1 above. Specify the frequencies present in the output. Each answer should be in the range $[-\pi, \pi]$.

For which of the four systems is the set of output frequencies equal to the set of input frequencies? Is that system both linear and time-invariant?