filtering in Chapters 3–5 and in our examination of systems described by linear constant-coefficient differential or difference equations, and we will gain a further appreciation for its utility in Chapter 6, in which we examine filtering and time-versus-frequency issues in more detail. In addition, the multiplication properties in continuous and discrete time are essential to our development of sampling in Chapter 7 and communications in Chapter 8.

Chapter 5 Problems

The first section of problems belongs to the basic category and the answers are provided in the back of the book. The remaining three sections contain problems belonging to the basic, advanced, and extension categories, respectively.

BASIC PROBLEMS WITH ANSWERS

5.1. Use the Fourier transform analysis equation (5.9) to calculate the Fourier transforms of:

(a) $(\frac{1}{2})^{n-1}u[n-1]$ **(b)** $(\frac{1}{2})^{|n-1|}$

Sketch and label one period of the magnitude of each Fourier transform.

5.2. Use the Fourier transform analysis equation (5.9) to calculate the Fourier transforms of:

(a) $\delta[n-1]+\delta[n+1]$ **(b)** $\delta[n+2]-\delta[n-2]$

Sketch and label one period of the magnitude of each Fourier transform.

5.3. Determine the Fourier transform for $-\pi \le \omega < \pi$ in the case of each of the following periodic signals:

(a) $\sin(\frac{\pi}{3}n+\frac{\pi}{4})$ **(b)** $2+\cos(\frac{\pi}{6}n+\frac{\pi}{8})$

5.4. Use the Fourier transform synthesis equation (5.8) to determine the inverse Fourier transforms of:

(a) $X_1(e^{j\omega}) = \sum_{k=-\infty}^{\infty}\{2\pi\delta(\omega-2\pi k)+\pi\delta(\omega-\frac{\pi}{2}-2\pi k)+\pi\delta(\omega+\frac{\pi}{2}-2\pi k)\}$

(b) $X_2(e^{j\omega}) = \begin{cases} 2j, & 0<\omega\le\pi \\ -2j, & -\pi<\omega\le 0 \end{cases}$

5.5. Use the Fourier transform synthesis equation (5.8) to determine the inverse Fourier transform of $X(e^{j\omega}) = |X(e^{j\omega})|e^{j\sphericalangle X(e^{j\omega})}$, where

$$|X(e^{j\omega})| = \begin{cases} 1, & 0\le|\omega|<\frac{\pi}{4} \\ 0, & \frac{\pi}{4}\le|\omega|\le\pi \end{cases} \quad \text{and } \sphericalangle X(e^{j\omega}) = -\frac{3\omega}{2}.$$

Use your answer to determine the values of n for which $x[n]=0$.

5.6. Given that $x[n]$ has Fourier transform $X(e^{j\omega})$, express the Fourier transforms of the following signals in terms of $X(e^{j\omega})$. You may use the Fourier transform properties listed in Table 5.1.

(a) $x_1[n] = x[1-n]+x[-1-n]$

(b) $x_2[n] = \frac{x^*[-n]+x[n]}{2}$

(c) $x_3[n] = (n-1)^2x[n]$

5.7. For each of the following Fourier transforms, use Fourier transform properties (Table 5.1) to determine whether the corresponding time-domain signal is (i) real, imaginary, or neither and (ii) even, odd, or neither. Do this without evaluating the inverse of any of the given transforms.

(a) $X_1(e^{j\omega}) = e^{-j\omega}\sum_{k=1}^{10}(\sin k\omega)$

(b) $X_2(e^{j\omega}) = j\sin(\omega)\cos(5\omega)$

(c) $X_3(e^{j\omega}) = A(\omega) + e^{jB(\omega)}$ where

$$A(\omega) = \begin{cases} 1, & 0 \leq |\omega| \leq \frac{\pi}{8} \\ 0, & \frac{\pi}{8} < |\omega| \leq \pi \end{cases} \quad \text{and } B(\omega) = -\frac{3\omega}{2} + \pi.$$

5.8. Use Tables 5.1 and 5.2 to help determine $x[n]$ when its Fourier transform is

$$X(e^{j\omega}) = \frac{1}{1 - e^{-j\omega}}\left(\frac{\sin \frac{3}{2}\omega}{\sin \frac{\omega}{2}}\right) + 5\pi\delta(\omega), \quad -\pi < \omega \leq \pi$$

5.9. The following four facts are given about a real signal $x[n]$ with Fourier transform $X(e^{jw})$:

1. $x[n] = 0$ for $n > 0$.
2. $x[0] > 0$.
3. $\mathcal{Im}\{X(e^{j\omega})\} = \sin\omega - \sin 2\omega$.
4. $\frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 \, d\omega = 3$.

