

5.1 PROPERTIES OF THE DISCRETE-TIME FOURIER TRANSFORM

Property	Aperiodic Signal	Fourier Transform
	$x[n]$	$X(\omega)$ } periodic with
	$y[n]$	$Y(\omega)$ } period 2π
Linearity	$ax[n] + by[n]$	$aX(\omega) + bY(\omega)$
Time Shifting	$x[n - n_0]$	$e^{-j\omega n_0} X(\omega)$
Frequency Shifting	$e^{j\omega_0 n} x[n]$	$X(\omega - \omega_0)$
Conjugation	$x^*[n]$	$X^*(-\omega)$
Time Reversal	$x[-n]$	$X(-\omega)$
Time Expansion	$x^{(k)}[n] = \begin{cases} x[n/k], & \text{if } n = \text{multiple of } k \\ 0, & \text{if } n \neq \text{multiple of } k \end{cases}$	$X(\omega/k)$
Convolution	$x[n] * y[n]$	$X(\omega)Y(\omega)$
Multiplication	$x[n]y[n]$	$\frac{1}{2\pi} \int_{2\pi} X(\theta)Y(\omega - \theta) d\theta$
Differencing in Time	$x[n] - x[n - 1]$	$(1 - e^{-j\omega})X(\omega)$
Accumulation	$\sum_{k=-\infty}^n x[k]$	$\frac{1}{1 - e^{-j\omega}} X(\omega)$
		$+ \pi X(0) \sum_{k=-\infty}^{+\infty} \delta(\omega - 2\pi k)$
Differentiation in Frequency	$nx[n]$	$j \frac{dX(\omega)}{d\omega}$
Conjugate Symmetry for	$x[n]$ real	$\begin{cases} X(\omega) = X^*(-\omega) \\ \text{Re}\{X(\omega)\} = \text{Re}\{X(-\omega)\} \\ \text{Im}\{X(\omega)\} = -\text{Im}\{X(-\omega)\} \end{cases}$