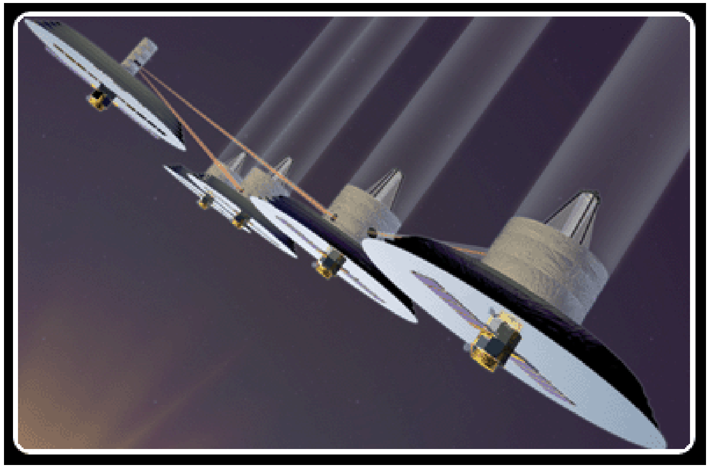
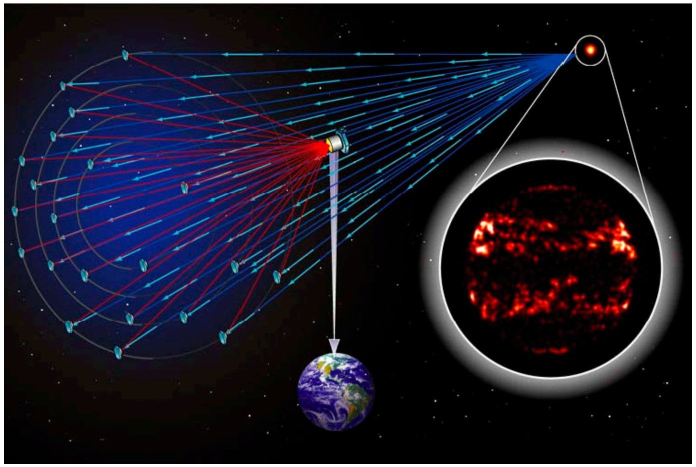
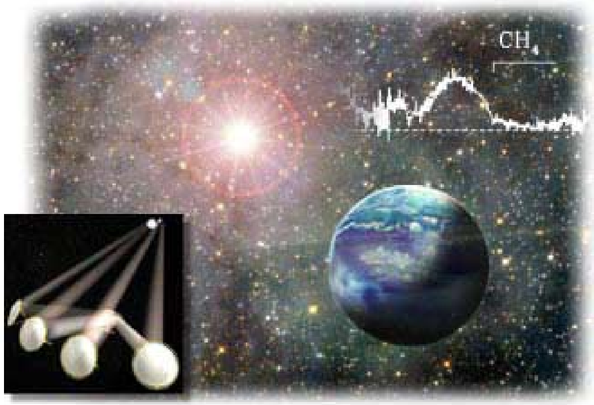
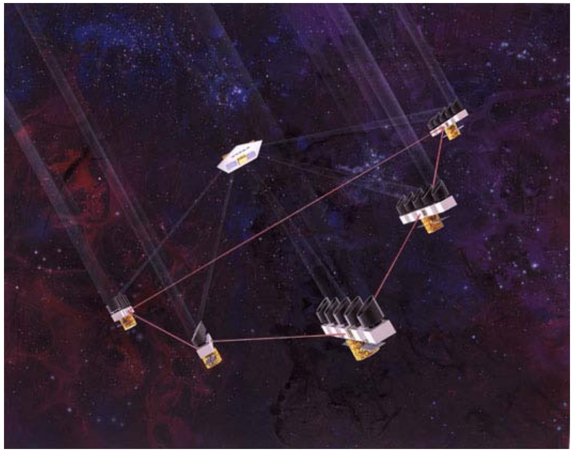




