Formation Control of the MAXIM L_2 Libration Orbit Mission

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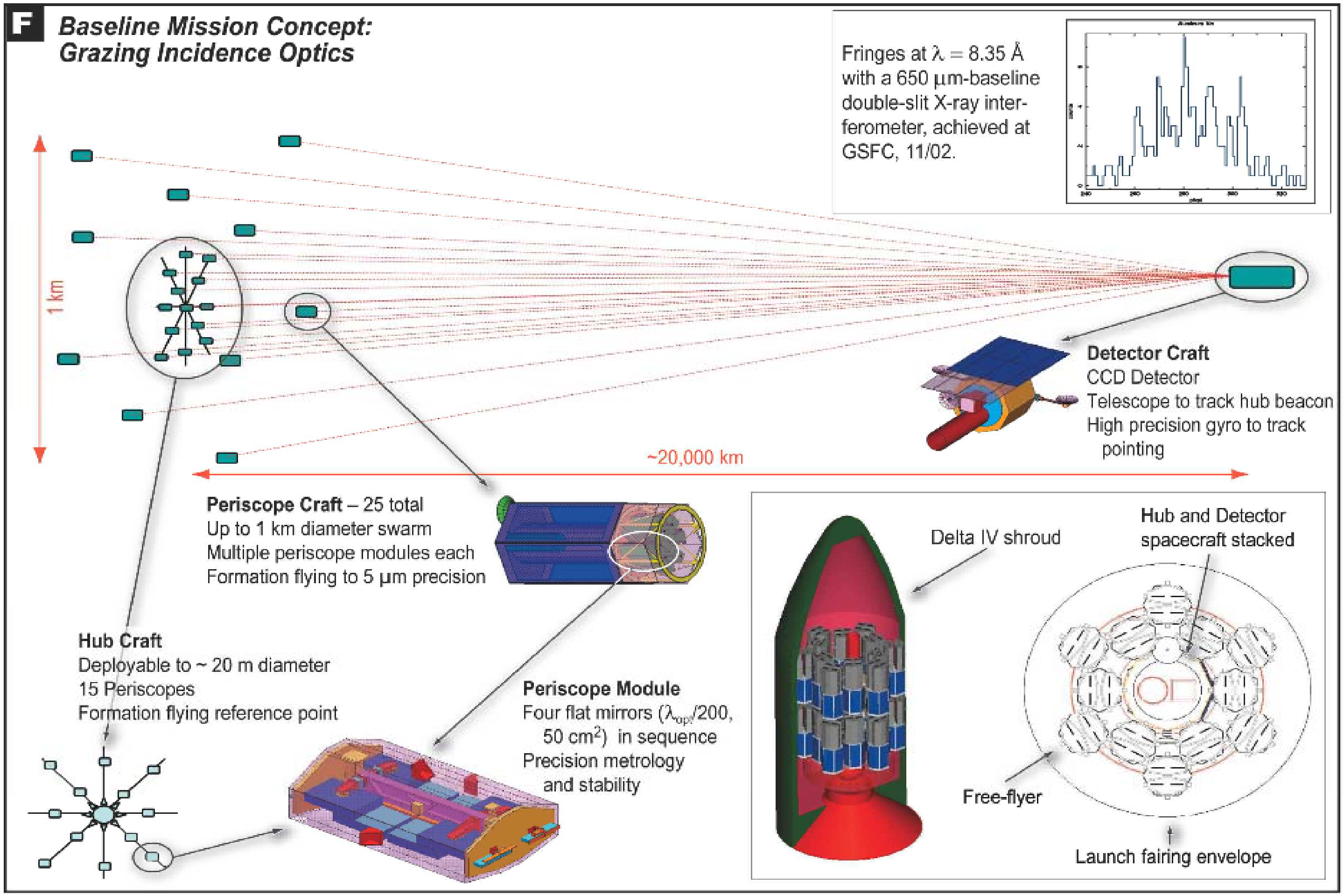
Agenda

- MAXIM Introduction
- MAXIM Formation
- Formation Assumptions
- Formation Definition
- Control – Discrete and Continuous
- Results
- Summary

MAXIM Overview

- The MAXIM concept for NASA's Black Hole Imager mission utilizes interferometric techniques at the short wavelengths of X-rays
- Very long optical baselines are needed to achieve high-precision angular resolution images



MAXIM Formation Overview

- Multiple free-flying spacecraft comprise a sparse aperture providing collecting area of ~ $1000 cm^2$.
- Images are generated through interference patterns gathered from the multiple satellites housing the optical elements that form the aperture.
- The interference patterns or fringes are observed only if the path lengths are controlled to great precision.
- The challenge is to control this path length in the presence of environmental and spacecraft disturbances driving the need for active control systems.
- We focus on the dynamics and control of formation flight in a full ephemeris modeling of the libration orbit to incorporate all gravitational perturbations and solar radiation pressure.
- Analysis focuses on amount and duration of the control effort versus science observation requirements as measured in the formation optics plane

MAXIM Formation Assumptions

✓MAXIM formation components;

Hub (1.3 x 2 meters , 331kg) , Freeflyer (periscope) (1.3 x 2 meters, 304kg) , and the Detector (varying area 1.9 m^2 to 5.6 m^2 , 619kg)

✓Optics Plane:

- Hub and Freeflyers form a physical configuration perpendicular to detector-hub line of sight (LOS) to a target.
- Associates physical configuration to science requirements derived from a Fourier transform of the image plane, the UV plane.

✓Observation duration is 100,000 secs

✓Controller options:

- Off during observation and on to realign and maintain the formation
- Continuously on during observations

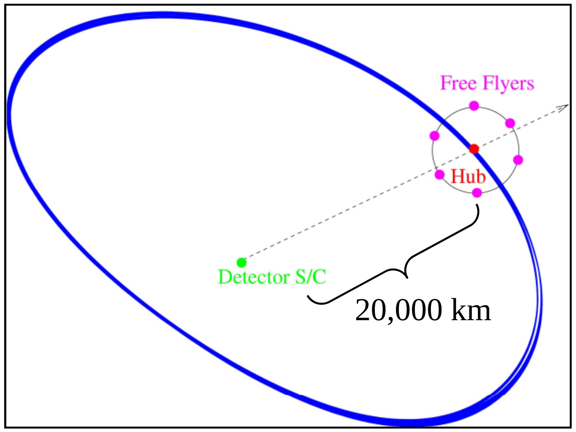
✓ Inertial target of 45^0 elevation and 45^0 azimuth

