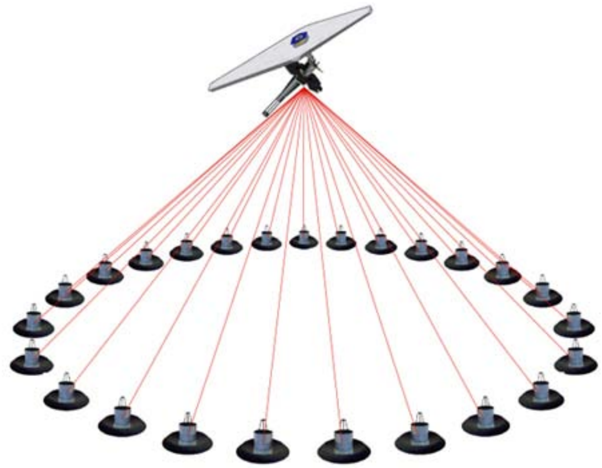
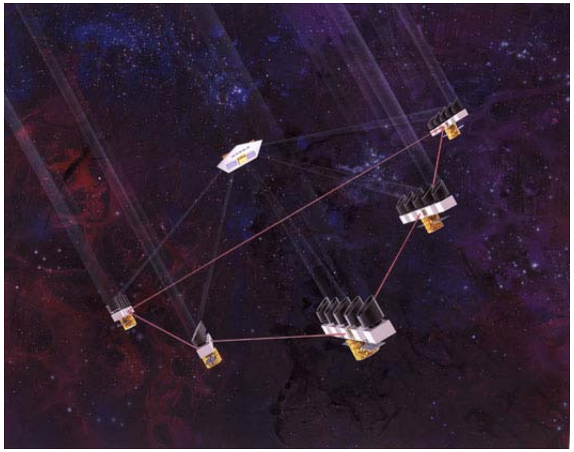
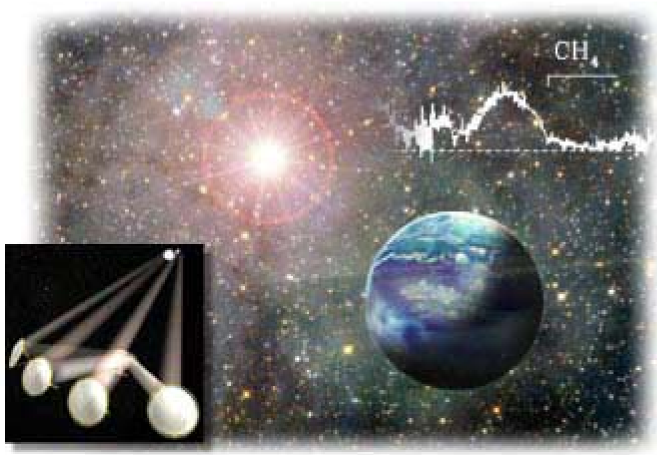