Determine $x[n]$.

5.10. Use Tables 5.1 and 5.2 in conjunction with the fact that

$$X(e^{j0}) = \sum_{n=-\infty}^{\infty} x[n]$$

to determine the numerical value of

$$A = \sum_{n=0}^{\infty} n\left(\frac{1}{2}\right)^n.$$

5.11. Consider a signal $g[n]$ with Fourier transform $G(e^{j\omega})$. Suppose

$$g[n] = x_{(2)}[n],$$

where the signal $x[n]$ has a Fourier transform $X(e^{j\omega})$. Determine a real number α such that $0 < \alpha < 2\pi$ and $G(e^{j\omega}) = G(e^{j(\omega - \alpha)})$.

5.12. Let

$$y[n] = \left(\frac{\sin \frac{\pi}{4}n}{\pi n}\right)^2 * \left(\frac{\sin \omega_c n}{\pi n}\right),$$

where $*$ denotes convolution and $|\omega_c| \leq \pi$. Determine a stricter constraint on ω_c

which ensures that

$$y[n] = \left(\frac{\sin \frac{\pi}{4} n}{\pi n}\right)^2.$$

5.13. An LTI system with impulse response $h_1[n] = (\frac{1}{3})^n u[n]$ is connected in parallel with another causal LTI system with impulse response $h_2[n]$. The resulting parallel interconnection has the frequency response

$$H(e^{j\omega}) = \frac{-12 + 5e^{-j\omega}}{12 - 7e^{-j\omega} + e^{-j2\omega}}.$$

Determine $h_2[n]$.

5.14. Suppose we are given the following facts about an LTI system S with impulse response $h[n]$ and frequency response $H(e^{j\omega})$:

1. $(\frac{1}{4})^n u[n] \longrightarrow g[n]$, where $g[n] = 0$ for $n \geq 2$ and $n < 0$.
2. $H(e^{j\pi/2}) = 1$.
3. $H(e^{j\omega}) = H(e^{j(\omega - \pi)})$.

Determine $h[n]$.

5.15. Let the inverse Fourier transform of $Y(e^{j\omega})$ be

$$y[n] = \left(\frac{\sin \omega_c n}{\pi n}\right)^2,$$

where $0 < \omega_c < \pi$. Determine the value of ω_c which ensures that

$$Y(e^{j\pi}) = \frac{1}{2}.$$

5.16. The Fourier transform of a particular signal is

$$X(e^{j\omega}) = \sum_{k=0}^{3} \frac{(1/2)^k}{1 - \frac{1}{4} e^{-j(\omega - \pi/2k)}}.$$

It can be shown that

$$x[n] = g[n]q[n],$$

where $g[n]$ is of the form $\alpha^n u[n]$ and $q[n]$ is a periodic signal with period N.

(a) Determine the value of α.
(b) Determine the value of N.
(c) Is $x[n]$ real?

5.17. The signal $x[n] = (-1)^n$ has a fundamental period of 2 and corresponding Fourier series coefficients a_k. Use duality to determine the Fourier series coefficients b_k of the signal $g[n] = a_n$ with a fundamental period of 2.

5.18. Given the fact that

$$a^{|n|} \overset{\mathfrak{F}}{\longleftrightarrow} \frac{1 - a^2}{1 - 2a\cos\omega + a^2}, \quad |a| < 1,$$

use duality to determine the Fourier series coefficients of the following continuous-time signal with period $T = 1$:

$$x(t) = \frac{1}{5 - 4\cos(2\pi t)}.$$

5.19. Consider a causal and stable LTI system S whose input $x[n]$ and output $y[n]$ are related through the second-order difference equation

$$y[n] - \frac{1}{6}y[n-1] - \frac{1}{6}y[n-2] = x[n].$$

(a) Determine the frequency response $H(e^{j\omega})$ for the system S.
(b) Determine the impulse response $h[n]$ for the system S.

5.20. A causal and stable LTI system S has the property that

$$\left(\frac{4}{5}\right)^n u[n] \longrightarrow n\left(\frac{4}{5}\right)^n u[n].$$

(a) Determine the frequency response $H(e^{j\omega})$ for the system S.
(b) Determine a difference equation relating any input $x[n]$ and the corresponding output $y[n]$.