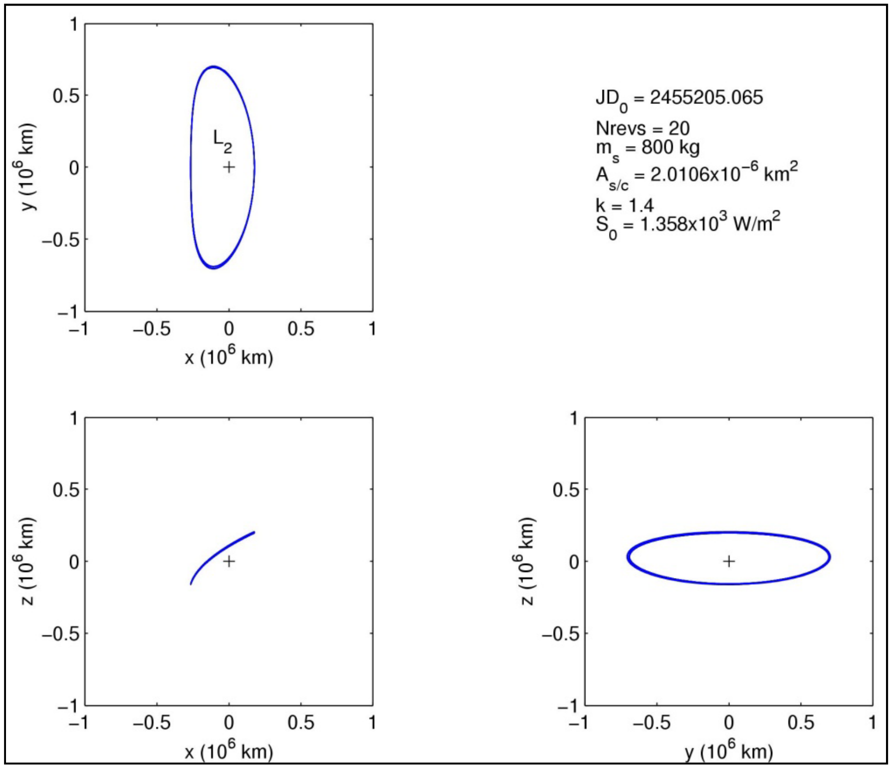
✓ Tolerance of radial distance of a Freeflyer from Hub less than 5 microns

✓ Detector at 20,000km, six freeflyers at the maximum nominal radial distance of 500 meters from the Hub.

MAXIM Halo Orbit

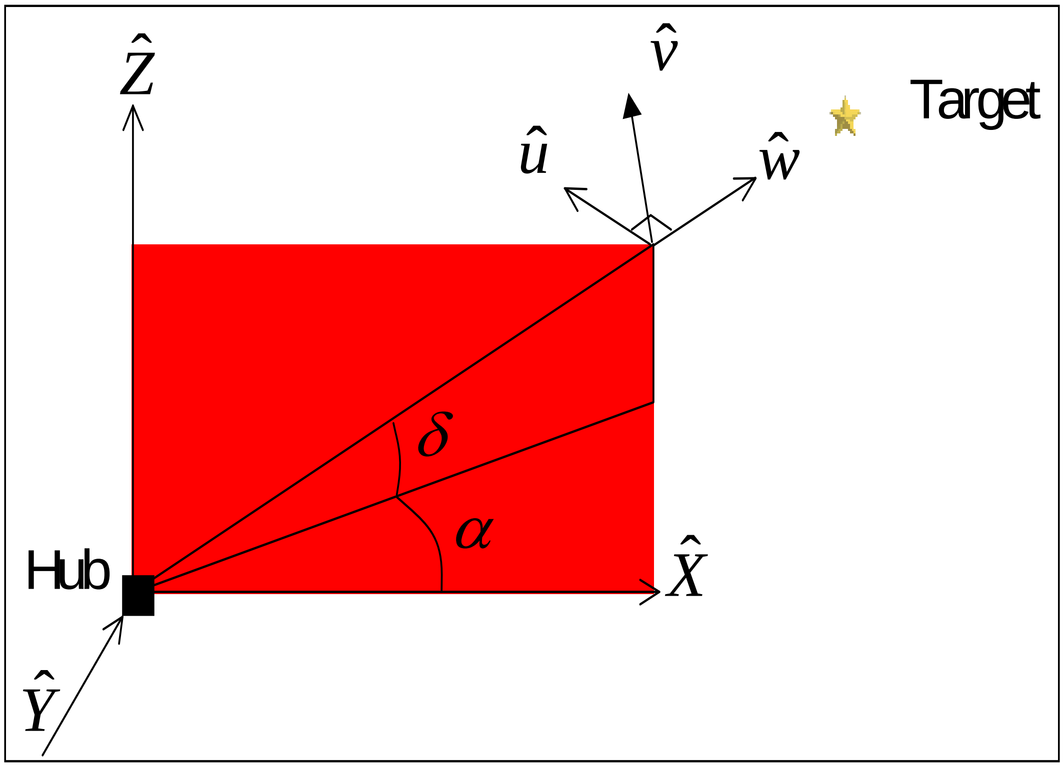
- MAXIM L_2 libration orbit is a typical mission
- A_y = 700,000 km and A_z =200,000 km
- Halo orbit computed with a full Ephemeris model
  - ✓ Sun, Earth, Moon point mass
  - ✓ Solar Radiation Pressure
- Hub follows Halo orbit





MAXIM Frame Definition

The MAXIM hub spacecraft is located at the X,Y,Z origin and the angles α, δ provide the alignment toward the target.



$$\hat{w} = C_\alpha C_\delta \hat{X} + S_\alpha C_\delta \hat{Y} + S_\delta \hat{Z}$$

$$\hat{u} = \frac{\hat{Z} \times \hat{w}}{\left|\hat{Z} \times \hat{w}\right|}$$

$$\hat{v} = \hat{w} \times \hat{u}$$

Direction Cosines for conversion between Optics frame and Inertial Frame

$$ {}^{I}C^{U} = \begin{bmatrix} -S_\alpha & -C_\alpha S_\delta & C_\alpha C_\delta \\ C_\alpha & -S_\alpha S_\delta & S_\alpha C_\delta \\ 0 & C_\delta & S_\delta \end{bmatrix} $$

MAXIM Control Strategies

- Our investigation takes a global view of the large-scale formation flying problem.

- Previous Research:
  - Near Earth - minimized gravitational perturbation - no close tracking of a reference solution - or use of non-linear (adaptive) 2-body problems
  - Multi-body systems - CRTBP only or controller effectiveness is demonstrated relative to the linear dynamics, not the full nonlinear system - Evolution approximated from the linear dynamics of the integrated lissajous trajectory
  - Naturally occurring formations derived from center manifold analysis, as well as a discrete impulsive control approach to maintain a prescribed formation plane

- Continuous control approach

  Obtain a rough analytical approximation of center manifold motion and determine how continuous optimal control and exact feedback linearization compares, in terms of cost, to the discrete station-keeping approach.