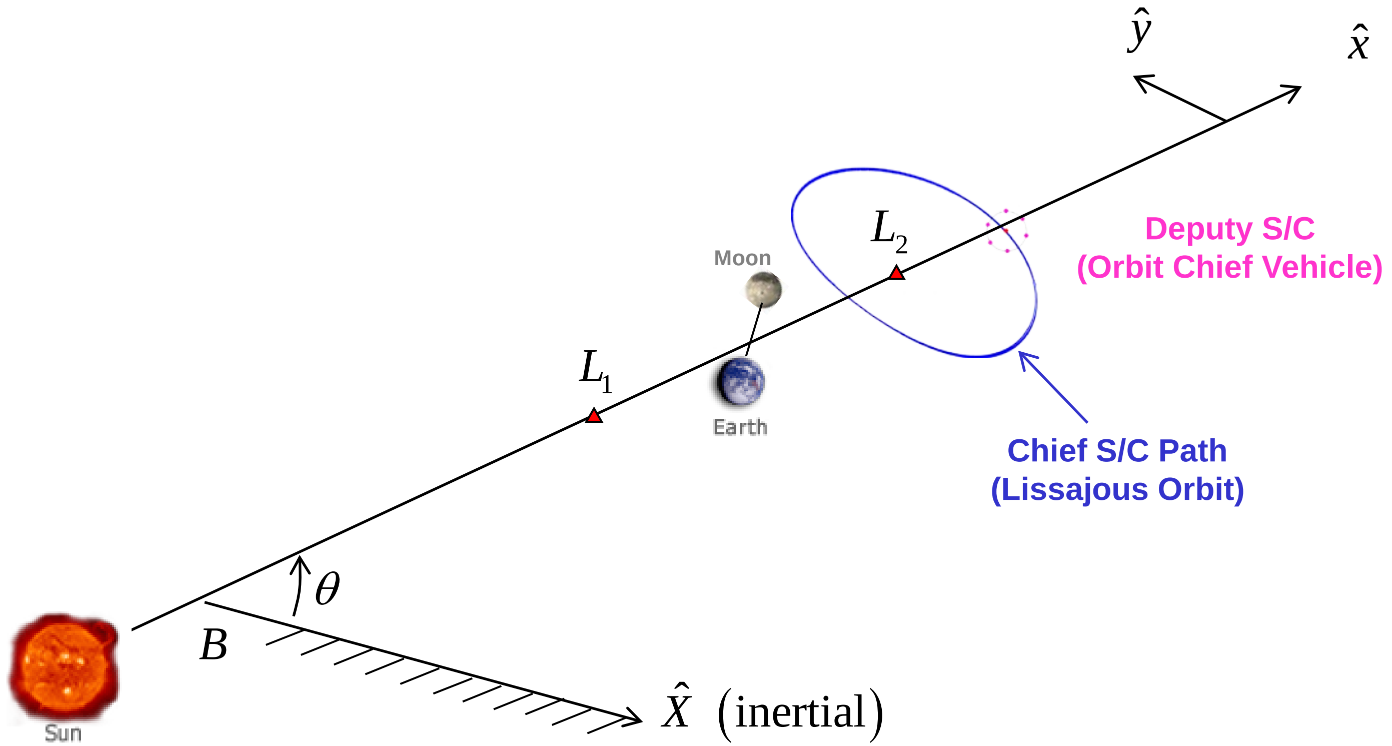
FORMATIONS NEAR THE LIBRATION POINTS: DESIGN STRATEGIES USING NATURAL AND NON-NATURAL ARCS

K. C. Howell and B. G. Marchand





Formations Near the Libration Points



EPHEM = Sun + Earth + Moon Motion From Ephemeris w/ SRP
CR3BP = Sun + Earth/Moon barycenter Motion Assumed Circular w/o SRP

Control Methodologies Considered in both the CR3BP and EPHEM Models

- Continuous Control
  - Linear Control
    - State Feedback with Control Input Lower Bounds
    - Optimal Control → Linear Quadratic Regulator (LQR)
  - Nonlinear Control
    - Input Feedback Linearization (State Tracking)
    - Output Feedback Linearization (Constraint Tracking)
      - Spherical + Aspherical Formations (i.e. Parabolic, Hyperbolic, etc.)
- Discrete Control
  - Nonlinear Optimal Control
    - Impulsive
    - Constant Thrust Arcs
  - Impulsive Targeter Schemes
    - State and Range+Range Rate
  - Natural Formations → Impulsive Deployment
  - Hybrid Formations → Blending Natural and Non-Natural Motions