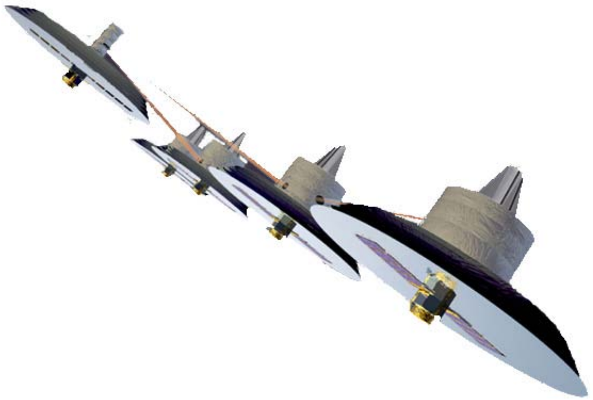


CONTROL STRATEGIES FOR FORMATION FLIGHT IN THE VICINITY OF THE LIBRATION POINTS

K.C. Howell and B.G. Marchand

Purdue University

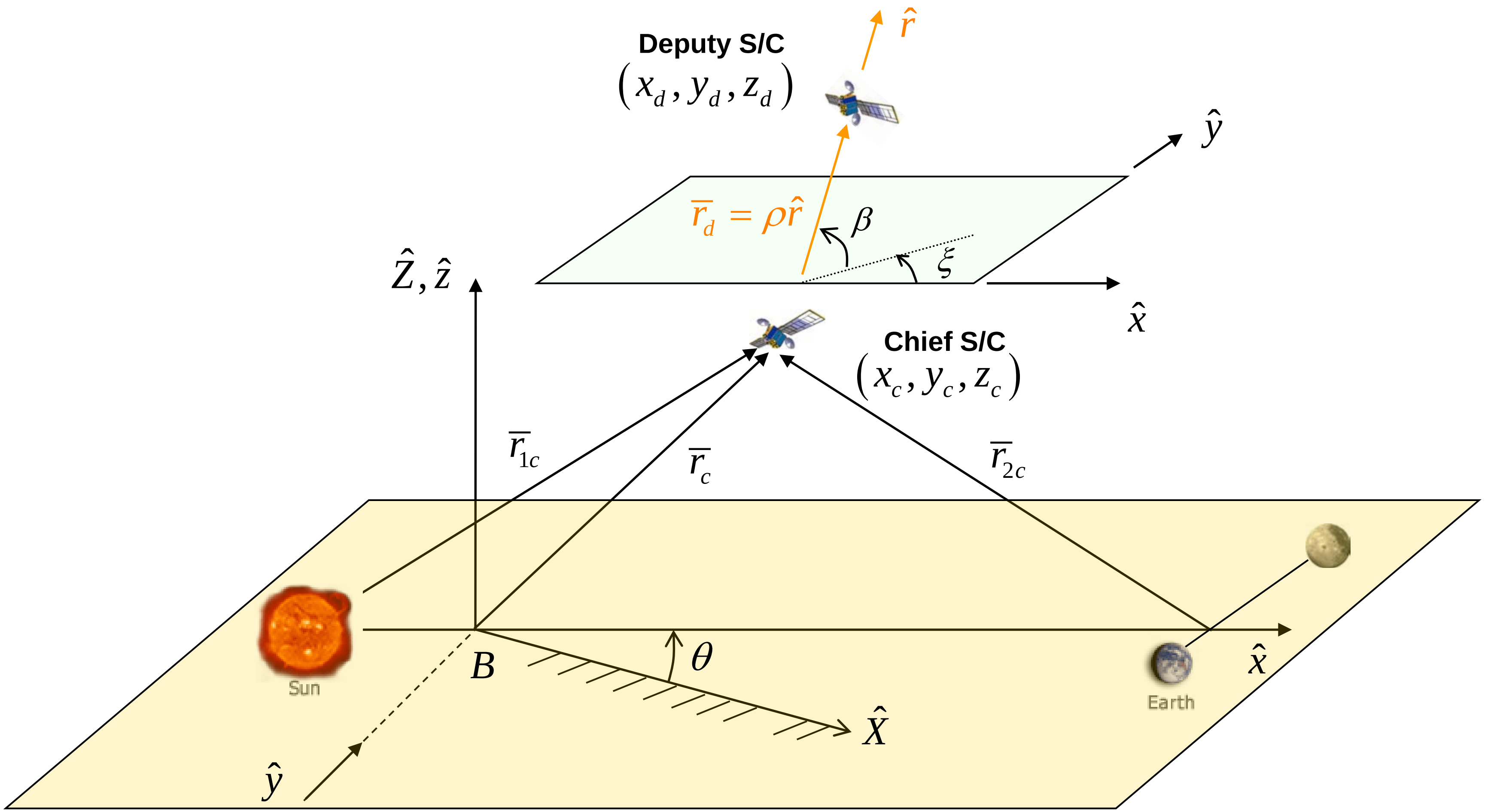




Previous Work on Formation Flight

- Multi-S/C Formations in the 2BP
  - Small Relative Separation (10 m – 1 km)
    - Model Relative Dynamics via the C-W Equations
    - Formation Control
      - LQR for Time Invariant Systems
      - Feedback Linearization
      - Lyapunov Based and Adaptive Control
- Multi-S/C Formations in the 3BP
  - Consider Wider Separation Range
    - Nonlinear model with complex reference motions
      - Periodic, Quasi-Periodic, Stable/Unstable Manifolds
    - Formation Control via simplified LQR techniques and "Gain Scheduling"-type methods.

2-S/C Formation Model in the Sun-Earth-Moon System



Dynamical Model

Nonlinear EOMs:

$$\ddot{\bar{r}}_c(t) = \bar{f}\left[\bar{r}_c(t)\right] + 2J\dot{\bar{r}}_c(t) + K\bar{r}_c(t) + \bar{u}_c(t)$$

$$\ddot{\bar{r}}_d(t) = \bar{f}\left[\bar{r}_c(t) + \bar{r}_d(t)\right] - \bar{f}\left[\bar{r}_c(t)\right] + 2J\dot{\bar{r}}_d(t) + K\bar{r}_d(t) + \bar{u}_d(t)$$

Linear System:

$$\underbrace{\begin{bmatrix} \dot{\bar{r}}_d(t) - \dot{\bar{r}}_d^{\circ}(t) \\ \ddot{\bar{r}}_d(t) - \ddot{\bar{r}}_d^{\circ}(t) \end{bmatrix}}_{\delta\dot{\bar{x}}_d(t)} = \underbrace{\begin{bmatrix} 0 & I \\ \Omega\left(\bar{r}_c(t), \bar{r}_d^{\circ}(t)\right) & 2J \end{bmatrix}}_{A(t)} \underbrace{\begin{bmatrix} \bar{r}_d(t) - \bar{r}_d^{\circ}(t) \\ \dot{\bar{r}}_d(t) - \dot{\bar{r}}_d^{\circ}(t) \end{bmatrix}}_{\delta\bar{x}_d(t)} + \underbrace{\begin{bmatrix} 0 \\ I \end{bmatrix}}_{B} \underbrace{\left(\bar{u}_d(t) - \bar{u}_d^{\circ}(t)\right)}_{\delta\bar{u}_d(t)}$$

Reference Motions

- Fixed Relative Distance and Orientation
  - Chief-Deputy Line Fixed Relative to the Rotating Frame