BASIC PROBLEMS

5.21. Compute the Fourier transform of each of the following signals:
(a) $x[n] = u[n-2] - u[n-6]$
(b) $x[n] = (\frac{1}{2})^{-n}u[-n-1]$
(c) $x[n] = (\frac{1}{3})^{|n|}u[-n-2]$
(d) $x[n] = 2^n \sin(\frac{\pi}{4}n)u[-n]$
(e) $x[n] = (\frac{1}{2})^{|n|}\cos(\frac{\pi}{8}(n-1))$
(f) $x[n] = \begin{cases} n, & -3 \le n \le 3 \\ 0, & \text{otherwise} \end{cases}$
(g) $x[n] = \sin(\frac{\pi}{2}n) + \cos(n)$
(h) $x[n] = \sin(\frac{5\pi}{3}n) + \cos(\frac{7\pi}{3}n)$
(i) $x[n] = x[n-6]$, and $x[n] = u[n] - u[n-5]$ for $0 \le n \le 5$
(j) $x[n] = (n-1)(\frac{1}{3})^{|n|}$
(k) $x[n] = (\frac{\sin(\pi n/5)}{\pi n})\cos(\frac{7\pi}{2}n)$

5.22. The following are the Fourier transforms of discrete-time signals. Determine the signal corresponding to each transform.
(a) $X(e^{j\omega}) = \begin{cases} 1, & \frac{\pi}{4} \le |\omega| \le \frac{3\pi}{4} \\ 0, & \frac{3\pi}{4} \le |\omega| \le \pi, 0 \le |\omega| < \frac{\pi}{4} \end{cases}$
(b) $X(e^{j\omega}) = 1 + 3e^{-j\omega} + 2e^{-j2\omega} - 4e^{-j3\omega} + e^{-j10\omega}$
(c) $X(e^{j\omega}) = e^{-j\omega/2}$ for $-\pi \le \omega \le \pi$
(d) $X(e^{j\omega}) = \cos^2\omega + \sin^2 3\omega$

(e) $X(e^{j\omega}) = \sum_{k=-\infty}^{\infty}(-1)^k\delta(\omega - \frac{\pi}{2}k)$
(f) $X(e^{j\omega}) = \frac{e^{-j\omega}-\frac{1}{5}}{1-\frac{1}{5}e^{-j\omega}}$
(g) $X(e^{j\omega}) = \frac{1-\frac{1}{3}e^{-j\omega}}{1-\frac{1}{4}e^{-j\omega}-\frac{1}{8}e^{-2j\omega}}$
(h) $X(e^{j\omega}) = \frac{1-(\frac{1}{3})^6 e^{-j6\omega}}{1-\frac{1}{3}e^{-j\omega}}$

5.23. Let $X(e^{j\omega})$ denote the Fourier transform of the signal $x[n]$ depicted in Figure P5.23. Perform the following calculations without explicitly evaluating $X(e^{j\omega})$:
(a) Evaluate $X(e^{j0})$.
(b) Find $\sphericalangle X(e^{j\omega})$.
(c) Evaluate $\int_{-\pi}^{\pi} X(e^{j\omega})d\omega$.
(d) Find $X(e^{j\pi})$.
(e) Determine and sketch the signal whose Fourier transform is $\mathcal{R}e\{x(\omega)\}$.
(f) Evaluate:
(i) $\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$
(ii) $\int_{-\pi}^{\pi} \left|\frac{dX(e^{j\omega})}{d\omega}\right|^2 d\omega$

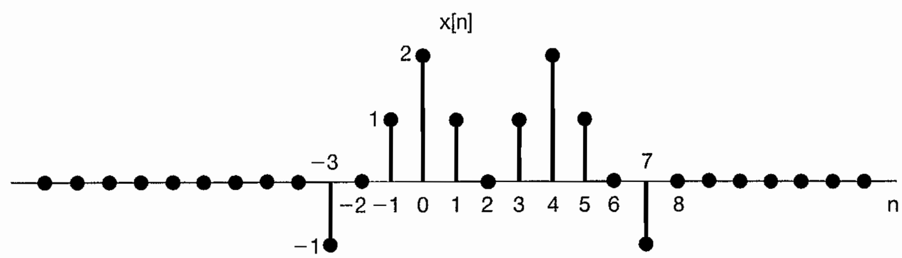


Fig P5.23

5.24. Determine which, if any, of the following signals have Fourier transforms that satisfy each of the following conditions:

1. $\mathcal{R}e\{X(e^{j\omega})\} = 0$.
2. $\mathcal{I}m\{X(e^{j\omega})\} = 0$.
3. There exists an integer α such that $e^{j\alpha\omega}X(e^{j\omega})$ is real.
4. $\int_{-\pi}^{\pi} X(e^{j\omega})d\omega = 0$.
5. $X(e^{j\omega})$ periodic.
6. $X(e^{j0}) = 0$.