MAXIM Control Strategies

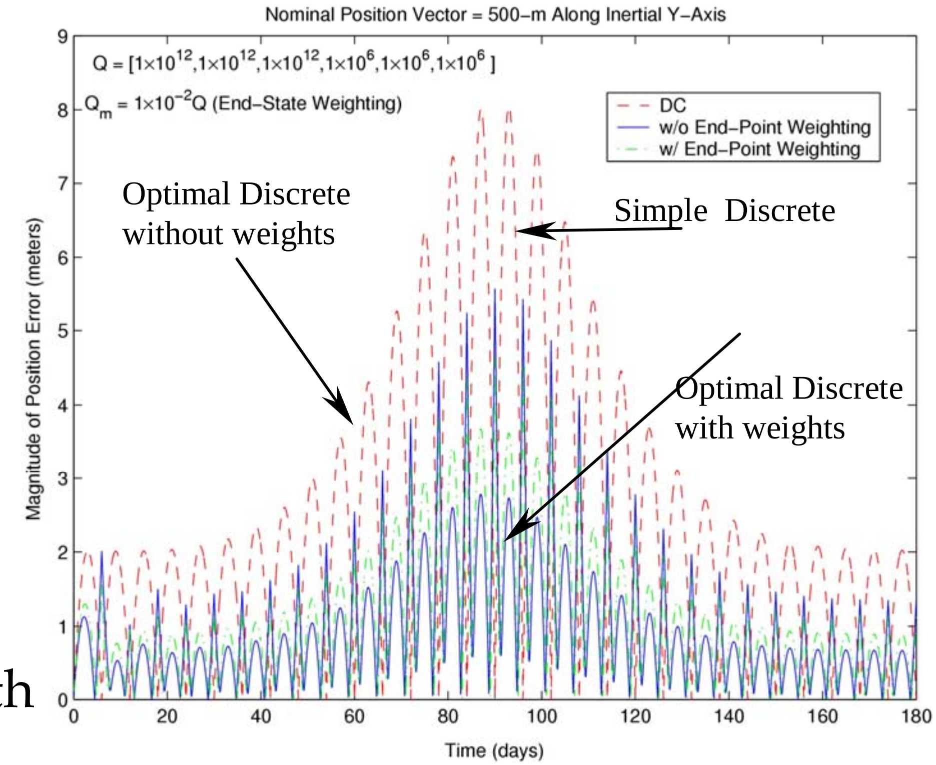
- Previous work demonstrates the efficiency and cost effectiveness of both input feedback linearization (IFL) and output feedback linearization (OFL) methods for formation control in the CRTBP.
- A linear quadratic regulator (LQR), derived from optimal control theory, yields essentially an identical error response and control acceleration history as the IFL approach.
- IFL controller is computationally much less intensive and, by comparison, conceptually simple.
- We address the properties of the IFL controller in defining the MAXIM formation control
- Analysis of position deviation of freeflyer or detector wrt Hub
- For a comparison, a discrete stationkeeping control approach is devised to force the orientation of the formation plane to remain fixed inertially.

MAXIM Discrete Control

- Accuracy of formation maintenance
- Simple DC can maintain formation
- Discrete LQR yields optimal magnitude of differential control impulse
- Simple: Target the end state
  Φ = STM
  δ = state perturbation
  Δv_0 = Impulsive ΔV at beginning
- Discrete Optimal Control:
  (Q_m) Weighted quadratic of end state error
  (Q) Weighted quadratic of state deviation along path
- Simple has greatest error along path

$$\begin{bmatrix} \delta \bar{r}_1 \\ \delta \bar{v}_1^- \end{bmatrix} = \Phi(t_1, t_0) \begin{bmatrix} \delta \bar{r}_0 \\ \delta \bar{v}_0^+ \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \delta \bar{r}_0 \\ \delta \bar{v}_0^- + \Delta \bar{v}_0 \end{bmatrix}$$

$$\Delta \bar{v}_0 = B^{-1}\left(\delta \bar{r}_1 - A \delta \bar{r}_0\right) - \delta \bar{v}_0^-$$



MAXIM Nominal Motion and Determination of Vehicle Position Relative to Optics-Frame

The nominal motion is in the local (spherical) coordinates while the control effort is formulated in the inertial focal frame.

Freeflyer / Detector Kinematics are written as

$$\bar{r}^{HD_i} = r\hat{d}_1$$

$$^{U}\dot{\bar{r}}^{HD_i} = \dot{r}\hat{d}_1 + r\dot{\nu}C_\varepsilon\hat{d}_2 + r\dot{\varepsilon}\hat{d}_3$$

$$\hat{d}_1 = C_\varepsilon C_\nu\hat{u} + C_\varepsilon S_\nu\hat{v} + S_\varepsilon\hat{w}$$

Cartesian coordinates to spherical:

$$\tilde{x} = rC_\nu C_\varepsilon$$

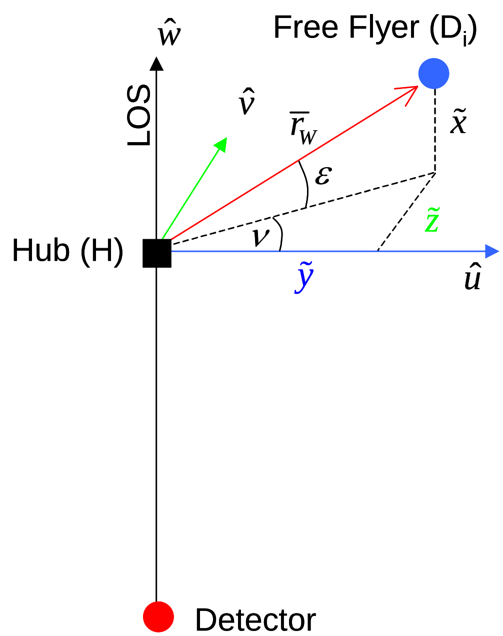
$$\tilde{y} = rS_\nu C_\varepsilon$$

$$\tilde{z} = rS_\varepsilon$$

$$\dot{\tilde{x}} = \dot{r}C_\nu C_\varepsilon - r\dot{\nu}S_\nu C_\varepsilon - r\dot{\varepsilon}C_\nu S_\varepsilon$$

$$\dot{\tilde{y}} = \dot{r}S_\nu C_\varepsilon + r\dot{\nu}C_\nu C_\varepsilon - r\dot{\varepsilon}S_\nu S_\varepsilon$$

$$\dot{\tilde{z}} = \dot{r}S_\varepsilon + r\dot{\varepsilon}C_\varepsilon$$



$$\nu = \tan^{-1}\left(\frac{\tilde{z}}{\tilde{y}}\right) \qquad \varepsilon = \tan^{-1}\left(\frac{\tilde{x}}{\sqrt{\tilde{y}^2 + \tilde{z}^2}}\right)$$

NOMINAL MOTION:

FF_1 :	$\nu^\circ = 0^\circ$,	$\varepsilon^\circ = 0^\circ$,	$\lvert\bar{r}_W\rvert = 500$ m
FF_2 :	$\nu^\circ = 60^\circ$,	$\varepsilon^\circ = 0^\circ$,	$\lvert\bar{r}_W\rvert = 500$ m
FF_3 :	$\nu^\circ = 120^\circ$,	$\varepsilon^\circ = 0^\circ$,	$\lvert\bar{r}_W\rvert = 500$ m
FF_4 :	$\nu^\circ = 180^\circ$,	$\varepsilon^\circ = 0^\circ$,	$\lvert\bar{r}_W\rvert = 500$ m
FF_5 :	$\nu^\circ = 240^\circ$,	$\varepsilon^\circ = 0^\circ$,	$\lvert\bar{r}_W\rvert = 500$ m
FF_6 :	$\nu^\circ = 300^\circ$,	$\varepsilon^\circ = 0^\circ$,	$\lvert\bar{r}_W\rvert = 500$ m
Detector:	$\nu^\circ = 0^\circ$,	$\varepsilon^\circ = -90^\circ$,	$\lvert\bar{r}_W\rvert = 20,000$ km