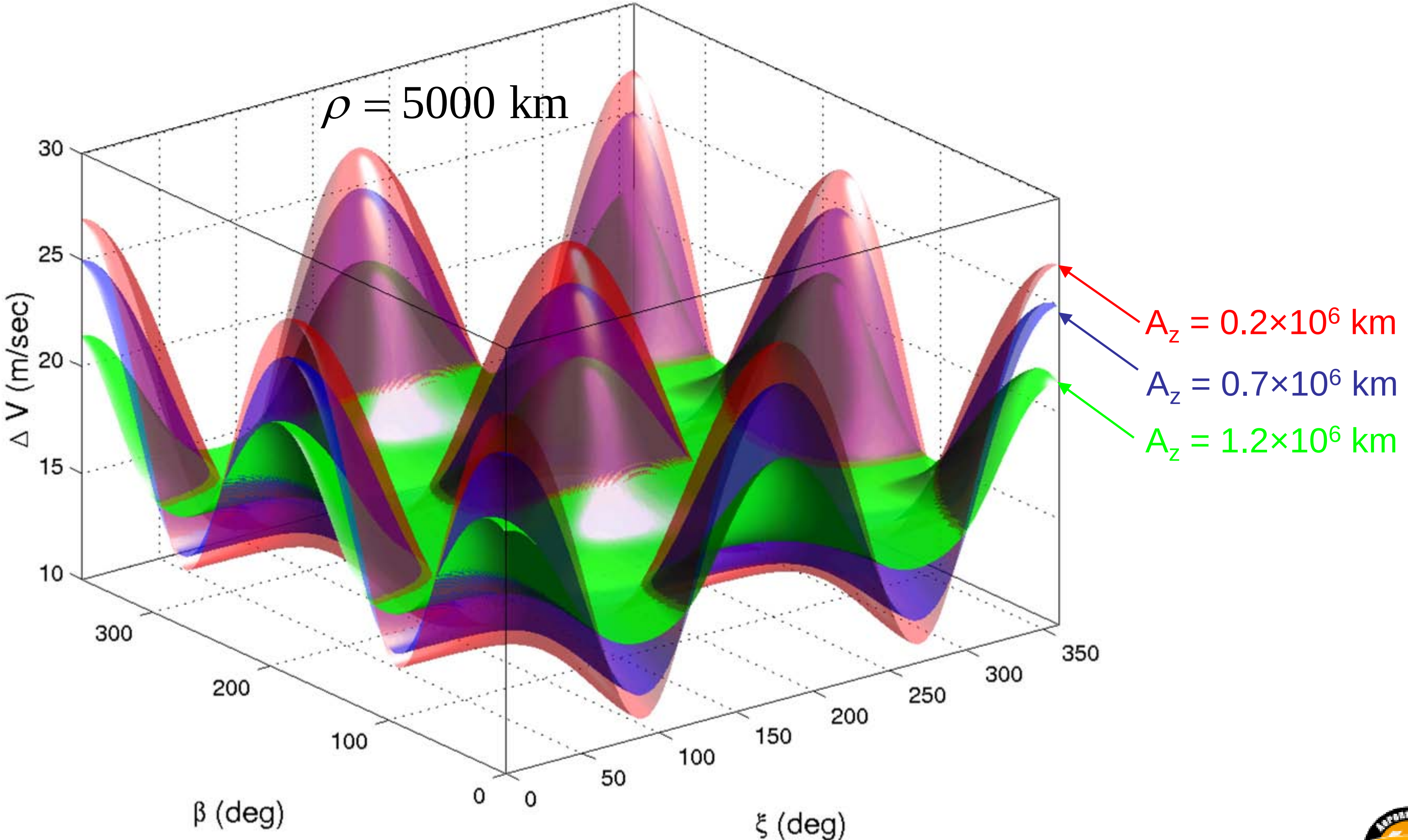
$$\bar{r}_d(t) = \bar{c} \quad \text{and} \quad \dot{\bar{r}}_d(t) = \bar{0}$$

  - Chief-Deputy Line Fixed Relative to the Inertial Frame

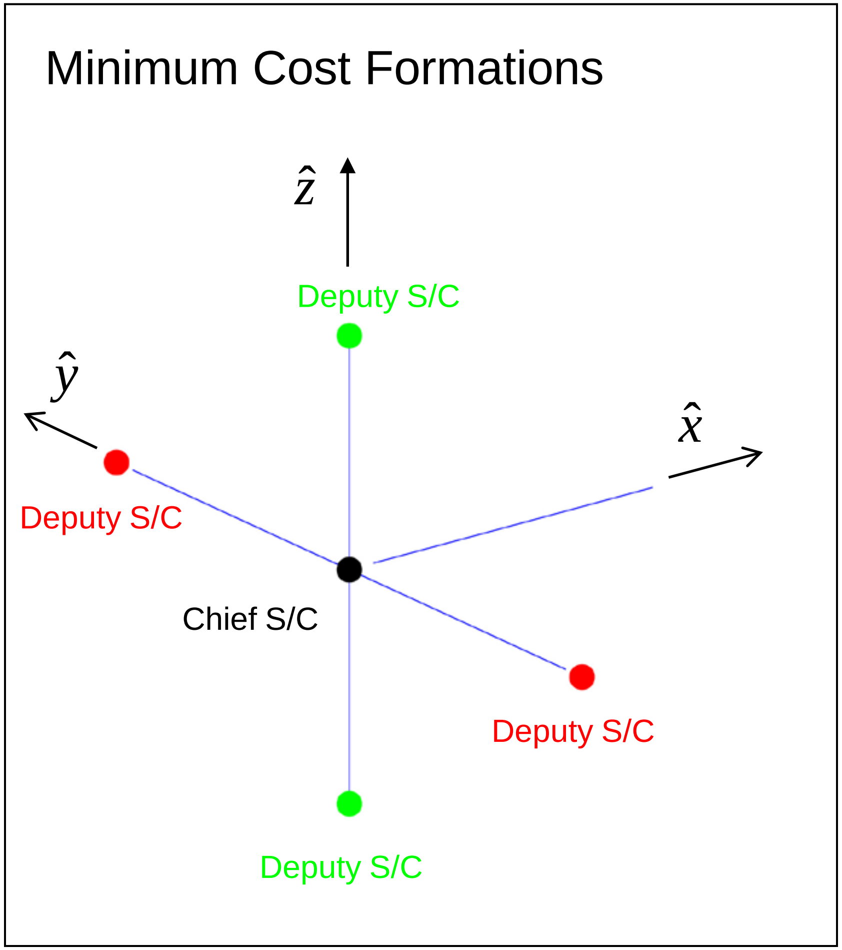
$$x_d(t) = x_{d0}\cos t + y_{d0}\sin t$$
$$y_d(t) = y_{d0}\cos t - x_{d0}\sin t$$
$$z_d(t) = z_{d0}$$

