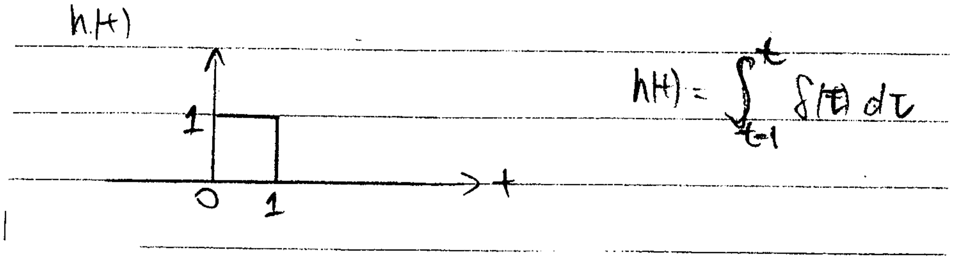
Exam 1 Sol'n   ECE 301   SP '07

Prob. 1

(a)



$$h(t) = \int_{t-1}^{t} \delta(\tau)\, d\tau$$

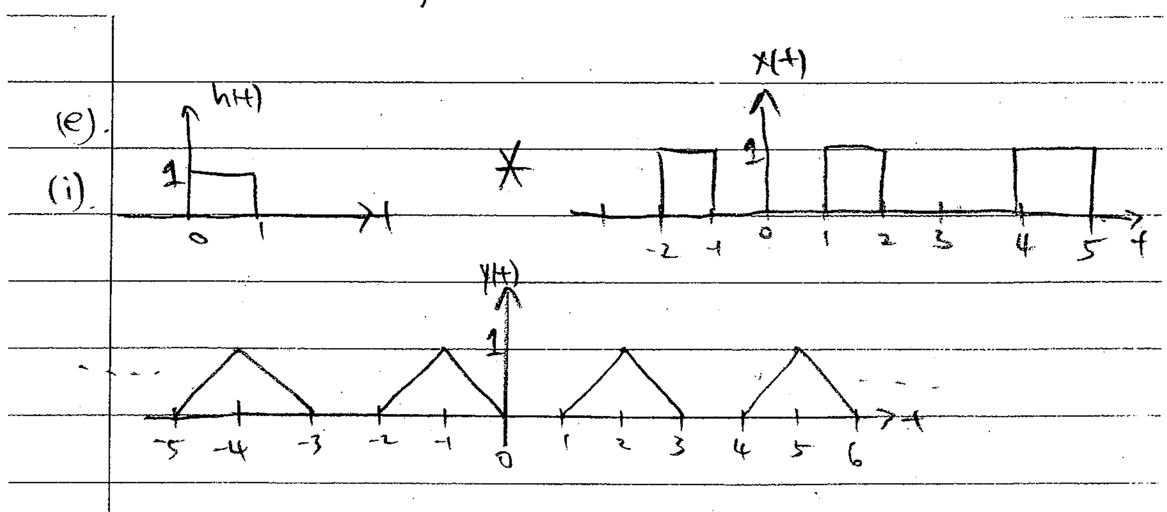
(b) Causal because $h(t) = 0$ for $t < 0$

(c) Stable because $\int_{-\infty}^{\infty} |h(t)|\, dt = 1 < \infty$

(d)
$$a_k = \frac{\sin\left(k\pi \frac{1}{3}\right)}{k\pi} \underbrace{e^{-jk\frac{2\pi}{3}\left(\frac{3}{2}\right)}}_{= (-1)^k}$$

$\frac{\tau}{T}$ t_0 shift

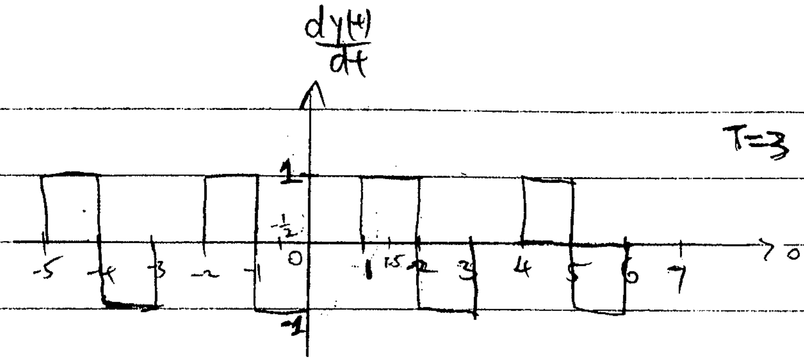
(e)



(e)-(ii): Compute FS coeffs for dy/dt first

$$\frac{dy}{dt} = \sum_{k=-\infty}^{\infty} d_k e^{jk\frac{2\pi}{3}t}, \quad \text{then } b_k = \frac{d_k}{jk\frac{2\pi}{3}}$$

1 (e) (ii)



$$d_k = \frac{\sin\left(k\pi\frac{1}{3}\right)}{k\pi} e^{-jk\frac{2\pi}{3}\left(\frac{3}{2}\right)} - \frac{\sin\left(k\pi\frac{1}{3}\right)}{k\pi} e^{-jk\frac{2\pi}{3}\left(-\frac{1}{2}\right)}$$

again: $b_k = \dfrac{d_k}{jk\frac{2\pi}{3}}$

(e)-(iii) $y(t)$ is neither real & even or real & odd, so the FS coefficients b_k are complex-valued in general

