# Practical D/A Conversion



**In practice:** digitally increase sampling rate by integer amount just prior to DAC so that ZOH (interpolating fn is rectangle) or linear Interpolation (interpolating fn is a triangle) works well

**Recall:** assume $\omega_s = \frac{2\pi}{T_s} > 2\omega_M$

$$x_a(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\, h_{LP}(t - nT_s)$$

$$h_{LP}(t) = \sin\left(\frac{\omega_s}{2} t\right) \Big/ \frac{\omega_s t}{2} \quad \text{OR}$$

$$h_{LP}(t) = \frac{\sin\left(\frac{\omega_s}{2} t\right)}{\frac{\omega_s}{2} t} \, \frac{\sin\left(\left(\frac{\omega_s}{2} - \omega_M\right) t\right)}{\left(\frac{\omega_s}{2} - \omega_M\right) t}$$

Change dummy variable of summation and define $x[n] = x_a(nT_s)$

$$x_a(t) = \sum_{k=-\infty}^{\infty} x[k]\, h_{LP}(t - kT_s)$$

- Now, "sample" $x(t)$ at a rate $\omega_{s_{new}} = L\,\omega_s \Rightarrow$ replace $t$ by $n\frac{T_s}{L}$
- sample at $L$ times original rate

$$y[n] = x_a\left(n\frac{T_s}{L}\right) = \sum_{k=-\infty}^{\infty} x[k]\, h_{LP}\left(n\frac{T_s}{L} - kT_s\right)$$

$$= \sum_{k=-\infty}^{\infty} x[k]\, h_{LP}\left((n-kL)\frac{T_s}{L}\right)$$

where, for example: $\left(\omega_s = \frac{2\pi}{T_s}\right)$

$\Rightarrow \omega_s T_s = 2\pi$

$$h[n] = h_{LP}\left(n\frac{T_s}{L}\right)$$

$$= \frac{\sin\left(\frac{\omega_s}{2} n \frac{T_s}{L}\right)}{\frac{\omega_s}{2} n \frac{T_s}{L}} = \frac{\sin\left(\frac{\pi}{L} n\right)}{\frac{\pi}{L} n}$$

Thus:

$$y[n] = x_a\left(n\frac{T_s}{L}\right) = \sum_{k=-\infty}^{\infty} x[k] \frac{\sin\left(\frac{\pi}{L}(n-kL)\right)}{\frac{\pi}{L}(n-kL)}$$

- Center a DT sinc function at every integer multiple of $L$ $\Rightarrow$ at $n=kL$ scale by $x[k]$ $\left(= x_a(kT_s)\right)$

- This is something we can do in Matlab
- i.e., this is something we can do in software or digital hardware just prior to D/A conversion so that ZOH or linear Interpolation (linearly connect successive samples) works well
- Still may want to do preemphasis $\Rightarrow$ See Fig. 7.8 in Text
- One possible implementation:

: **note**: $h[n-kL] = h[n] * \delta[n-kL]$

- **Thus**:

$$y[n] = \left\{ \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-kL] \right\} * h[n]$$

- This mimics the theoretical way to reconstruct the signal: See Fig. 7.4

$$x_a(t) = \left\{ \sum_{k=-\infty}^{\infty} x_a(kT_s)\,\delta(t-kT_s) \right\} * h_{LP}(t)$$

- Here, to increase sampling DIGITALLY in discrete-time prior to D/A conversion

$$y[n] = x_a\left(n\frac{T_s}{L}\right) = \left\{ \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-kL] \right\} * h[n]$$

$x[k] = x_a(kT_s)$ $\quad$ $h[n] = h_{LP}\left(n\frac{T_s}{L}\right)$

and $\delta(t)$ is Dirac-Delta function

$\delta[n]$ is Kronecker Delta function

## Ideal D/A:

$$x_a(t) = \sum_{n=-\infty}^{\infty} \underbrace{x_a(nT_s)}_{x[n]} \frac{\sin\left(\frac{\pi}{T_s}(t-nT_s)\right)}{\frac{\pi}{T_s}(t-nT_s)}$$

## Digital Up-Sampling:

$$y[n] = x_a\left(n\frac{T_s}{L}\right)$$ (replace summation $n$ variable above by $k$)

$$= \sum_{k=-\infty}^{\infty} x[k] \frac{\sin\left(\frac{\pi}{T_s}\left(n\frac{T_s}{L} - kT_s\right)\right)}{\frac{\pi}{T_s}\left(n\frac{T_s}{L} - kT_s\right)}$$

$$= \sum_{k=-\infty}^{\infty} x[k] \frac{\sin\left(\frac{\pi}{L}(n-kL)\right)}{\frac{\pi}{L}(n-kL)}$$