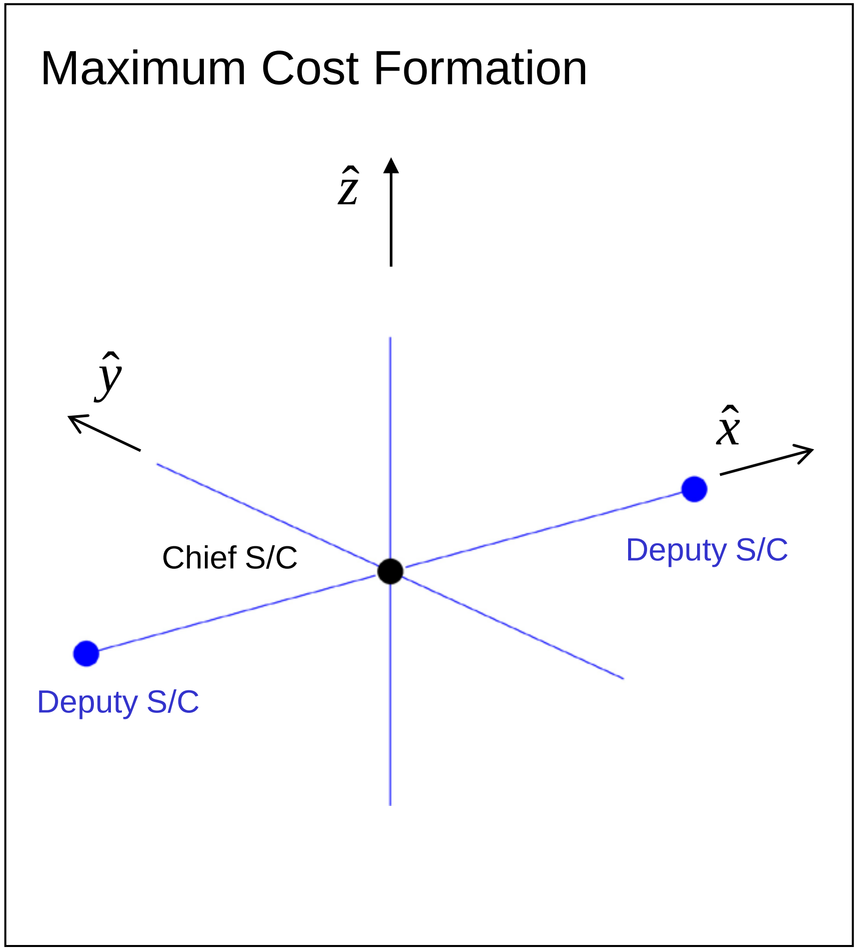
- Fixed Relative Distance, No Orientation Constraints
- Natural Formations (Center Manifold)
  - Deputy evolves along a quasi-periodic 2-D Torus that envelops the chief spacecraft's halo orbit (bounded motion)

Nominal Formation Keeping Cost
(Configurations Fixed in the Rotating Frame)

