Discrete-Time Signals (DT signals)

- typically obtained by sampling an analog signal at equi-spaced instants in time

$$x[n] = x(t)\Big|_{t=nT_s} = x(nT_s) \qquad -\infty < n < \infty$$

$$\left.\begin{aligned} &\vdots \\ x[-1] &= x(-T_s) \\ x[0] &= x(0) \\ x[1] &= x(T_s) \\ x[2] &= x(2T_s) \\ x[3] &= x(3T_s) \\ &\vdots \end{aligned}\right\}$$

the samples of the CT signal are stored in an array.

CT = continuous time

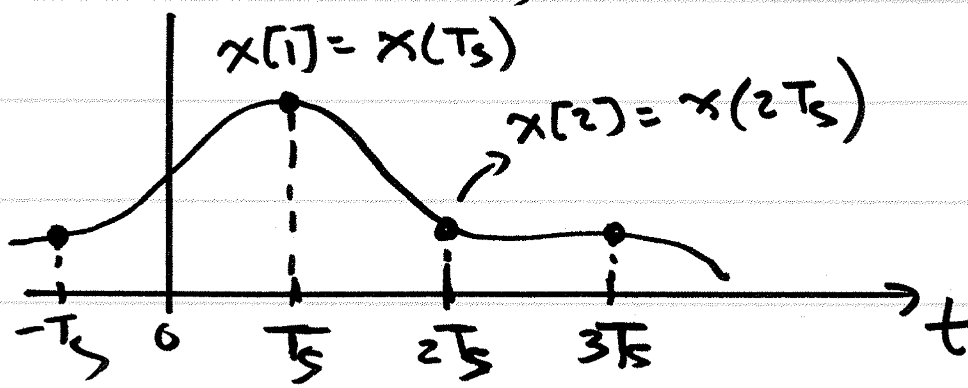
THUS:

$x[n]$

n must be an integer

(n is "like" an index location)

- $x[n+\frac{1}{2}]$ is meaningless $\Rightarrow$ doesn't make sense
- corresponds to $x\left((n+\frac{1}{2})T_s\right) = x\left(nT_s + \frac{T_s}{2}\right)$
- we have a sample at nT_s and $(n+1)T_s$ but not at halfway in between



$x[1+\frac{1}{2}] = x[\frac{3}{2}]$ doesn't make sense

- also, how can you have an index location for an array (vector) that is not an integer?
- $\frac{1}{T_s}$ = sampling rate = no. of samples per sec.

DT sinewaves:

$$x[n] = e^{j\omega_0 n} \quad \forall n \text{ integer}$$

for all

- suppose DT sinewave was obtained sampling a CT sinewave

$$x[n] = e^{j2\pi f_0 t}\Big|_{t=nT_s} = e^{j(2\pi f_0 T_s)n}$$

$$= e^{j\omega_0 n} \qquad \omega_0 = 2\pi f_0 T_s$$

- because n is an integer, DT sinewaves are very different from CT sinewaves
- two <u>major</u> differences

1. CT sinewaves are unique as long as frequencies are different
   $\Rightarrow$ not true for DT sinewaves

- consider: $e^{j(\omega_0 + l 2\pi)n}$
  - where l is an integer
- recall: $e^{j\theta} = \cos\theta + j\sin\theta$

$$e^{j2\pi} = \underbrace{\cos(2\pi)}_{1} + j\underbrace{\sin(2\pi)}_{0} = 1$$

- THUS:

$$e^{j(\omega_0 + l2\pi)n} = e^{j\omega_0 n} e^{j2\pi l n}$$

$$= e^{j\omega_0 n}\left(e^{j2\pi}\right)^{ln} = e^{j\omega_0 n}$$

- Any two DT sinewaves whose frequencies are separated by an integer multiple of 2π are the SAME sinewave.

2. CT sinewaves are <u>always</u> periodic

$\Rightarrow$ not always true for DT sinewaves

- the period for a DT sinewave has to be an integer $N \Rightarrow x[n] = x[n+N]$ $\forall n$

$$e^{j\omega_0(n+N)} = e^{j\omega_0 n} e^{j\omega_0 N}$$

$$= e^{j\omega_0 n} e^{jm2\pi} \Rightarrow \omega_0 N = m\,2\pi$$

$$\Rightarrow \frac{\omega_0}{2\pi} = \frac{m}{N}$$ } must be rational for DT sinewave to be periodic

- If this condition holds:

$$e^{j\omega_0 n} = e^{j 2\pi \frac{m}{N}(n+N)} = e^{j 2\pi \frac{m}{N} n}$$

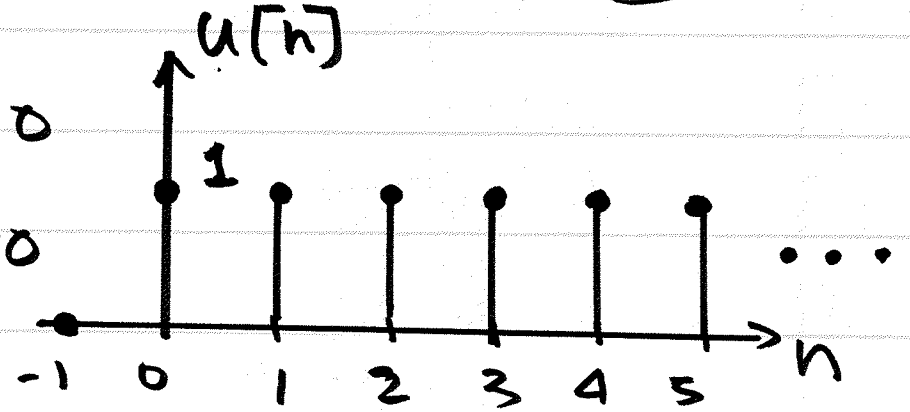
- divide out ~~any~~ greatest common divisor between m and N $\Rightarrow$ resulting period is called the fundamental period $N_0 = \dfrac{N}{\gcd(m, N)}$

