CT LTI System Examples

System 1

$$y(t) = \int_{-\infty}^{t} e^{-a(t-\tau)} x(\tau)\, d\tau$$

Let: $x(t) = \delta(t) \Rightarrow y(t) = h(t)$

$$h(t) = \int_{-\infty}^{t} e^{-a(t-\tau)} \delta(\tau)\, d\tau$$

$$= \int_{-\infty}^{t} e^{-a(t-0)} \delta(\tau)\, d\tau$$

} Sifting Property of Dirac-Delta Function

$$= e^{-at} \int_{-\infty}^{t} \delta(\tau)\, d\tau$$

$$= e^{-at} u(t)$$

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System 2:

$$y(t) = -\frac{1}{T}\int_{t-T}^{t} (t-\tau-T)\,x(\tau)\,d\tau$$

Let $x(t) = \delta(t) \Rightarrow y(t) = h(t)$

$$h(t) = -\frac{1}{T}\int_{t-T}^{t} (t-\tau-T)\,\delta(\tau)\,d\tau$$

$$= -\frac{1}{T}\int_{t-T}^{t} (t-0-T)\,\delta(\tau)\,d\tau$$

$$= -\frac{1}{T}(t-T)\int_{t-T}^{t} \delta(\tau)\,d\tau \Bigg\} \quad \begin{matrix} t>0 \text{ \& } t-T<0 \\ t<T \end{matrix}$$

so $= 1$ if $0<t<T$

$= 0$ otherwise

$$= -\frac{1}{T}(t-T)\{u(t)-u(t-T)\}$$

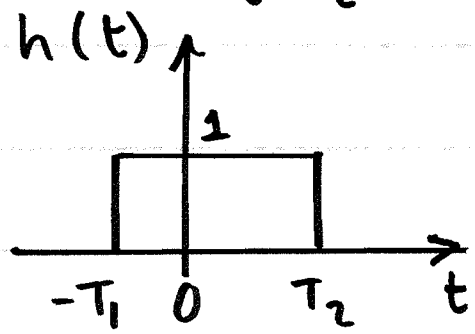
$$\text{rect}\left(\frac{t-\frac{T}{2}}{T}\right)$$

- **Common Examples:**

- **Integrator:** $y(t) = \int_{t-T_2}^{t+T_1} x(\tau)\, d\tau \qquad T_1 \geq 0, \; T_2 \geq 0$

Impulse response? $h(t) = ?$ let $x(t) = \delta(t)$

$$h(t) = \int_{t-T_2}^{t+T_1} \delta(\tau)\, d\tau = 1 \quad \text{if} \begin{cases} t+T_1 > 0 \Rightarrow t > -T_1 \\ t-T_2 < 0 \Rightarrow t < T_2 \end{cases}$$



$h(t) \neq 0$ for $t < 0$

causal? No, unless $T_1 = 0$

stable? Yes $\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$

as long as $T_2 < \infty$

- **Special case:** $T_1 = 0$ and $T_2 = \infty$

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau \Rightarrow h(t) = u(t) \Rightarrow \text{causal, but not stable}$$