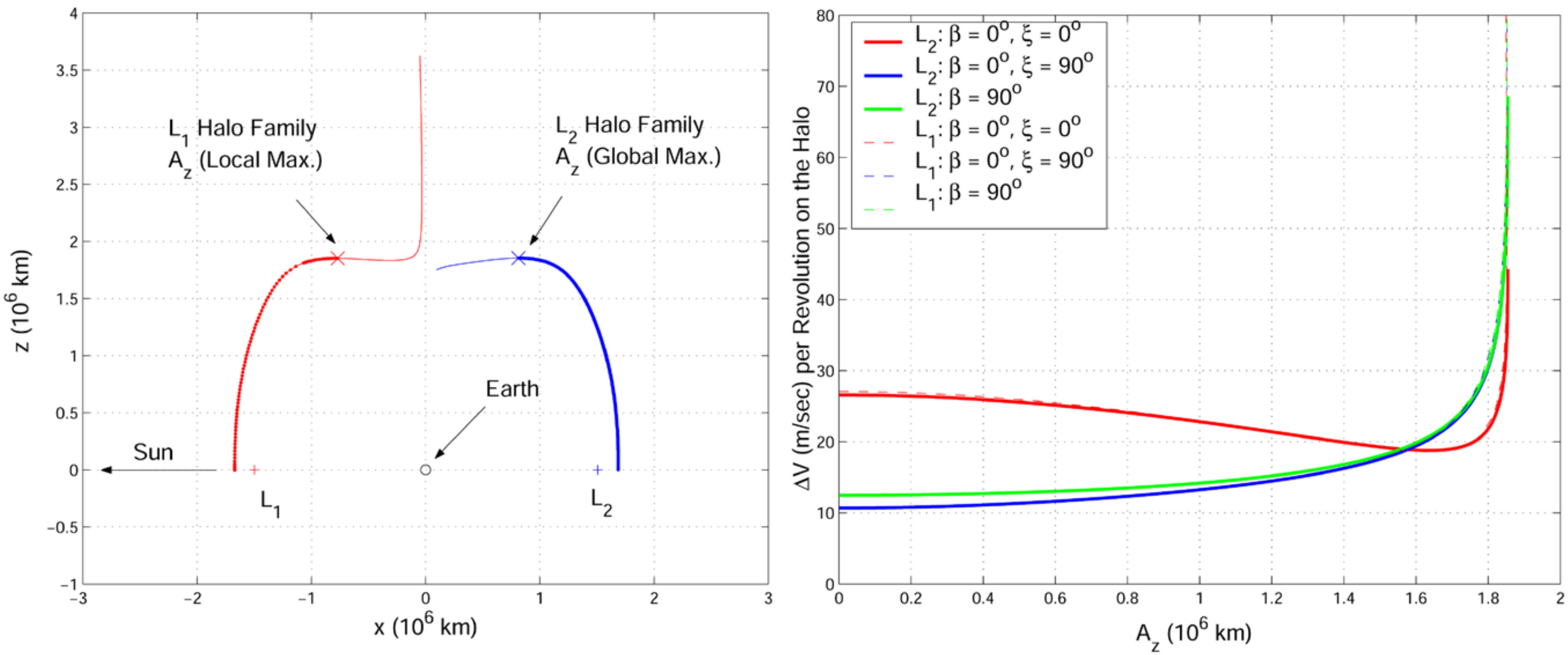


Max./Min. Cost Formations
(Configurations Fixed in the Rotating Frame)