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MAXIM IFL Controller Development

Control of Equations of Motion (EOM) in Ephemeris Frame Wrt Earth (P_2)

- EOM for Freeflyer/detector $^{I}\ddot{\bar{r}}_{I}^{P_2D_i} = \bar{f}\left(\bar{r}_I^{P_2D_i}, {}^{I}\dot{\bar{r}}_I^{P_2D_i}\right) + \bar{u}_I^{(D_i)}(t)$
- EOM for Hub $^{I}\ddot{\bar{r}}_{I}^{P_2H} = \bar{f}\left(\bar{r}_I^{P_2H}, {}^{I}\dot{\bar{r}}_I^{P_2H}\right)$

- Controller is selected as type of response as a critical damped

- Control in the local frame $^{U}\ddot{\bar{r}}_{U}^{HD_i} = \left\{{}^{U}C^{I}\right\}\Delta\bar{f}_I + \left\{{}^{U}C^{I}\right\}\bar{u}_I^{(D_i)}(t) = \left\{{}^{U}C^{I}\right\}\Delta\bar{f}_I + \tilde{u}^{(D_i)}(t)$

- Controller eliminates system dynamics terms yields critical response control

$$\tilde{u}^{(D_i)}(t) = -\left\{{}^{U}C^{I}\right\}\Delta\bar{f}_I^{(D_i)} - 2\omega_n\left({}^{U}\dot{\bar{r}}_U^{HD_i} - \dot{\bar{r}}^*\right) - \omega_n^2\left(\bar{r}_U^{HD_i} - \bar{r}^*\right)$$

$$^{I}\ddot{\bar{r}}_I^{HD_i} = \Delta\bar{f}_I + \bar{u}_I^{(D_i)}(t) \quad \rightarrow \quad \left\{{}^{I}C^{U}\right\}{}^{U}\ddot{\bar{r}}_U^{HD_i} = \Delta\bar{f}_I + \bar{u}_I^{(D_i)}(t)$$

- Once control determined in optics frame, rotate into inertial frame for controller

$$\bar{u}_I^{(D_i)}(t) = \left\{{}^{I}C^{U}\right\}\tilde{u}^{(D_i)}(t)$$

(Note: Full state feedback for IFL and no constraints)

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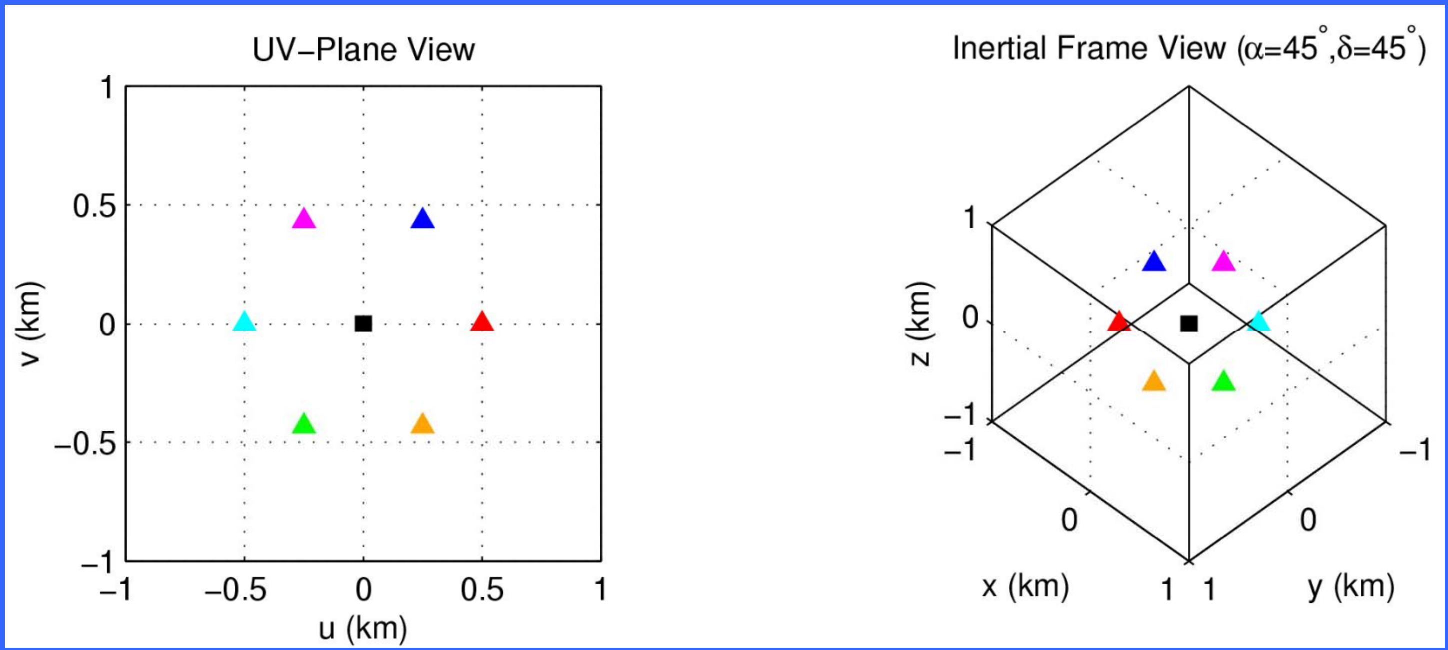
MAXIM Freeflyer Placement

Freeflyers at a maximum 500 meters from hub evenly spaced in azimuth at 60 degrees

Optics Plane View

Inertial View



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MAXIM Maintenance – Thrust Profiles

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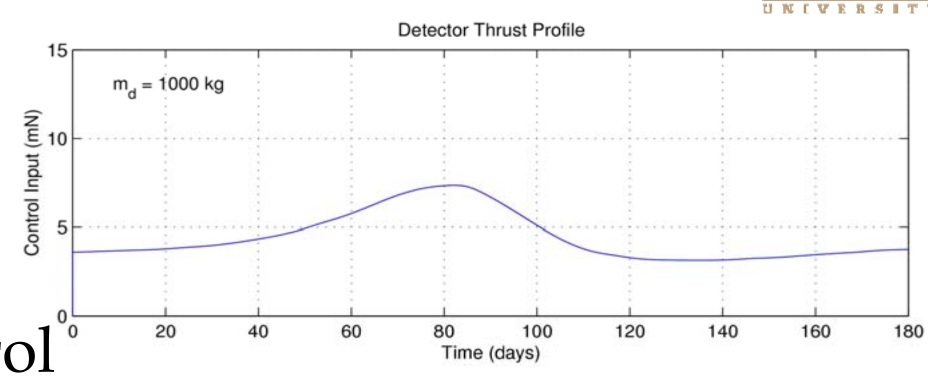
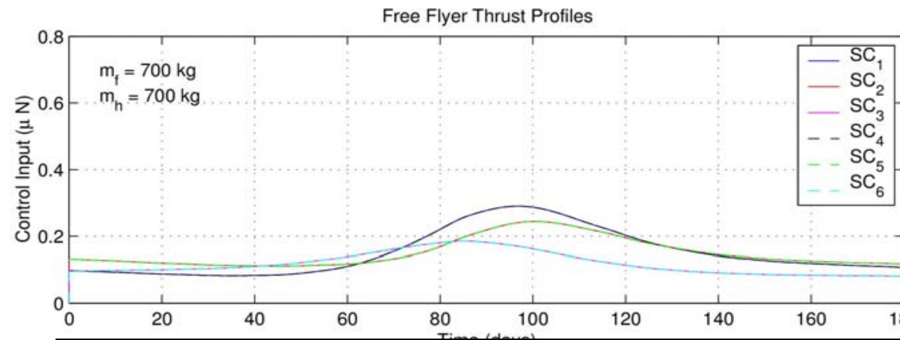
Detector < 7 mN