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IMPULSIVE FORMATION KEEPING: TARGETER METHODS

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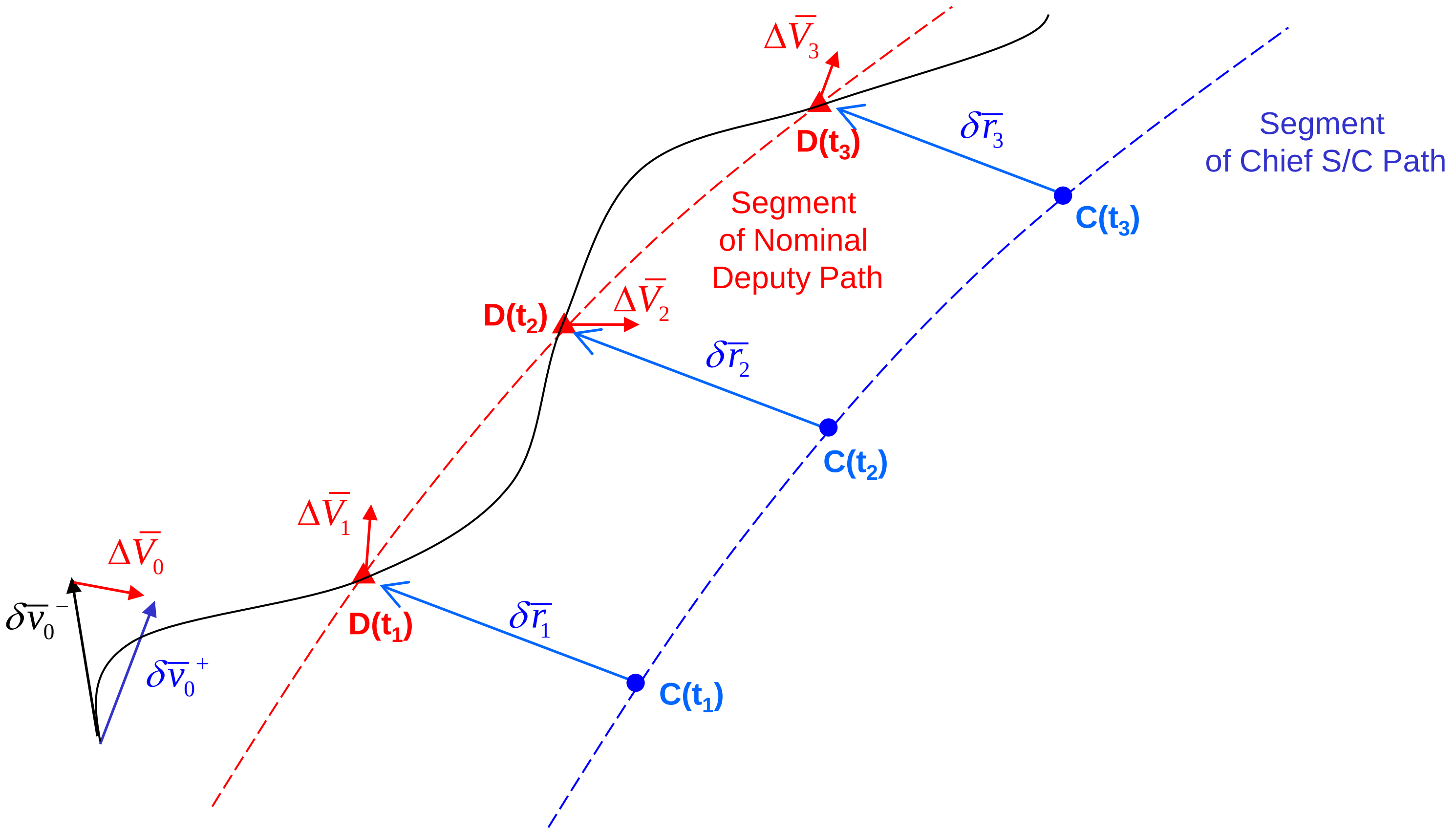
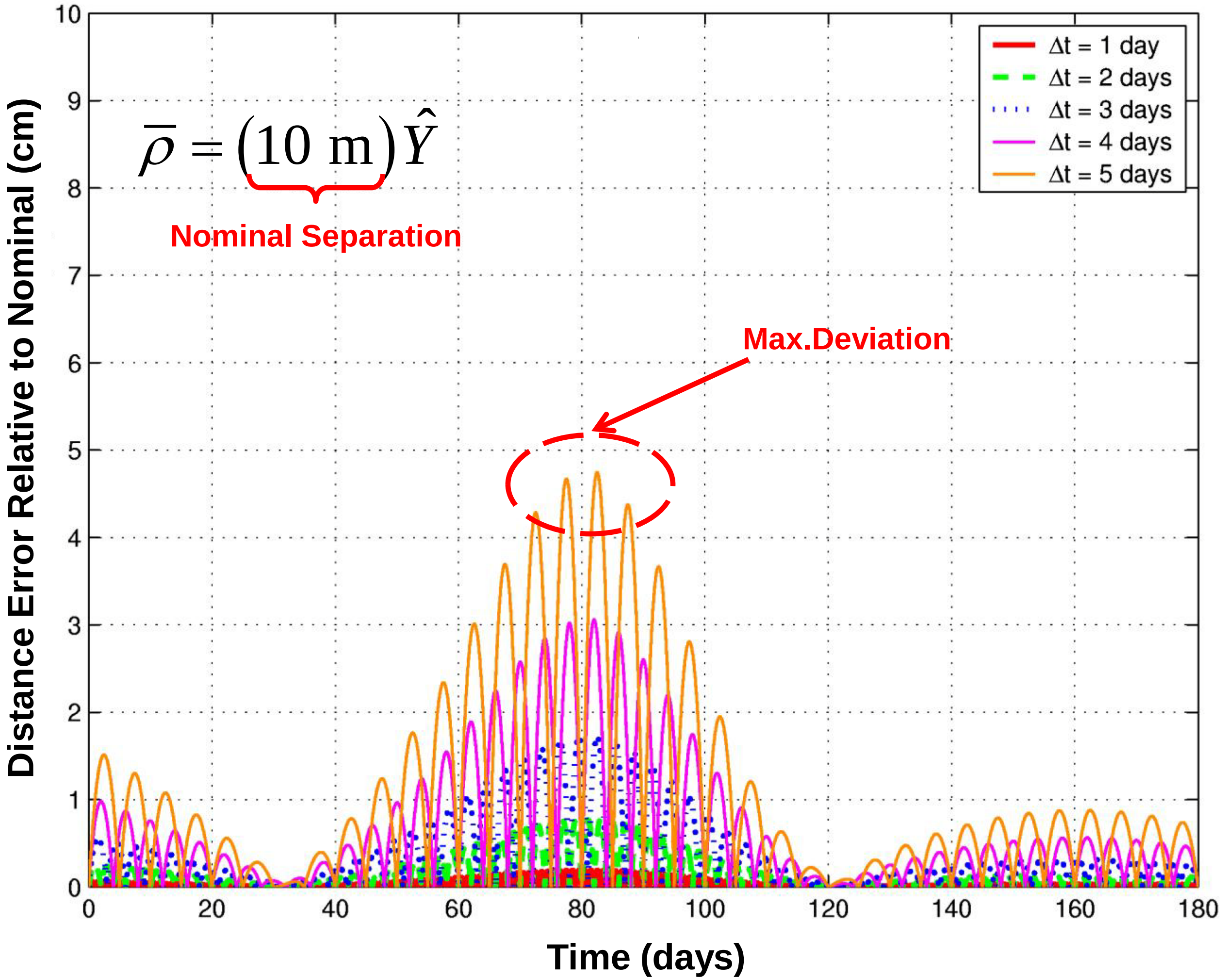