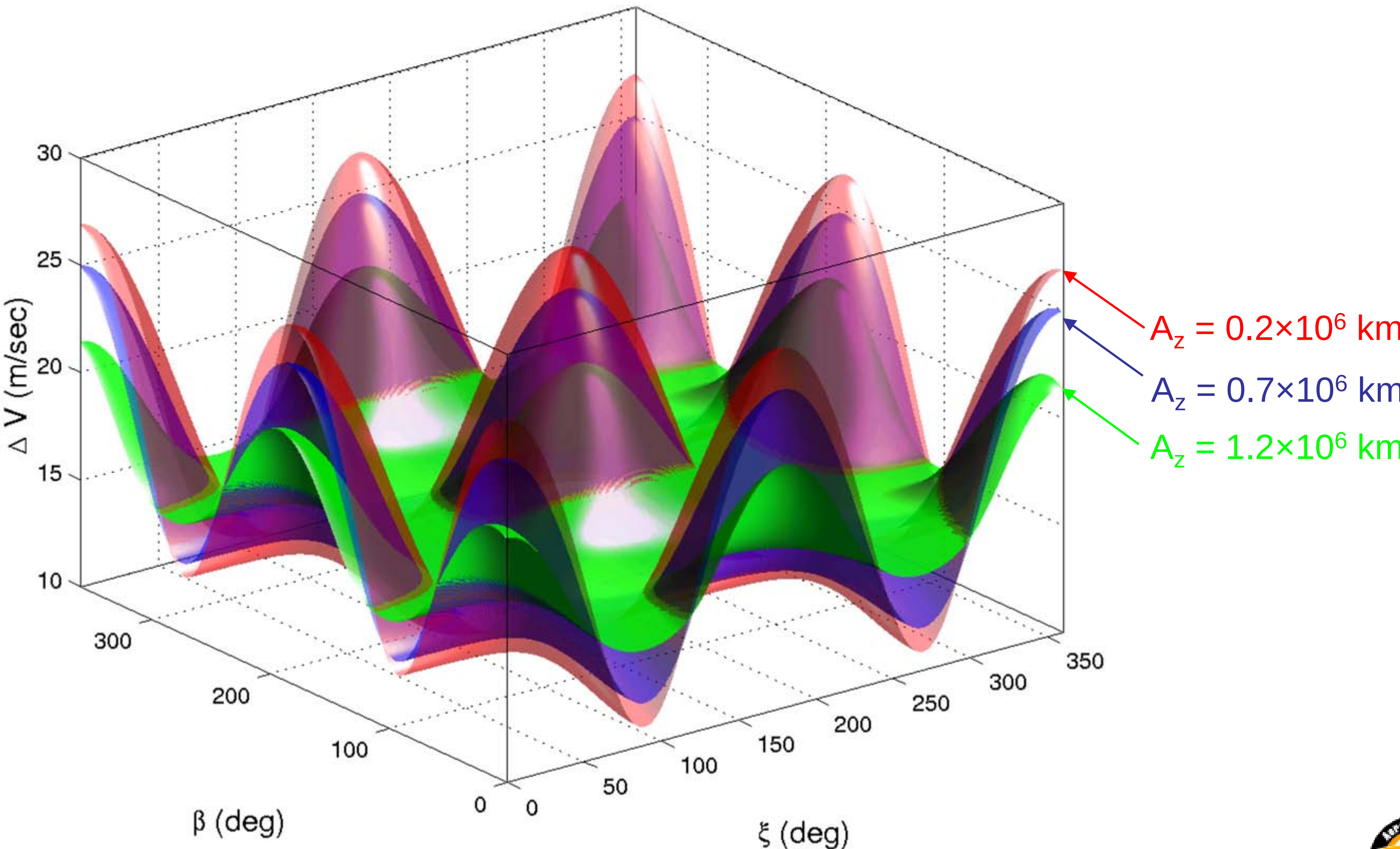




Formation Keeping Cost Variation Along the SEM L_1 and L_2 Halo Families (Configurations Fixed in the Rotating Frame)

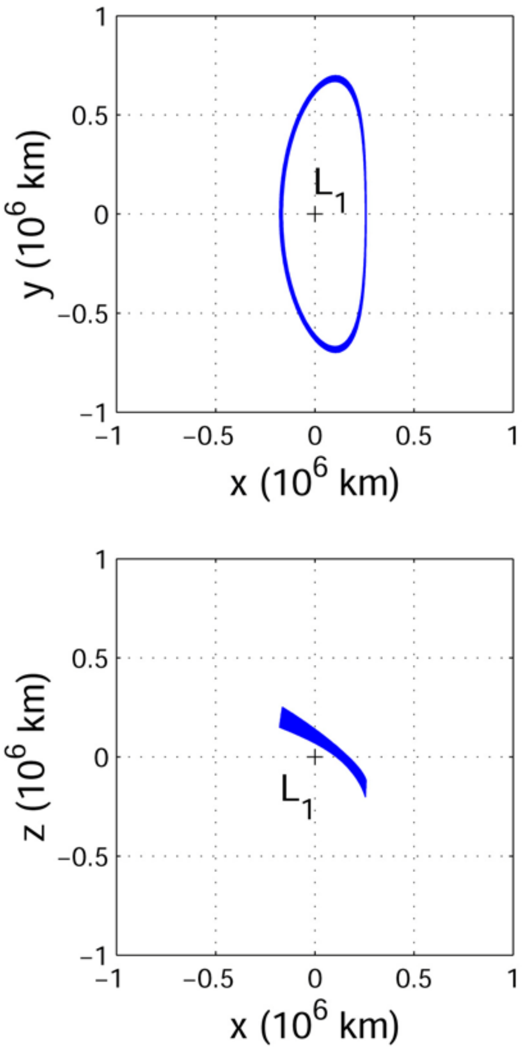


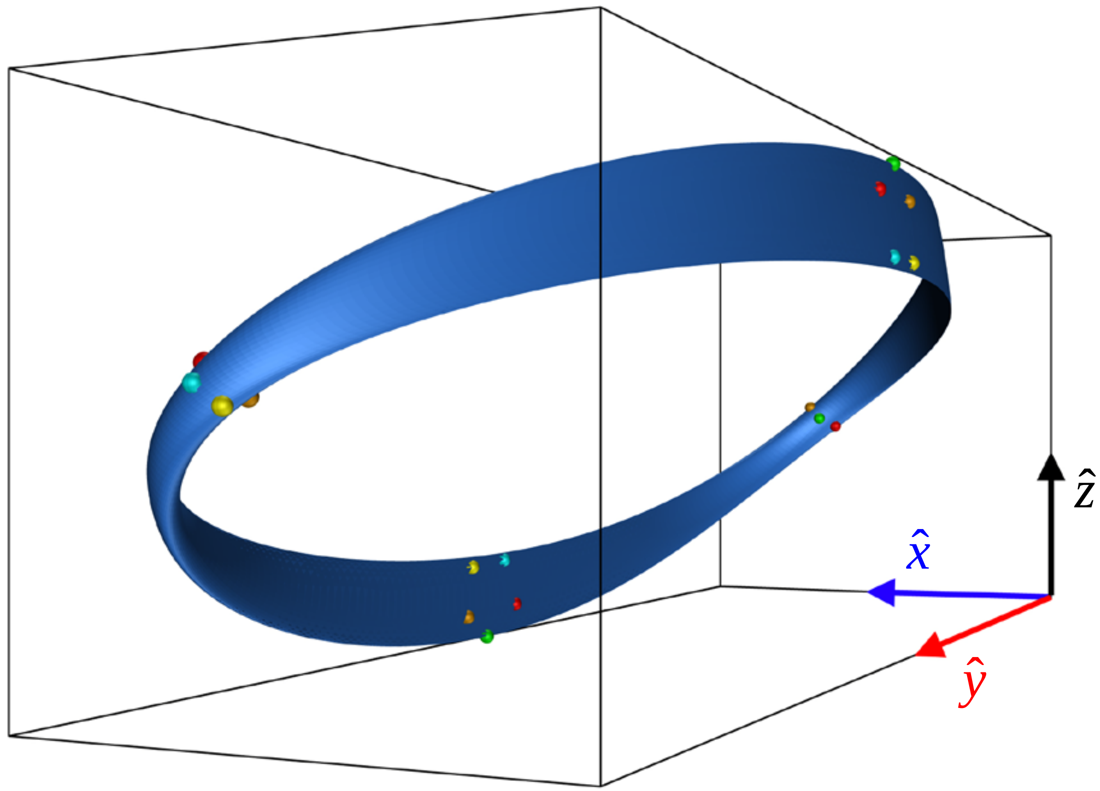
Nominal Formation Keeping Cost
(Configurations Fixed in the Rotating Frame)