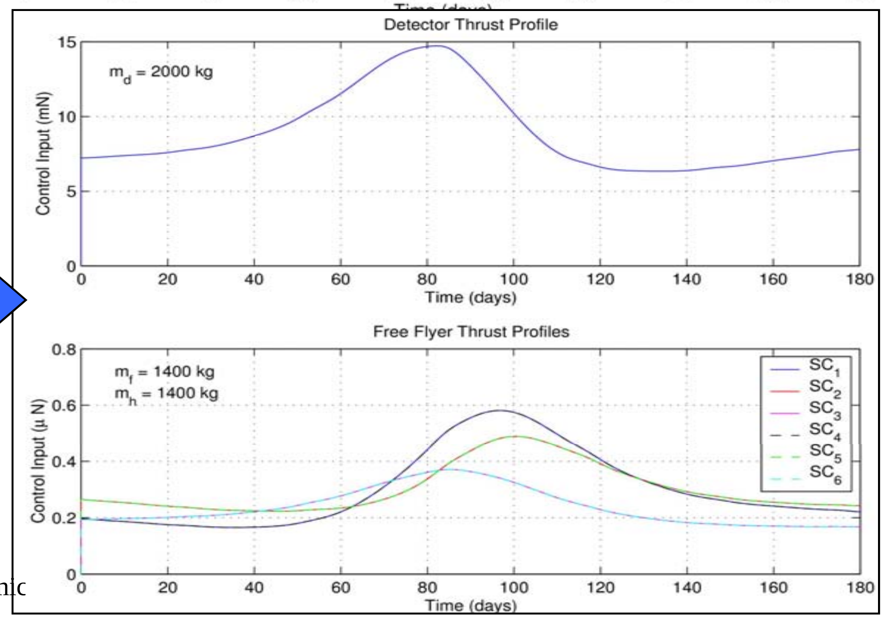


• 180 day IFL continuous control

Freeflyer ~ tenths of μN



Thrust Profiles proportional to spacecraft mass, e.g. 2:1



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MAXIM Maintenance and Recovery

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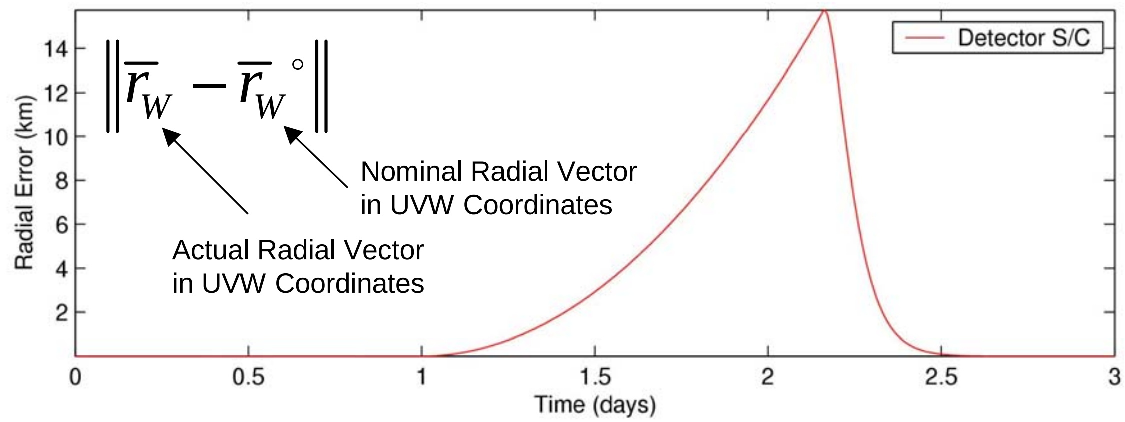
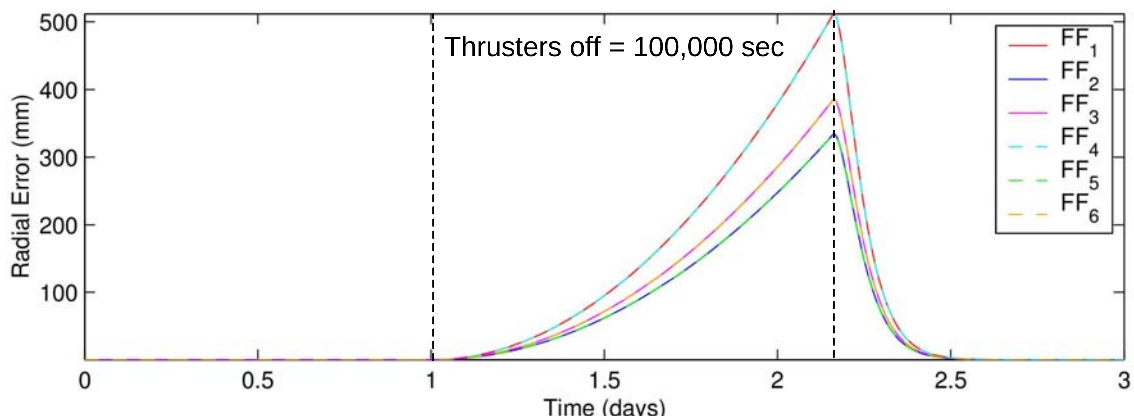
- Maintenance for 1 day
- Control off during observation of 100,000 seconds
- Increase in radial errors of detector and freeflyer
- Recovery back to original positions in ½ day