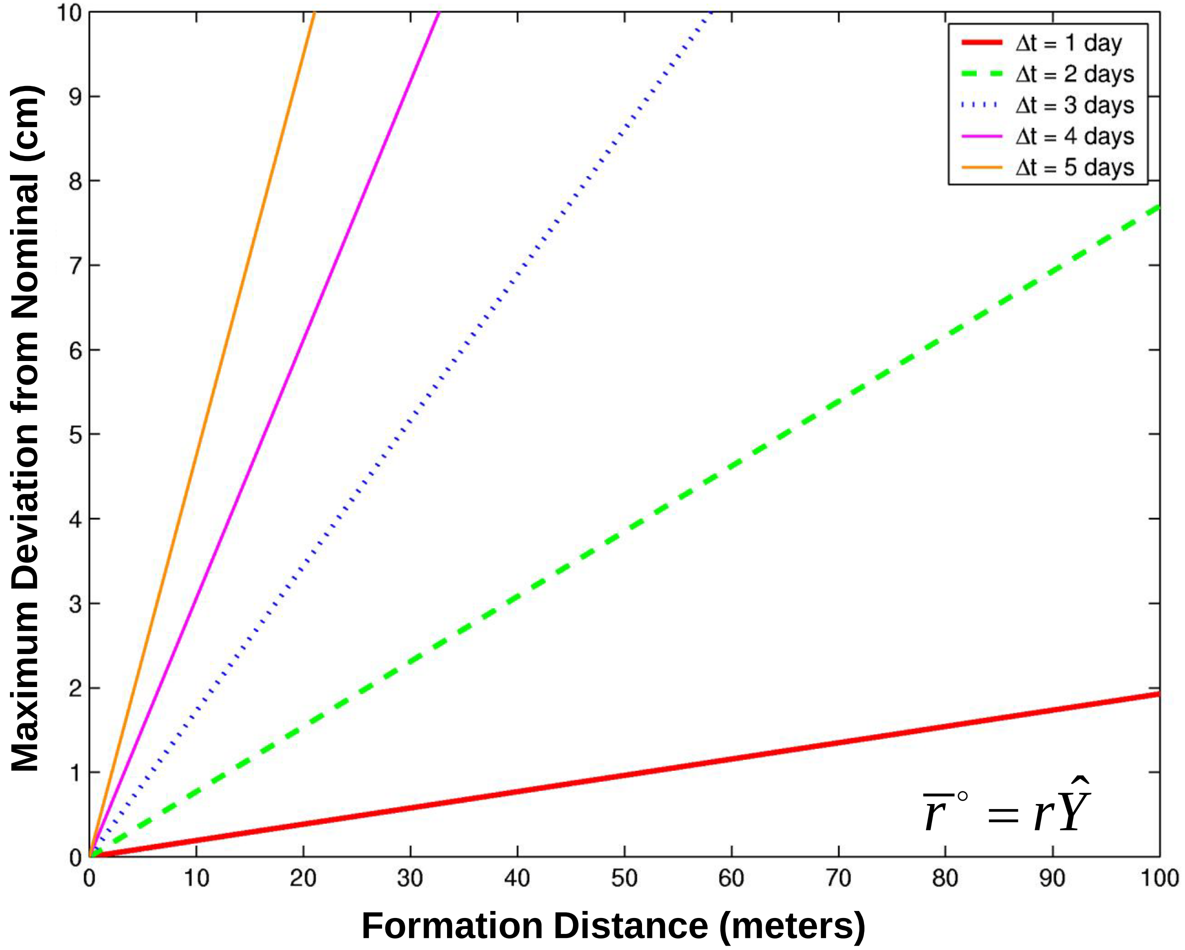
State Targeter: Impulsive Control Law Formulation