(a) $x[n]$ as in Figure P5.24(a)
(b) $x[n]$ as in Figure P5.24(b)
(c) $x[n] = (\frac{1}{2})^n u[n]$
(d) $x[n] = (\frac{1}{2})^{|n|}$
(e) $x[n] = \delta[n-1] + \delta[n+2]$
(f) $x[n] = \delta[n-1] + \delta[n+3]$
(g) $x[n]$ as in Figure P5.24(c)
(h) $x[n]$ as in Figure P5.24(d)
(i) $x[n] = \delta[n-1] - \delta[n+1]$

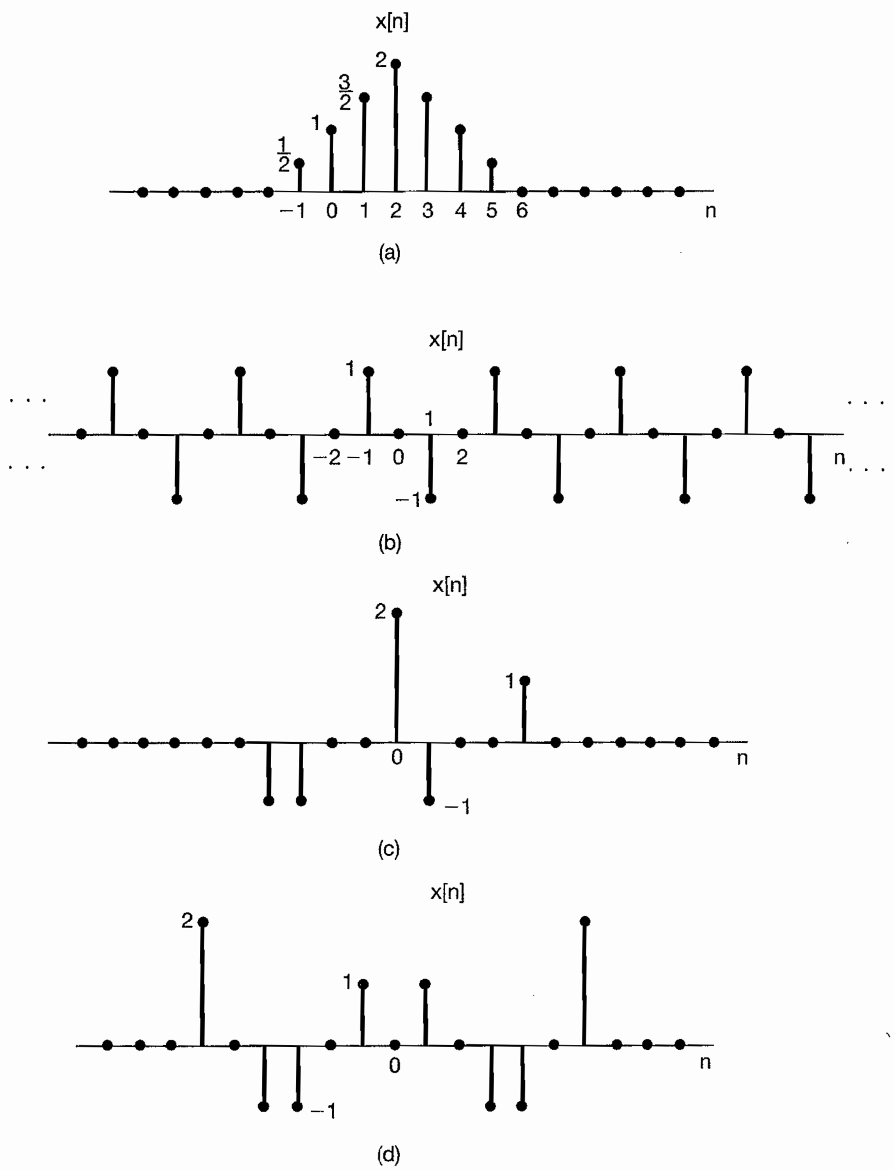


Fig P5.24

5.25. Consider the signal depicted in Figure P5.25. Let the Fourier transform of this signal be written in rectangular form as