Quasi-Periodic Configurations

(Natural Formations Along the Center Manifold)





$\Delta V_{NOMINAL} = 0$

Controllers Considered

- LQR

$$\min J = \frac{1}{2}\int_0^{t_f}\left(\delta\overline{x}_d(t)^T Q\delta\overline{x}_d(t)+\delta\overline{u}_d(t)^T R\delta\overline{u}_d(t)\right)d$$

$$\delta\overline{u}_d(t) = -R^{-1}B^T P(t)\delta\overline{x}_d(t)$$

$$\dot{P}(t) = -A(t)^T P(t) - P(t)A(t) + P(t)B(t)R^{-1}B(t)^T P(t) - Q$$

- Input Feedback Linearization

$$\dot{\overline{x}}(t) = \overline{f}\left(\overline{x}(t)\right)+\overline{u}(t)$$

$$\overline{u}(t) = -\overline{f}\left(\overline{x}(t)\right)+\overline{g}\left(\overline{x}(t)\right)$$

- Output Feedback Linearization

$$\dot{\overline{x}}(t) = \overline{f}\left(\overline{x}(t)\right)+\overline{u}(t)$$

$$\overline{y}(t) = \begin{bmatrix} r \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \left(\overline{r}^T\overline{r}\right)^{1/2} \\ \dfrac{\overline{r}^T\dot{\overline{r}}}{r} \end{bmatrix}$$

$$\ddot{r} = -\left(\overline{r}^T\overline{r}\right)^{-\frac{3}{2}}\left(\overline{r}^T\dot{\overline{r}}\right)^2 + \left(\overline{r}^T\overline{r}\right)^{-\frac{1}{2}}\left(\dot{\overline{r}}^T\dot{\overline{r}}+\overline{r}^T\ddot{\overline{r}}\right) = g(r)$$

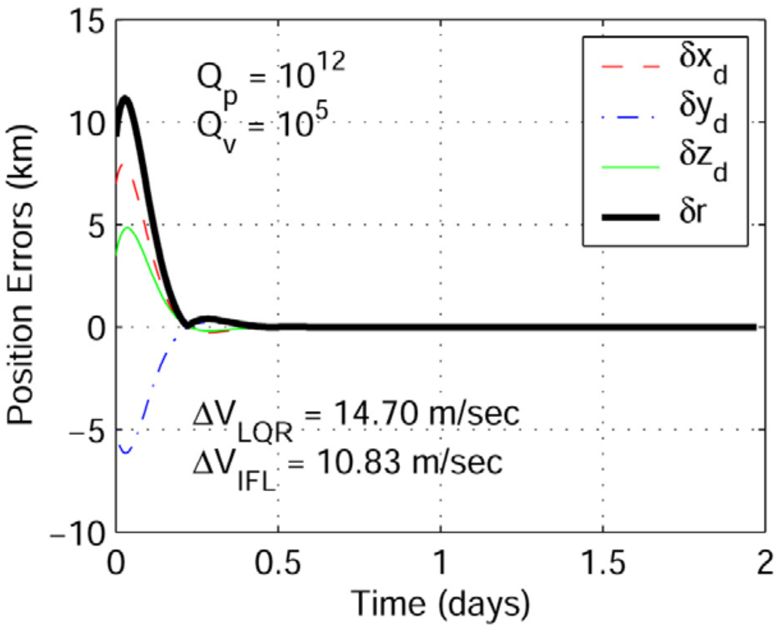
$$\overline{u}(t) = \left(\frac{g(\overline{r})}{r} - \frac{\dot{\overline{r}}^T\dot{\overline{r}}}{r^2}\right)\overline{r} - 2J\dot{\overline{r}} - K\overline{r} - \overline{f}(\overline{r})$$

Dynamic Response to Injection Error

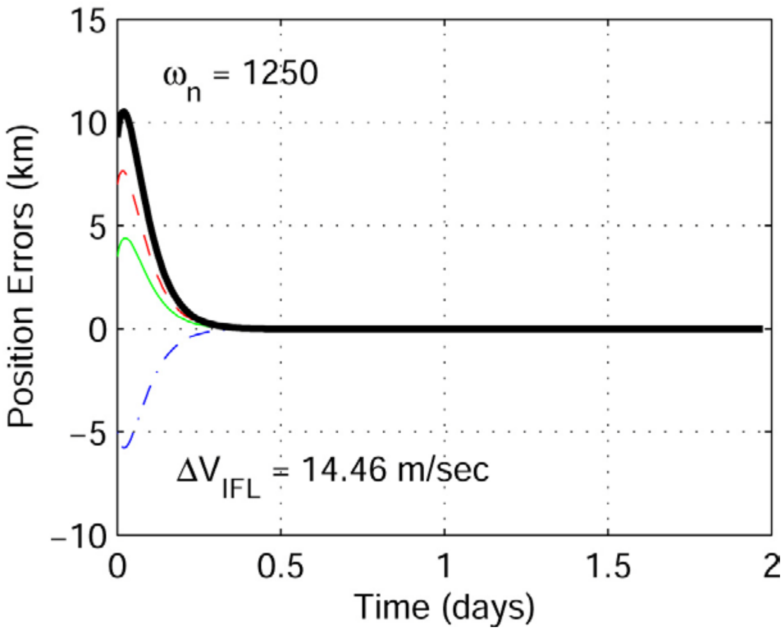
$$\rho = 5000 \text{ km},\ \xi = 90^\circ,\ \beta = 0^\circ$$