$$\left\| \bar{r}_W - \bar{r}_W^{\circ} \right\|$$



✓ Error growth is not linear

✓ Peak error of 15 km for detector

✓ Peak errors range from 300mm to 550mm for freeflyer

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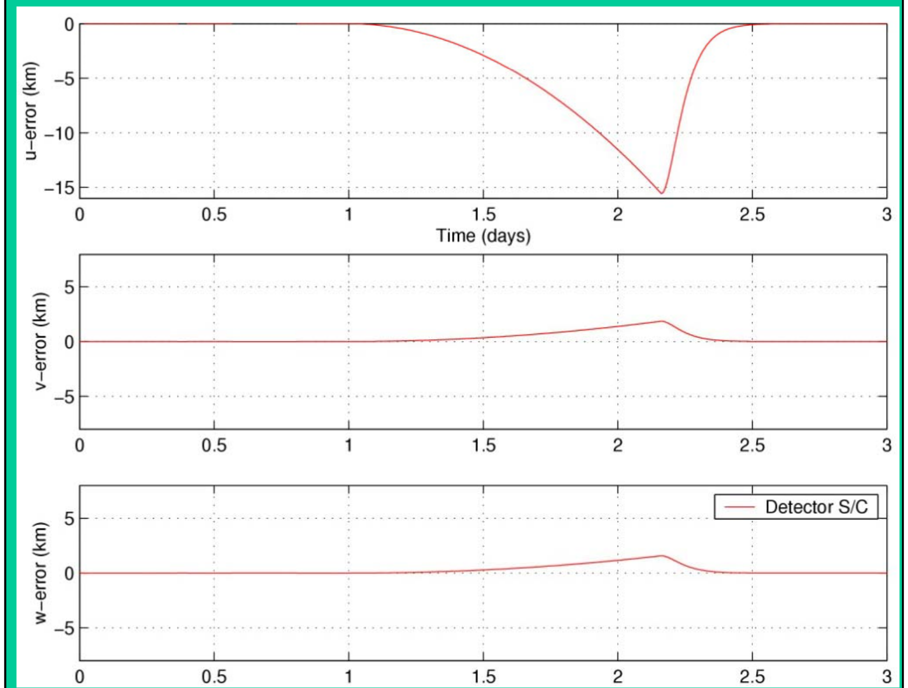
MAXIM Maintenance and Recovery

Deviation in the Optics Plane During Observation With Control Off

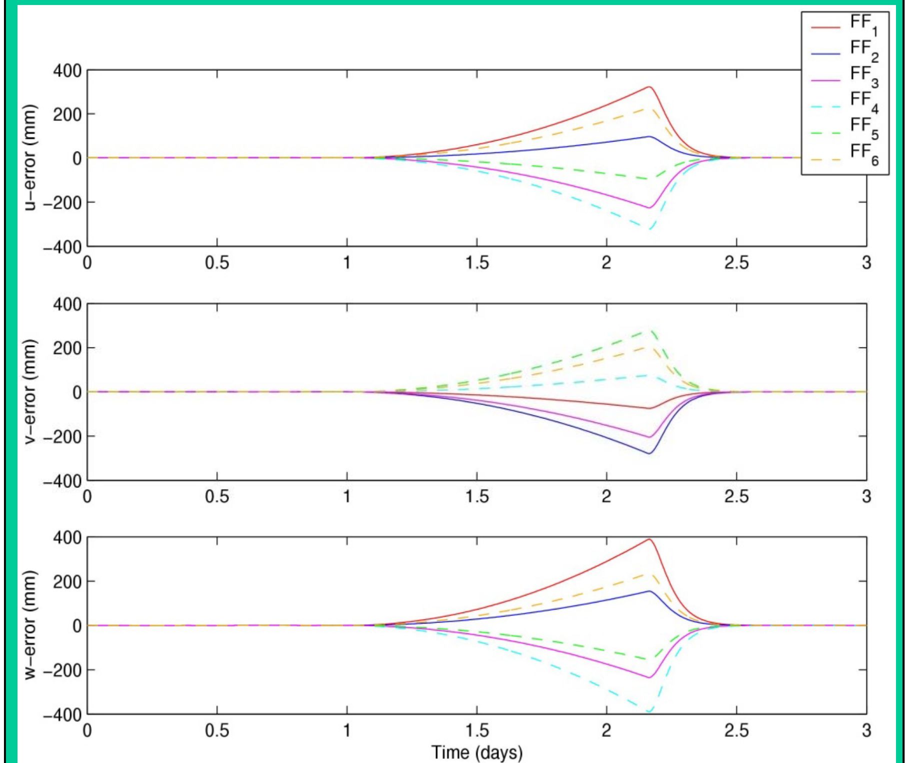
Detector

Vertical Scale: u:15 km to 0 km
v: +/- 5 km
w:+/- 5 km



Freeflyer

Vertical Scale: +/- 400 mm
In all 3 components (u,v,w)



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MAXIM Maintenance and Recovery

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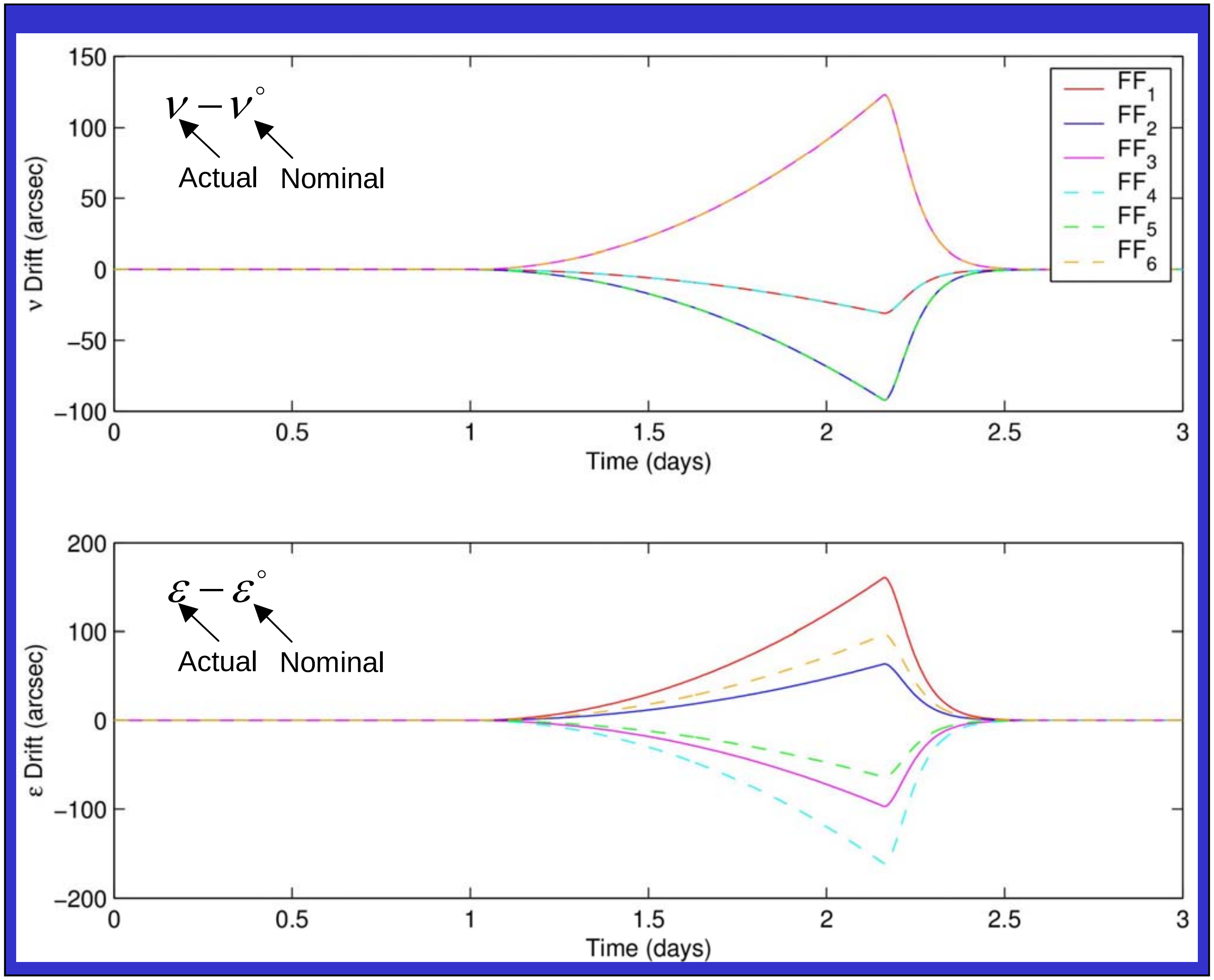
Freeflyer Errors As Pointing Errors (Arc-seconds)