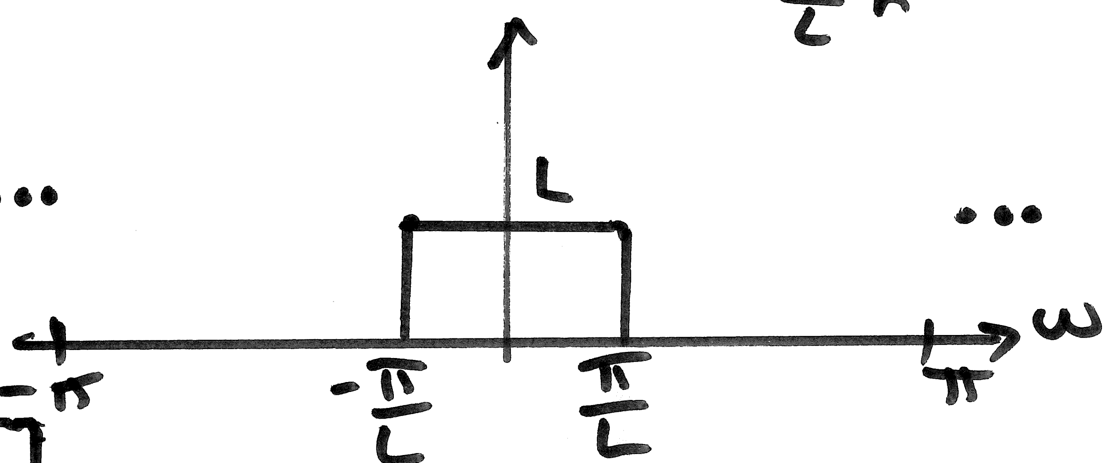
$$y[n] = \left\{ \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-kL] \right\} * \frac{\sin\left(\frac{\pi}{L} n\right)}{\frac{\pi}{L} n}$$

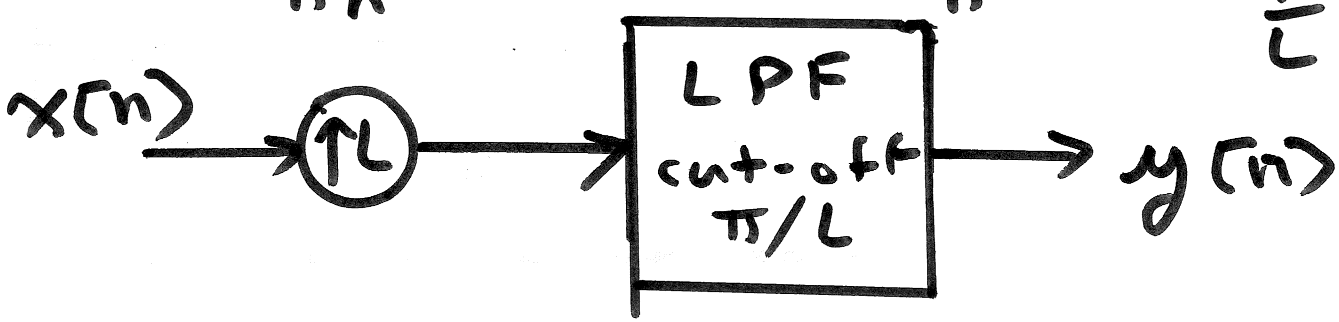
$$y[n] = x_a\left(n \frac{T_s}{L}\right) \qquad x[n] = x_a(nT_s)$$