$$\delta\bar{x}(0) = [7 \text{ km} \quad -5 \text{ km} \quad 3.5 \text{ km} \quad 1 \text{ mps} \quad -1 \text{ mps} \quad 1 \text{ mps}]^T$$

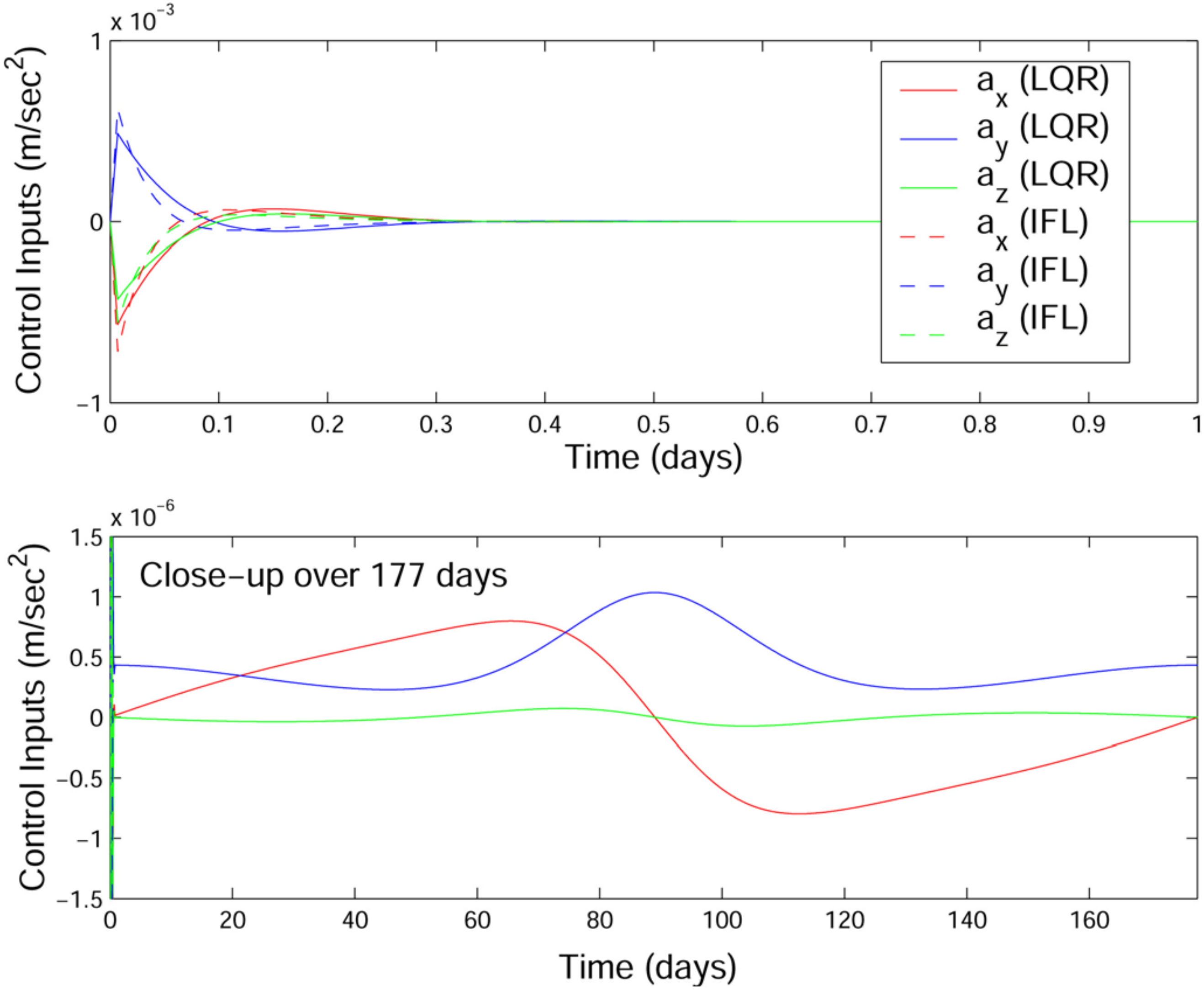
LQR Controller



IFL Controller



Control Acceleration Histories



Conclusions

- Natural vs. Forced Formations
  - The nominal formation keeping costs in the CR3BP are very low, even for relatively large non-naturally occurring formations.
- Above the nominal cost, standard LQR and FL approaches work well in this problem.
  - Both LQR & FL yield essentially the same control histories but FL method is computationally simpler to implement.
- The required control accelerations are extremely low. However, this may change once other sources of error and uncertainty are introduced.
  - Low Thrust Delivery
  - Continuous vs. Discrete Control
- Complexity increases once these results are transferred into the ephemeris model.