$$X(e^{j\omega}) = A(\omega) + jB(\omega).$$

Sketch the function of time corresponding to the transform

$$Y(e^{j\omega}) = [B(\omega) + A(\omega)e^{j\omega}].$$

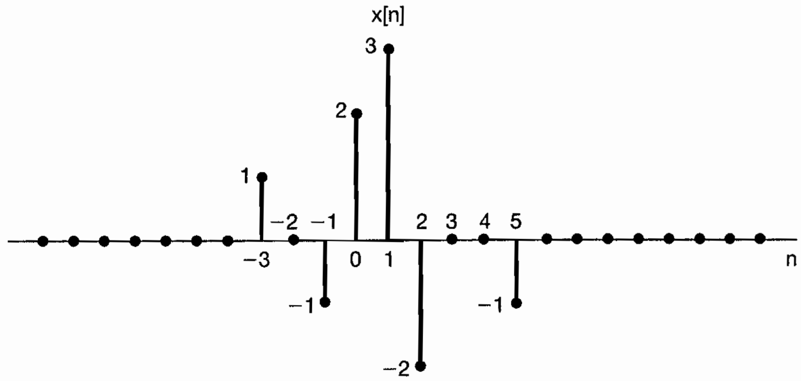


Fig P5.25

5.26. Let $x_1[n]$ be the discrete-time signal whose Fourier transform $X_1(e^{j\omega})$ is depicted in Figure P5.26(a).

(a) Consider the signal $x_2[n]$ with Fourier transform $X_2(e^{j\omega})$, as illustrated in Figure P5.26(b). Express $x_2[n]$ in terms of $x_1[n]$. [*Hint:* First express $X_2(e^{j\omega})$ in terms of $X_1(e^{j\omega})$, and then use properties of the Fourier transform.]

(b) Repeat part (a) for $x_3[n]$ with Fourier transform $X_3(e^{j\omega})$, as shown in Figure P5.26(c).

(c) Let

$$\alpha = \frac{\sum_{n=-\infty}^{\infty} n x_1[n]}{\sum_{n=-\infty}^{\infty} x_1[n]}.$$

This quantity, which is the center of gravity of the signal $x_1[n]$, is usually referred to as the *delay time* of $x_1[n]$. Find α. (You can do this without first determining $x_1[n]$ explicitly.)

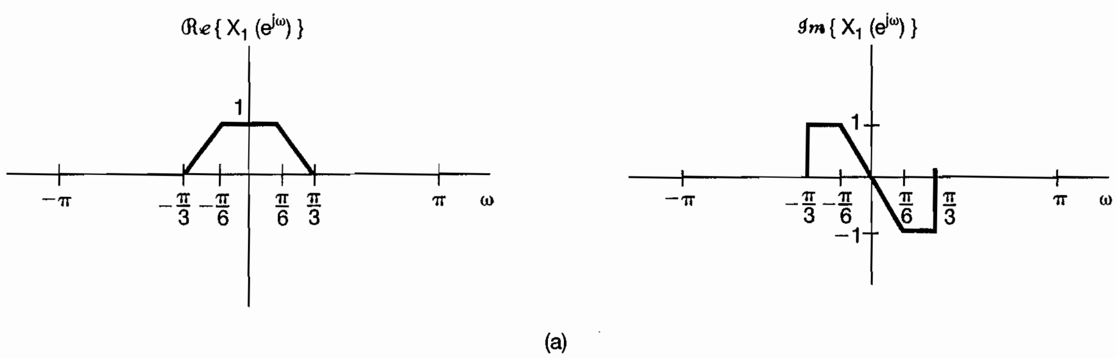
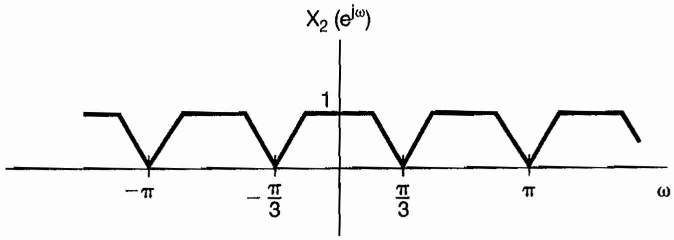
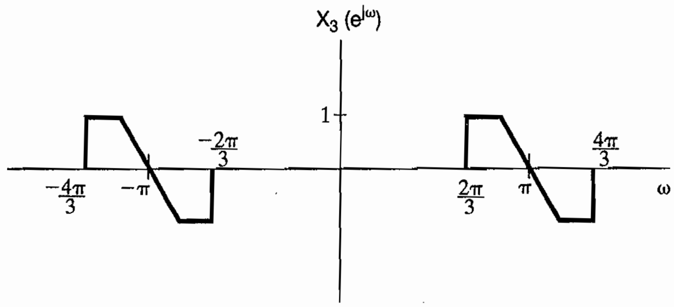