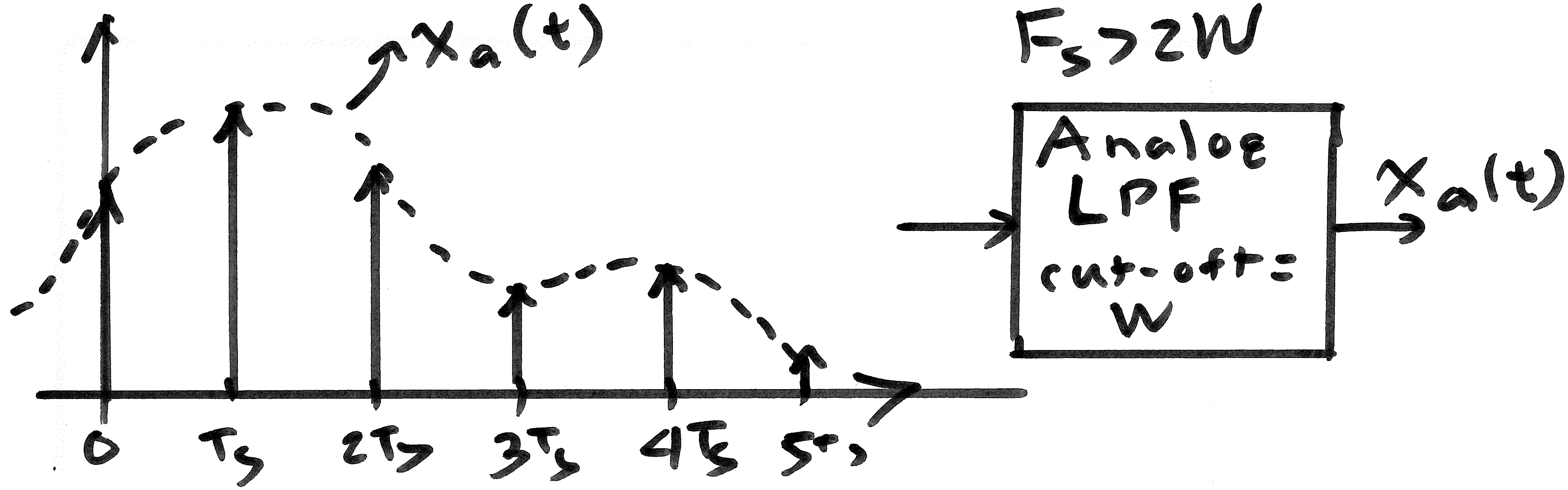
<u>Since</u>:
$$\frac{\sin\left(\frac{\pi}{L}(n-kL)\right)}{\frac{\pi}{L}(n-kL)} = \delta[n-kL] * \frac{\sin\left(\frac{\pi}{L} n\right)}{\frac{\pi}{L} n}$$

From Table 4.2

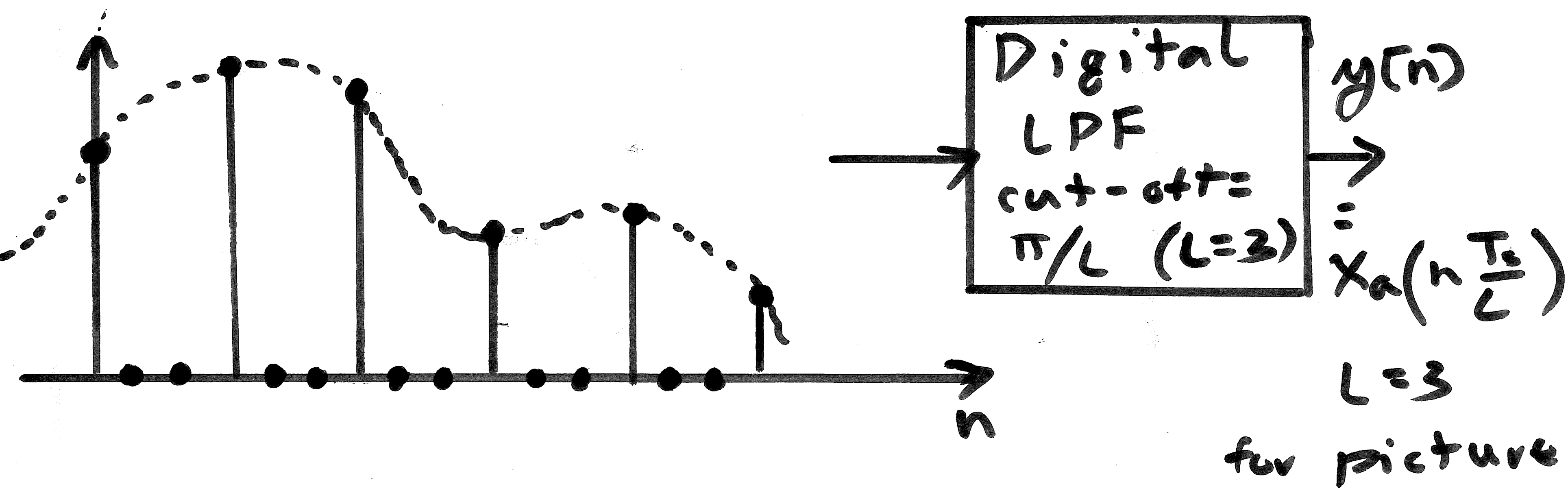
$$L\,\frac{\sin\left(\frac{\pi}{L} n\right)}{\pi n} \xleftrightarrow{\text{DTFT}} \cdots$$



