Examples

1. $y(n) = \frac{1}{2}\,[x(n) + x(n-1)]$

Let $x(n) = e^{j\omega n}$

$$\begin{aligned} y(n) &= \frac{1}{2}\left[e^{j\omega n} + e^{j\omega(n-1)}\right] \\ &= \frac{1}{2}\left[1 + e^{-j\omega}\right] e^{j\omega n} \end{aligned}$$

$$\therefore \quad H(e^{j\omega}) = \frac{1}{2}\,[1 + e^{-j\omega}]$$

Factor out the half-angle:

$$H(e^{j\omega}) = \frac{1}{2}\, e^{-j\omega/2} \left[e^{j\omega/2} + e^{-j\omega/2} \right]$$

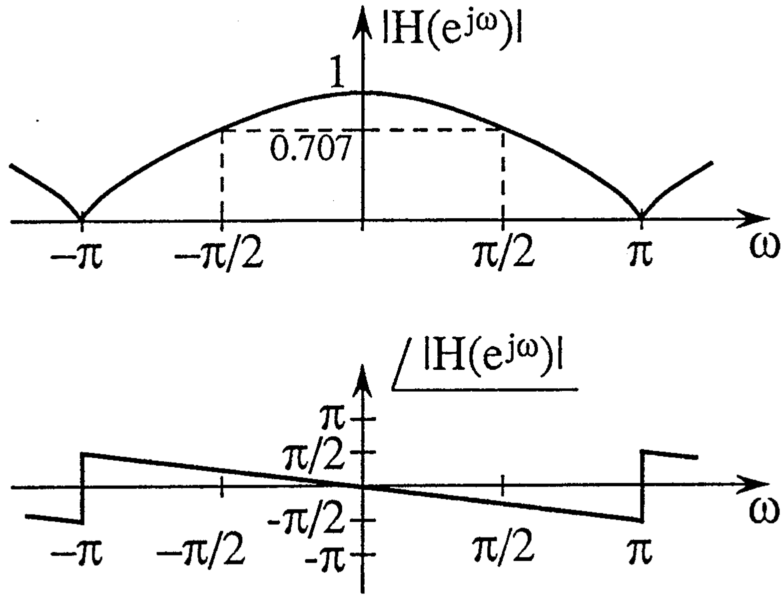
$$= e^{-j\omega/2} \cos(\omega/2)$$

$$|\, H(e^{j\omega}) \,| = |\, e^{-j\omega/2} \,| \;\; |\, \cos(\omega/2) \,|$$

$$= |\, \cos(\omega/2) \,|$$

$$\underline{/H(e^{j\omega})} = \underline{/e^{-j\omega/2}} + \underline{/\cos(\omega/2)}$$

$$= \begin{cases} -\omega/2, & \cos(\omega/2) \geq 0 \\ -\omega/2 \pm \pi, & \cos(\omega/2) < 0 \end{cases}$$



Note: even symmetry of $|H(e^{j\omega})|$

odd symmetry of $\angle H(e^{j\omega})$

periodicity of $H(e^{j\omega})$ with period 2π

low pass characteristic

2. $y(n) = \frac{1}{2}\,[x(n) - x(n-2)]$

Let $x(n) = e^{j\omega n}$

$$y(n) = \frac{1}{2}\left[e^{j\omega n} - e^{j\omega(n-2)}\right]$$

$$= \frac{1}{2}\left[1 - e^{-j\omega 2}\right] e^{j\omega n}$$

$$\therefore \quad H(e^{j\omega}) = \frac{1}{2}\left[1 - e^{-j\omega 2}\right]$$

$$= je^{-j\omega}\left[\frac{1}{j2}\left(e^{j\omega} - e^{-j\omega}\right)\right]$$

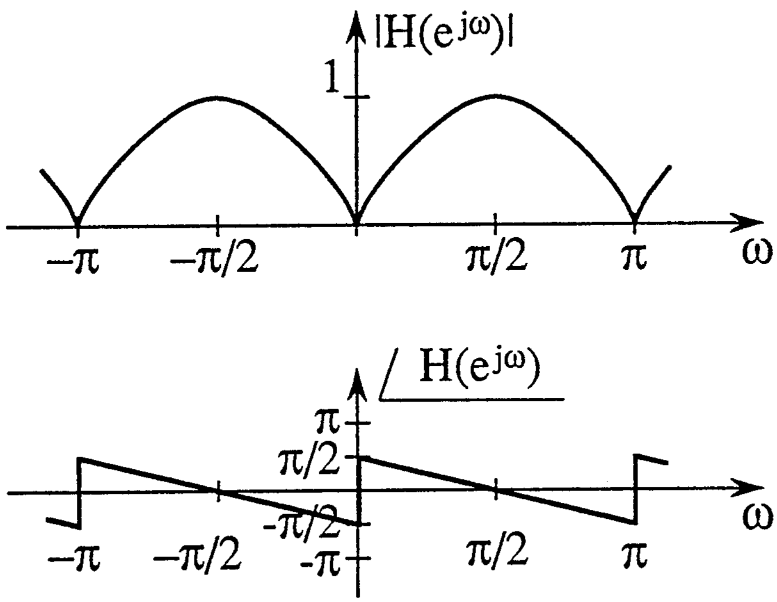
$$H(e^{j\omega}) = je^{-j\omega}\sin(\omega)$$