Fig P5.26a



(b)



(c)

Fig P5.26b,c

(d) Consider the signal $x_4[n] = x_1[n] * h[n]$, where

$$h[n] = \frac{\sin(\pi n/6)}{\pi n}.$$

Sketch $X_4(e^{j\omega})$.

5.27. (a) Let $x[n]$ be a discrete-time signal with Fourier transform $X(e^{j\omega})$, which is illustrated in Figure P5.27. Sketch the Fourier transform of

$$w[n] = x[n]p[n]$$

for each of the following signals $p[n]$:

(i) $p[n] = \cos \pi n$

(ii) $p[n] = \cos(\pi n/2)$

(iii) $p[n] = \sin(\pi n/2)$

(iv) $p[n] = \sum_{k=-\infty}^{\infty} \delta[n - 2k]$

(v) $p[n] = \sum_{k=-\infty}^{\infty} \delta[n - 4k]$

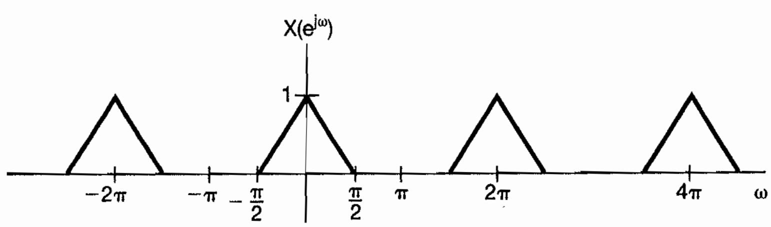


Fig P5.27

(b) Suppose that the signal $w[n]$ of part (a) is applied as the input to an LTI system with unit sample response

$$h[n] = \frac{\sin(\pi n/2)}{\pi n}.$$

Determine the output $y[n]$ for each of the choices of $p[n]$ in part (a).

5.28. The signals $x[n]$ and $g[n]$ are known to have Fourier transforms $X(e^{j\omega})$ and $G(e^{j\omega})$, respectively. Furthermore, $X(e^{j\omega})$ and $G(e^{j\omega})$ are related as follows:

$$\frac{1}{2\pi}\int_{-\pi}^{+\pi} X(e^{j\theta})G(e^{j(\omega-\theta)})d\theta = 1 + e^{-j\omega} \qquad \text{(P5.28–1)}$$

(a) If $x[n] = (-1)^n$, determine a sequence $g[n]$ such that its Fourier transform $G(e^{j\omega})$ satisfies eq. (P5.28–1). Are there other possible solutions for $g[n]$?

(b) Repeat the previous part for $x[n] = (\frac{1}{2})^n u[n]$.

5.29. (a) Consider a discrete-time LTI system with impulse response

$$h[n] = \left(\frac{1}{2}\right)^n u[n].$$

Use Fourier transforms to determine the response to each of the following input signals:

(i) $x[n] = (\frac{3}{4})^n u[n]$

(ii) $x[n] = (n+1)(\frac{1}{4})^n u[n]$

(iii) $x[n] = (-1)^n$

(b) Suppose that

$$h[n] = \left[\left(\frac{1}{2}\right)^n \cos\left(\frac{\pi n}{2}\right)\right] u[n].$$

Use Fourier transforms to determine the response to each of the following inputs:

(i) $x[n] = (\frac{1}{2})^n u[n]$

(ii) $x[n] = \cos(\pi n/2)$

(c) Let $x[n]$ and $h[n]$ be signals with the following Fourier transforms:

$$X(e^{j\omega}) = 3e^{j\omega} + 1 - e^{-j\omega} + 2e^{-j3\omega},$$
$$H(e^{j\omega}) = -e^{j\omega} + 2e^{-2j\omega} + e^{j4\omega}.$$

Determine $y[n] = x[n] * h[n]$.