Azimuthal angle (ν) maximum ~120

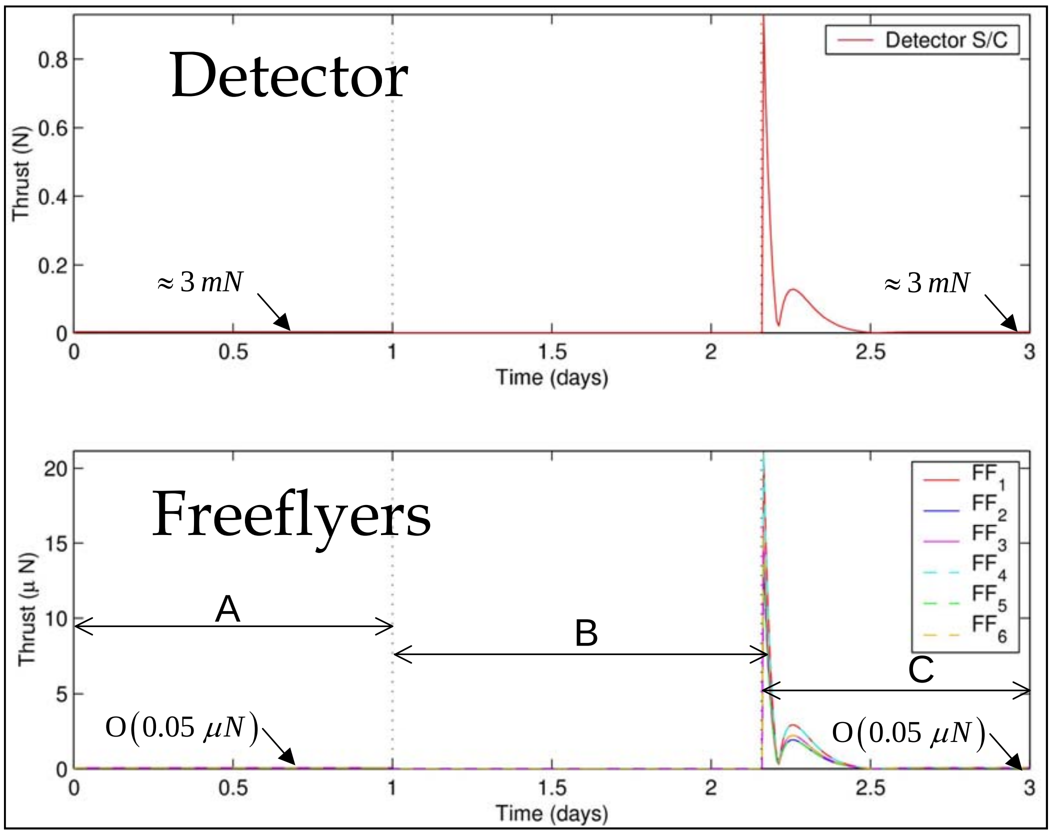
Out-of-plane (ε) Maximum ~120

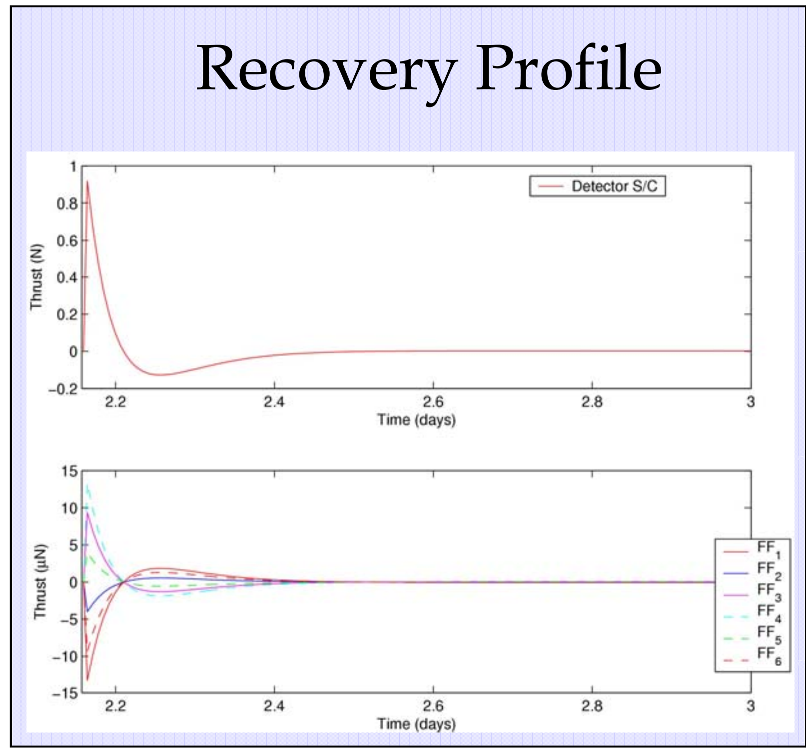
MAXIM Maintenance, Observation, and Recovery

Three day simulation with maintenance 1 day, 100,000 sec observation, and ½ days recovery

Maintenance:
Detector required $3e^{-3}$ N
Freeflyers required < 0.05μN

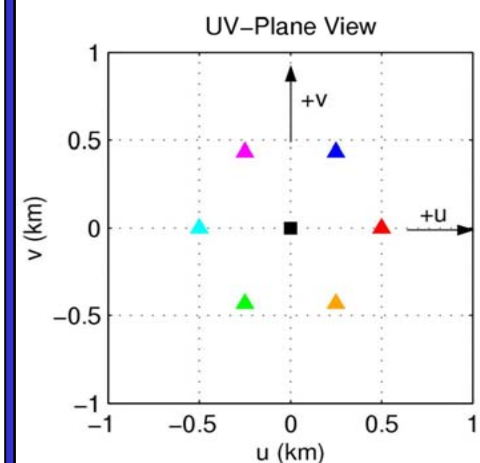
Recovery:
Detector required 1N
Freeflyers required < 15μN