- Table 1.1 in text summarizes differences between $e^{j\omega_0 t}$ and $e^{j\omega_0 n}$

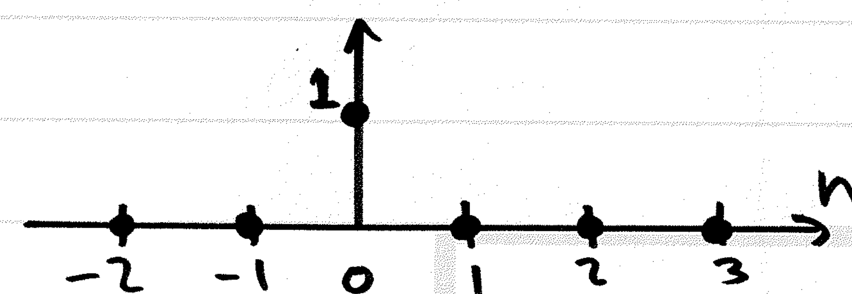
- See also Hmwk. Prob. 1.36

- See Fig. 1.27 on pg. 27

- **Basic DT signals**
  - unit step
  - $x[n] = u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$



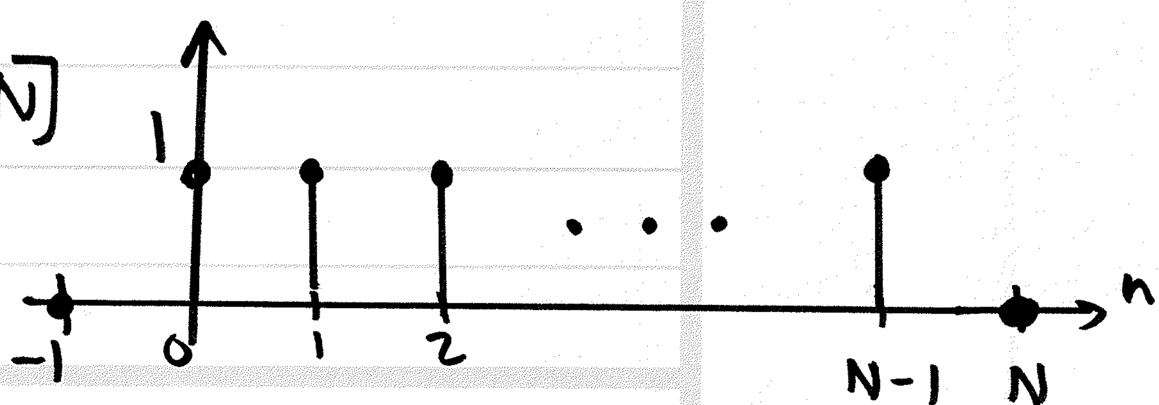
- Kronecker Delta Function (DT impulse)
  - $x[n] = \delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$



- DT rectangle

$x[n] = u[n] - u[n-N]$

"turned on" for N units of DT from $n=0$ to $n=N-1$

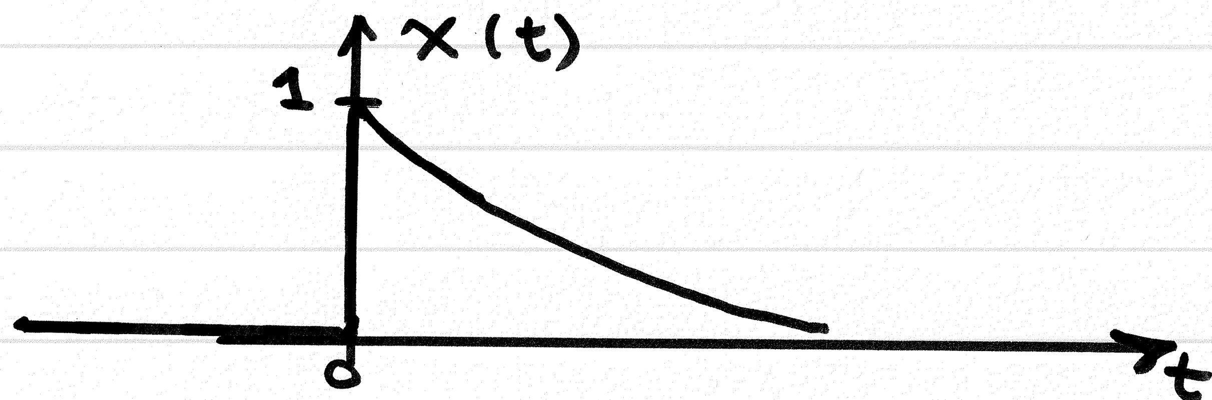


CT Exponential Signals and DT Geometric Signals/Sequences

- CT: $x(t) = e^{-at} u(t)$

where a can be complex-valued, in general

- If a is real-valued and $a > 0$



- Consider sampling $x(t)$ at equi-spaced intants in time $\Rightarrow$ every T_s seconds

$\Rightarrow$ replace t by nT_s, where n=integer

$$x[n] = x(t)\Big|_{t=nT_s} = x(nT_s) =$$

$$= e^{-anT_s}\, u(nT_s) = \left(e^{-aT_s}\right)^n u[n]$$

- $\Rightarrow \quad x[n] = \alpha^n u[n]$

  where: $\alpha = e^{-aT_s}$

- Sampling a CT exponential signal yields a DT geometric signal / sequence