5.30. In Chapter 4, we indicated that the continuous-time LTI system with impulse response

$$h(t) = \frac{W}{\pi}\operatorname{sinc}\left(\frac{Wt}{\pi}\right) = \frac{\sin Wt}{\pi t}$$

plays a very important role in LTI system analysis. The same is true of the discrete-time LTI system with impulse response

$$h[n] = \frac{W}{\pi}\operatorname{sinc}\left(\frac{Wn}{\pi}\right) = \frac{\sin Wn}{\pi n}.$$

(a) Determine and sketch the frequency response for the system with impulse response $h[n]$.

(b) Consider the signal

$$x[n] = \sin\left(\frac{\pi n}{8}\right) - 2\cos\left(\frac{\pi n}{4}\right).$$

Suppose that this signal is the input to LTI systems with the following impulse responses. Determine the output in each case.

(i) $h[n] = \frac{\sin(\pi n/6)}{\pi n}$

(ii) $h[n] = \frac{\sin(\pi n/6)}{\pi n} + \frac{\sin(\pi n/2)}{\pi n}$

(iii) $h[n] = \frac{\sin(\pi n/6)\sin(\pi n/3)}{\pi^2 n^2}$

(iv) $h[n] = \frac{\sin(\pi n/6)\sin(\pi n/3)}{\pi n}$

(c) Consider an LTI system with unit sample response

$$h[n] = \frac{\sin(\pi n/3)}{\pi n}.$$

Determine the output for each of the following inputs:

(i) $x[n] =$ the square wave depicted in Figure P5.30

(ii) $x[n] = \sum_{k=-\infty}^{\infty} \delta[n - 8k]$

(iii) $x[n] = (-1)^n$ times the square wave depicted in Figure P5.30

(iv) $x[n] = \delta[n + 1] + \delta[n - 1]$

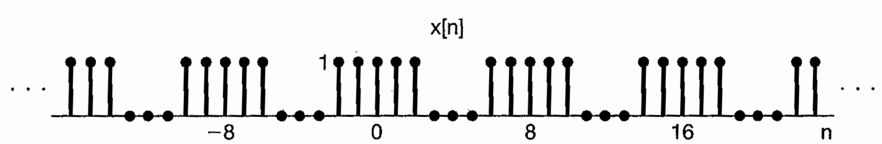


Fig P5.30

5.31. An LTI system S with impulse response $h[n]$ and frequency response $H(e^{j\omega})$ is known to have the property that, when $-\pi \leq \omega_0 \leq \pi$,

$$\cos \omega_0 n \longrightarrow \omega_0 \cos \omega_0 n.$$

(a) Determine $H(e^{j\omega})$.

(b) Determine $h[n]$.

5.32. Let $h_1[n]$ and $h_2[n]$ be the impulse responses of causal LTI systems, and let $H_1(e^{j\omega})$ and $H_2(e^{j\omega})$ be the corresponding frequency responses. Under these conditions, is the following equation true in general or not? Justify your answer.

$$\left[\frac{1}{2\pi}\int_{-\pi}^{\pi} H_1(e^{j\omega})d\omega\right]\left[\frac{1}{2\pi}\int_{-\pi}^{\pi} H_2(e^{j\omega})d\omega\right] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_1(e^{j\omega})H_2(e^{j\omega})d\omega.$$

5.33. Consider a causal LTI system described by the difference equation

$$y[n] + \frac{1}{2}y[n-1] = x[n].$$

(a) Determine the frequency response $H(e^{j\omega})$ of this system.
(b) What is the response of the system to the following inputs?
  (i) $x[n] = (\frac{1}{2})^n u[n]$
  (ii) $x[n] = (-\frac{1}{2})^n u[n]$
  (iii) $x[n] = \delta[n] + \frac{1}{2}\delta[n-1]$
  (iv) $x[n] = \delta[n] - \frac{1}{2}\delta[n-1]$
(c) Find the response to the inputs with the following Fourier transforms:
  (i) $X(e^{j\omega}) = \frac{1-\frac{1}{4}e^{-j\omega}}{1+\frac{1}{2}e^{-j\omega}}$
  (ii) $X(e^{j\omega}) = \frac{1+\frac{1}{2}e^{-j\omega}}{1-\frac{1}{4}e^{-j\omega}}$
  (iii) $X(e^{j\omega}) = \frac{1}{(1-\frac{1}{4}e^{-j\omega})(1+\frac{1}{2}e^{-j\omega})}$
  (iv) $X(e^{j\omega}) = 1 + 2e^{-3j\omega}$