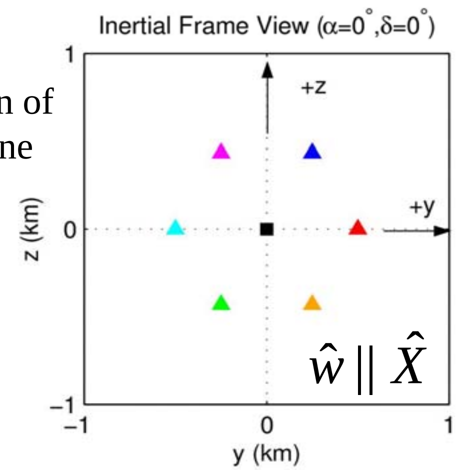


MAXIM Reorientation

- 90 degrees rotation about the z-axis
- Target initially along the inertial x-axis
- x-axis reoriented into y-axis direction
- Elevation angle set to zero



Initial Orientation of Optics Plane



NOMINAL MOTION:

$FF_1: \quad \nu^{\circ} = 0^{\circ}, \quad \varepsilon^{\circ} = 0^{\circ}, \quad |\bar{r}_W| = 500$ m

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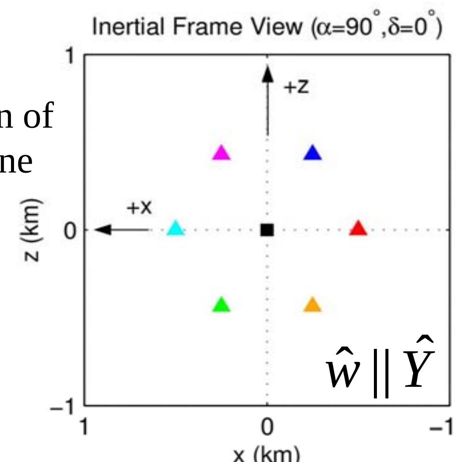
$FF_6: \quad \nu^{\circ} = 300^{\circ}, \quad \varepsilon^{\circ} = 0^{\circ}, \quad |\bar{r}_W| = 500$ m

Detector: $\nu^{\circ} = 0^{\circ}, \quad \varepsilon^{\circ} = -90^{\circ}, |\bar{r}_W| = 20,000$ km

Final Orientation of Optics Plane



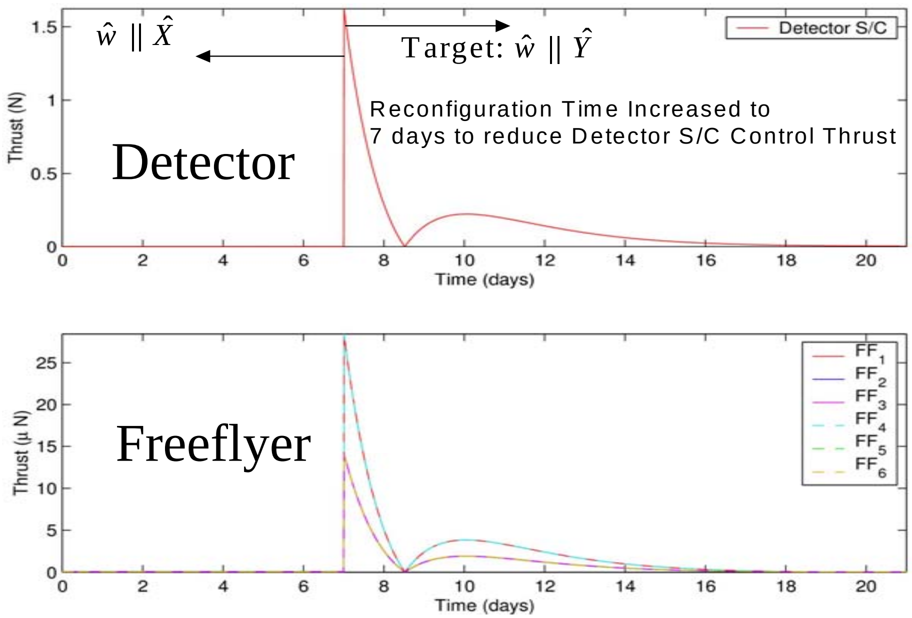
Initial Formation Orientation: $(\alpha,\delta) = (0^{\circ}, 0^{\circ})$

Target Formation Orientation: $(\alpha,\delta) = (90^{\circ}, 0^{\circ})$

MAXIM Reorientation

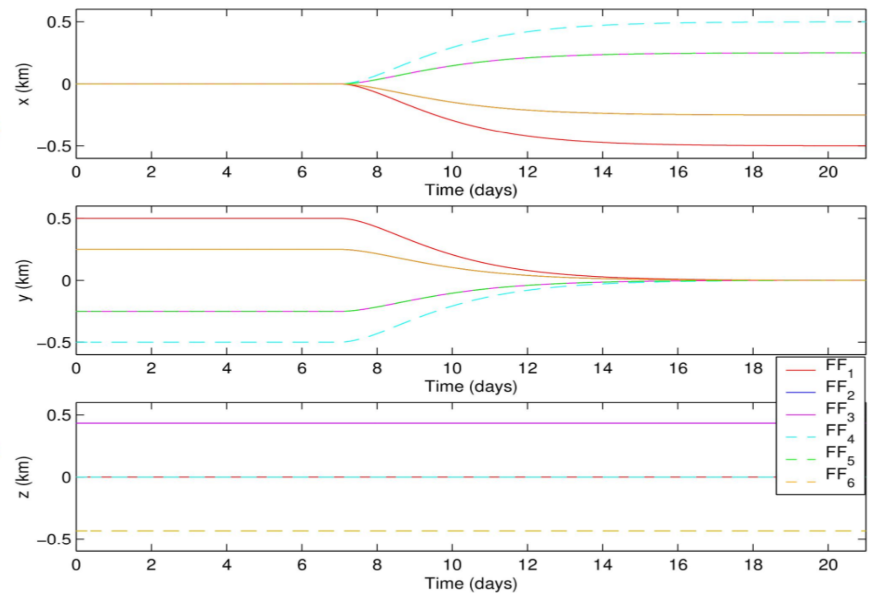
- 7 day Simulation
- Detector ~ 1.5 N
- Freeflyer ~ 2.5 μN

Thrust Levels



Freeflyer Displacement in Inertial Frame

Vertical Scale +/- 0.5 Km



Summary

- Two Approaches, Discrete and Continuous, Were Investigated for the Control of the Maxim Formation.
- Simple or Optimal Discrete or by Input Feedback Linearization (IFL) Control.
  - ✓ Discrete Control Approaches Continuous Time Interval Effort.
  - ✓ IFL Continuous Control Combines the Effect of Annihilating the Environmental Dynamics While Adding a Specific User-defined Critically Damped Response
- The Total Maintenance Control Effort Requires
  - ✓Detector Thrust Level that Ranges From 4 mN to 7 mN
  - ✓Freeflyer Thrust Levels of 0.1 μN to 0.3 μN.
- Formation Recovery
  - ✓Detector Thrust Less than 1 N
  - ✓Freeflyers Less than 15 μN
- These Efforts Do Not Include Navigation or Maneuver Errors or Navigation Measurement Updates.
- The Challenge Is Propulsion System Implementation and Required Power Levels as Current Propulsion Technology Can Meet Minimum Thrust Levels