(f) $w(t) = x(t)\cos\left(\frac{2\pi}{3}t\right)$

$$= \frac{1}{2}x(t)e^{j\frac{2\pi}{3}(1)t} + \frac{1}{2}x(t)e^{j\frac{2\pi}{3}(-1)t}$$

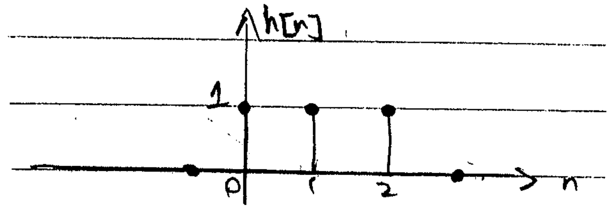
thus, from frequency shift property:

$$w_k = \frac{1}{2}a_{k-1} + \frac{1}{2}a_{k+1}$$

$$= \frac{1}{2}\frac{\sin\left([k-1]\pi\frac{1}{3}\right)}{[k-1]\pi}(-1)^{k-1} + \frac{1}{2}\frac{\sin\left([k+1]\pi\frac{1}{3}\right)}{[k+1]\pi}(-1)^{k+1}$$

Prob. 2

(a)



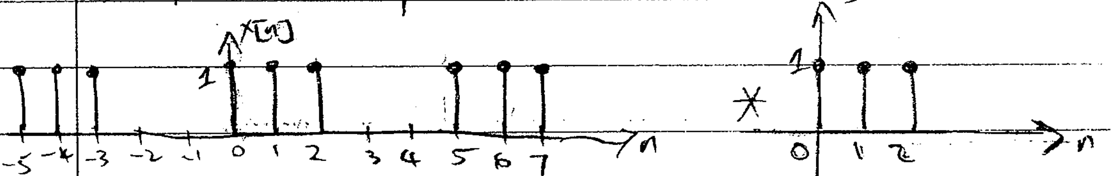
$$h[n] = \sum_{k=n-2}^{n} \delta[k] = \delta[n] + \delta[n-1] + \delta[n-2]$$

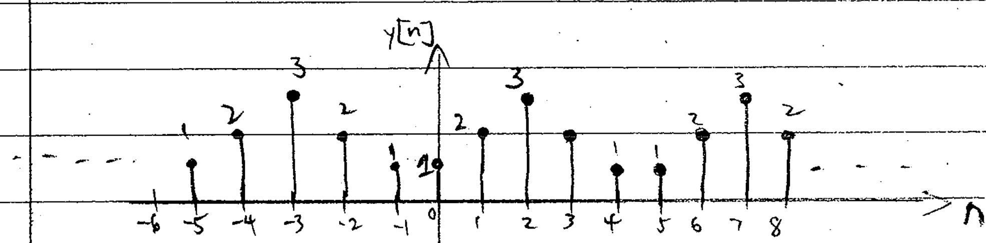
(b) Causal because $h[n] = 0$ for $n < 0$

(c) Stable because $\sum_{n=-\infty}^{\infty} |h[n]| = 3 < \infty$

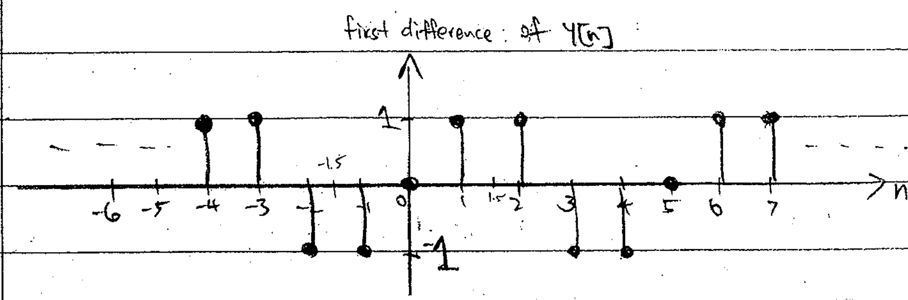
(d) $$a_k = \frac{1}{5} \frac{\sin\left(k\pi \frac{3}{5}\right)}{\sin\left(k\pi \frac{1}{5}\right)} e^{-jk\frac{2\pi}{5}\left(\frac{3-1}{2}\right)}$$

(e)(i) $y[n] = x[n] * h[n]$





(e)(ii)



N=5

(e)- (ii) first solve for FS coeffs., c_k, for difference $w[n] = y[n] - y[n-1]$

$$w[n] = \sum_{k=} c_k e^{jk\frac{2\pi}{5}n}$$

then: $$b_k = \frac{c_k}{\left(1 - e^{-jk\frac{2\pi}{5}}\right)}$$

- answer to (f):

$$c_k = \frac{1}{5} \frac{\sin\left(k\pi\frac{2}{5}\right)}{\sin\left(k\pi\frac{1}{5}\right)} e^{-jk\frac{2\pi}{5}\left(\frac{2-1}{2}\right)} \cdot e^{-jk\frac{2\pi}{5}(1)}$$

$$- \frac{1}{5} \frac{\sin\left(k\pi\frac{2}{5}\right)}{\sin\left(k\pi\frac{1}{5}\right)} e^{-jk\frac{2\pi}{5}\left(\frac{2-1}{2}\right)} e^{-jk\frac{2\pi}{5}(3)}$$

(e)-(iii) $y[n]$ is neither real & even or real & odd, thus FS coefficients b_k are complex-valued in general