5.34. Consider a system consisting of the cascade of two LTI systems with frequency responses

$$H_1(e^{j\omega}) = \frac{2 - e^{-j\omega}}{1 + \frac{1}{2}e^{-j\omega}}$$

and

$$H_2(e^{j\omega}) = \frac{1}{1 - \frac{1}{2}e^{-j\omega} + \frac{1}{4}e^{-j2\omega}}.$$

(a) Find the difference equation describing the overall system.
(b) Determine the impulse response of the overall system.

5.35. A causal LTI system is described by the difference equation

$$y[n] - ay[n-1] = bx[n] + x[n-1],$$

where a is real and less than 1 in magnitude.
(a) Find a value of b such that the frequency response of the system satisfies

$$|H(e^{j\omega})| = 1, \text{ for all } \omega.$$

This kind of system is called an *all-pass system,* as it does not attenuate the input $e^{j\omega n}$ for *any* value of ω. Use the value of b that you have found in the rest of the problem.
(b) Roughly sketch $\sphericalangle H(e^{j\omega})$, $0 \le \omega \le \pi$, when $a = \frac{1}{2}$.
(c) Roughly sketch $\sphericalangle H(e^{j\omega})$, $0 \le \omega \le \pi$, when $a = -\frac{1}{2}$.

(d) Find and plot the output of this system with $a = -\frac{1}{2}$ when the input is

$$x[n] = \left(\frac{1}{2}\right)^n u[n].$$

From this example, we see that a nonlinear change in phase can have a significantly different effect on a signal than the time shift that results from a linear phase.

5.36. (a) Let $h[n]$ and $g[n]$ be the impulse responses of two stable discrete-time LTI systems that are inverses of each other. What is the relationship between the frequency responses of these two systems?

(b) Consider causal LTI systems described by the following difference equations. In each case, determine the impulse response of the inverse system and the difference equation that characterizes the inverse.

(i) $y[n] = x[n] - \frac{1}{4}x[n-1]$

(ii) $y[n] + \frac{1}{2}y[n-1] = x[n]$

(iii) $y[n] + \frac{1}{2}y[n-1] = x[n] - \frac{1}{4}x[n-1]$

(iv) $y[n] + \frac{5}{4}y[n-1] - \frac{1}{8}y[n-2] = x[n] - \frac{1}{4}x[n-1] - \frac{1}{8}x[n-2]$

(v) $y[n] + \frac{5}{4}y[n-1] - \frac{1}{8}y[n-2] = x[n] - \frac{1}{2}x[n-1]$

(vi) $y[n] + \frac{5}{4}y[n-1] - \frac{1}{8}y[n-2] = x[n]$

(c) Consider the causal, discrete-time LTI system described by the difference equation

$$y[n] + y[n-1] + \frac{1}{4}y[n-2] = x[n-1] - \frac{1}{2}x[n-2]. \qquad \text{(P5.36–1)}$$

What is the inverse of this system? Show that the inverse is not causal. Find another causal LTI system that is an "inverse with delay" of the system described by eq. (P5.36–1). Specifically, find a causal LTI system such that the output $w[n]$ in Figure P5.36 equals $x[n-1]$.

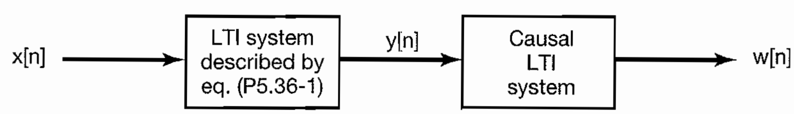


Fig P5.36

ADVANCED PROBLEMS

5.37. Let $X(e^{j\omega})$ be the Fourier transform of $x[n]$. Derive expressions in terms of $X(e^{j\omega})$ for the Fourier transforms of the following signals. (Do not assume that $x[n]$ is real.)

(a) $\mathcal{R}e\{x[n]\}$

(b) $x^*[-n]$

(c) $\mathcal{E}v\{x[n]\}$