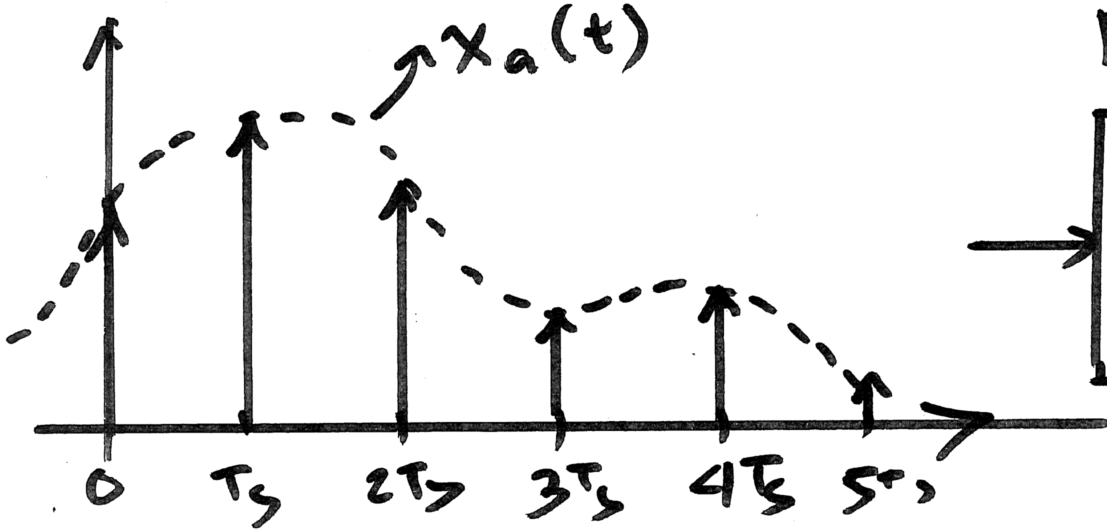





$x_a(t)$
$F_s > 2W$
Analog LPF cut-off = W
$x_a(t)$
0
$T_s$
$2T_s$
$3T_s$
$4T_s$
$5T_s$

Digital LPF cut-off = $\pi/L$ ($L=3$)
$y(n) = x_a\left(n\frac{T_s}{L}\right)$
n
$L=3$ for picture

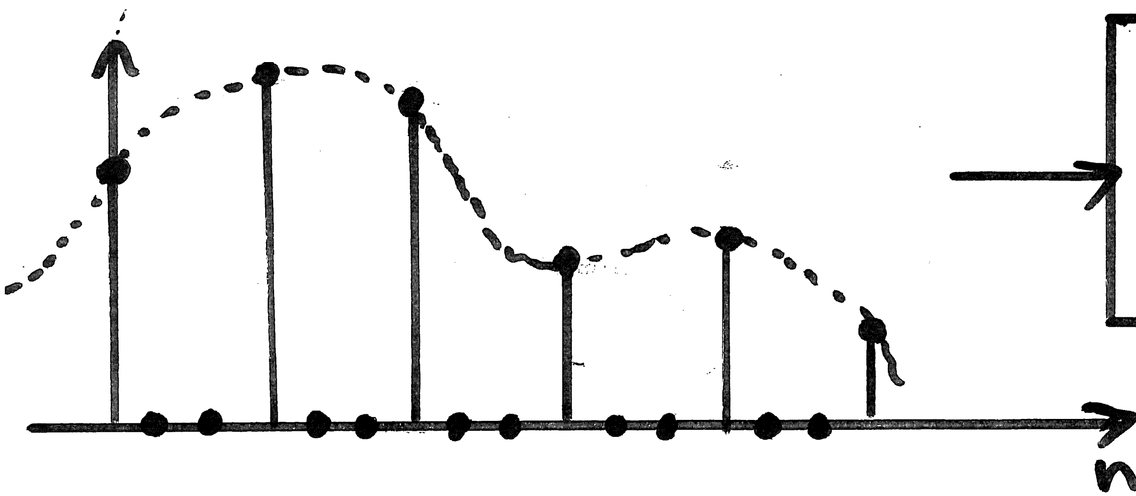



$F_s > 2W$

Analog LPF cut-off = W

$\longrightarrow$ Analog LPF $\longrightarrow x_a(t)$

$$h_{LP}(t) = \frac{\sin\left(\pi \frac{t}{T_s}\right)}{\pi \frac{t}{T_s}}$$




Digital LPF cut-off = $\pi/L$ $(L=3)$

$\longrightarrow$ Digital LPF $\longrightarrow y(n) = x_a\left(n \frac{T_s}{L}\right)$

$L=3$ for picture

$$h_{LP}[n] = h_{LP}(t)\Big|_{t = n\frac{T_s}{L}}$$

$$h_{LP}[n] = \frac{\sin\left(\frac{\pi}{L} n\right)}{\frac{\pi}{L} n}$$