$$|H(e^{j\omega})| = |j|\ |e^{-j\omega}|\ |\sin(\omega)|$$

$$= |\sin(\omega)|$$

$$\underline{/H(e^{j\omega})} = \underline{/j} + \underline{/e^{-j\omega}} + \underline{/\sin(\omega)}$$

$$= \begin{cases} \pi/2 - \omega, & \sin(\omega) \geq 0 \\ \pi/2 - \omega \pm \pi, & \sin(\omega) < 0 \end{cases}$$



|H(e jω)|
1
−π
−π/2
π/2
π
ω
H(e jω)
π
π/2
-π/2
-π
−π
−π/2
π/2
π
ω

This filter has a *bandpass* characteristic.

Consider $x(n) = \cos(\omega n)$

a. $\omega = 0$

x(n)	...	1	1	1	1	1	...
y(n)	...	0	0	0	0	0	...

↑ n=0

b. $\omega = \pi/2$

x(n)	...	1	0	-1	0	1	0	-1	0	...
y(n)	...	1	0	-1	0	1	0	-1	0	...

n=0

c. $\omega = \pi$

x(n)	...	1	-1	1	-1	1	-1	...
y(n)	...	0	0	0	0	0	0	...

n=0

3. $y(n) = x(n) - x(n-1) - y(n-1)$

Let $x(n) = e^{j\omega n}$, how do we find y(n)?

Assume desired form of output,

i.e. $y(n) = H(e^{j\omega})\, e^{j\omega n}$

$$H(e^{j\omega})\, e^{j\omega n} = e^{j\omega n} - e^{j\omega(n-1)} - H(e^{j\omega})\, e^{j\omega(n-1)}$$

$$H(e^{j\omega})\, [1 + e^{-j\omega}]\, e^{j\omega n} = [1 - e^{-j\omega}]\, e^{j\omega n}$$

$$H(e^{j\omega}) = \frac{[1 - e^{-j\omega}]}{[1 + e^{-j\omega}]}$$

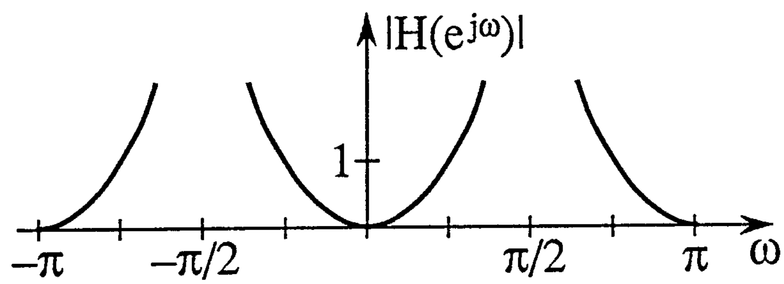
$$= \frac{je^{-j\omega/2}\left[\frac{1}{j2}\left(e^{j\omega/2} - e^{-j\omega/2}\right)\right]}{e^{-j\omega/2}\left[\frac{1}{2}\left(e^{j\omega/2} + e^{j\omega/2}\right)\right]}$$

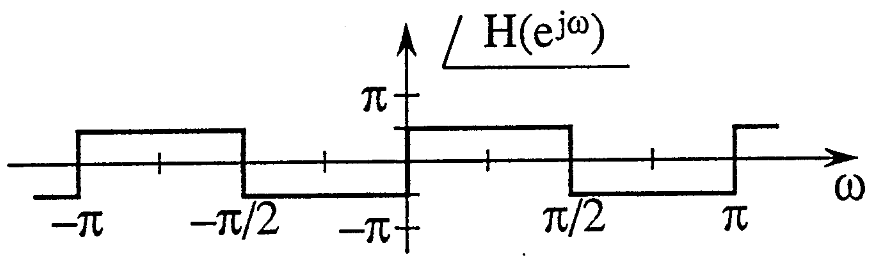
$$= j\,\frac{\sin(\omega/2)}{\cos(\omega/2)}$$

$$= j\,\tan(\omega/2)$$

$$|H(e^{j\omega})| = |\tan(\omega/2)|$$

$$\angle H(e^{j\omega}) = \begin{cases} \pi/2, & \tan(\omega/2) \geq 0 \\ \pi/2 \pm \pi, & \tan(\omega/2) < 0 \end{cases}$$





Comments

1. What happens at $\omega = \pi/2$?

2. Factoring out the half angle is possible only for a relatively restricted class of filters.