State Targeter: Radial Distance Error



State Targeter: Achievable Accuracy



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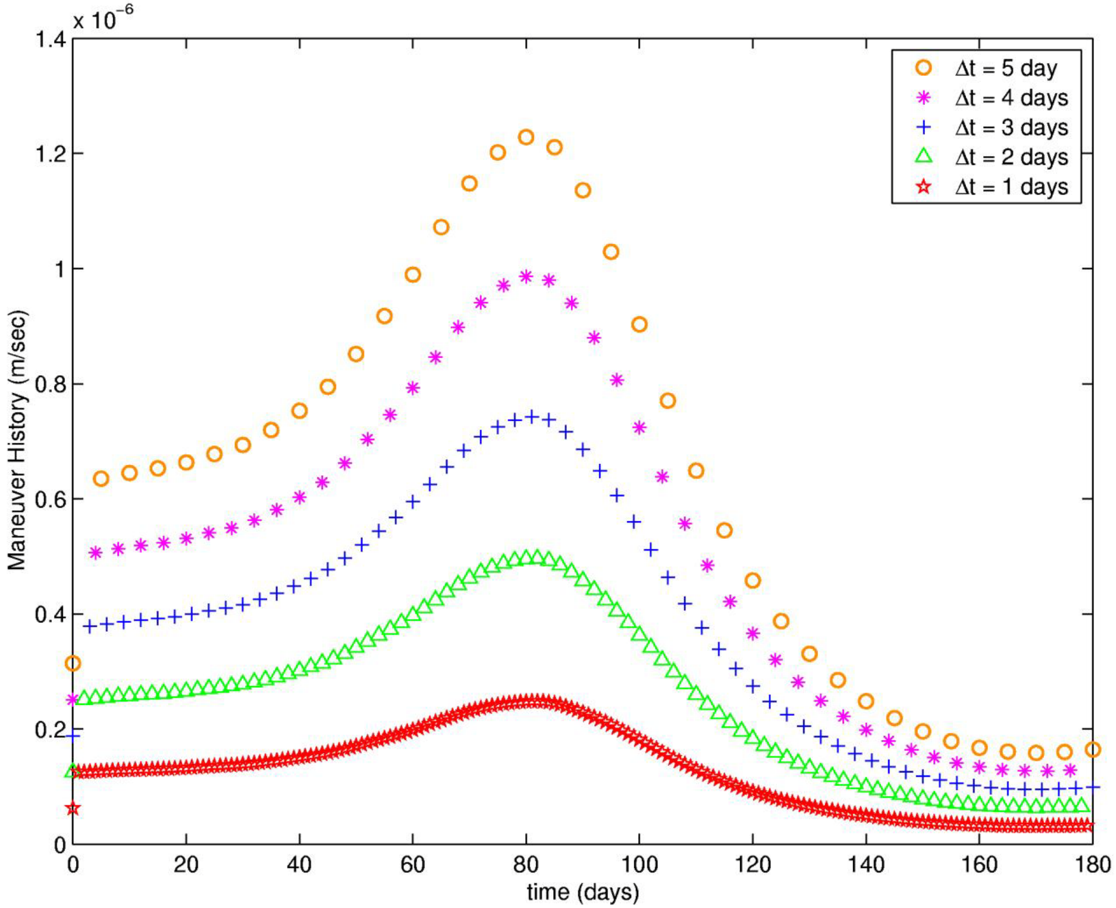
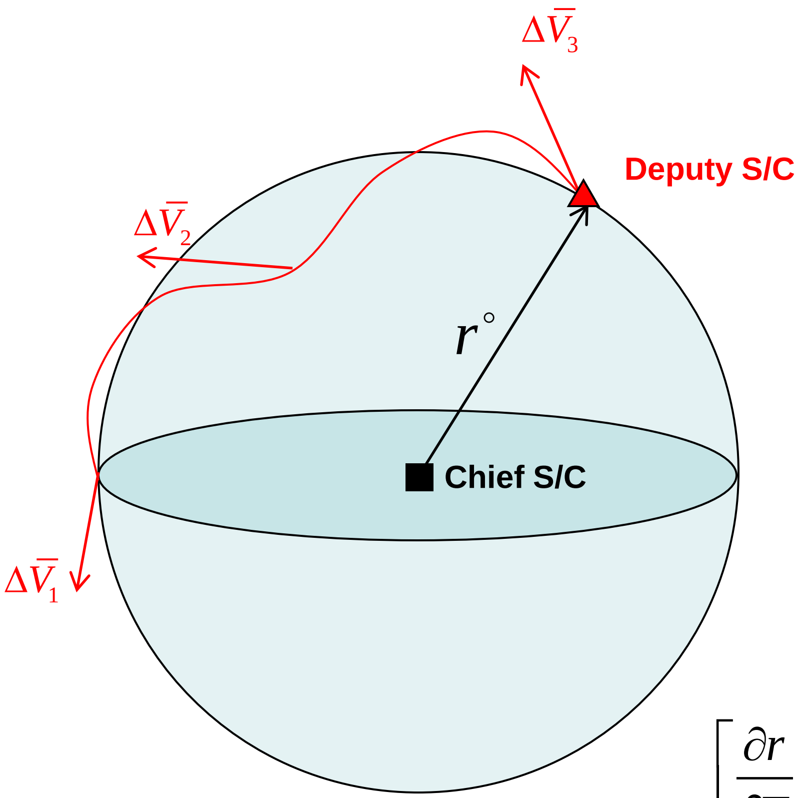
State Targeter: Maneuver Schedule



Range + Range Rate Targeter

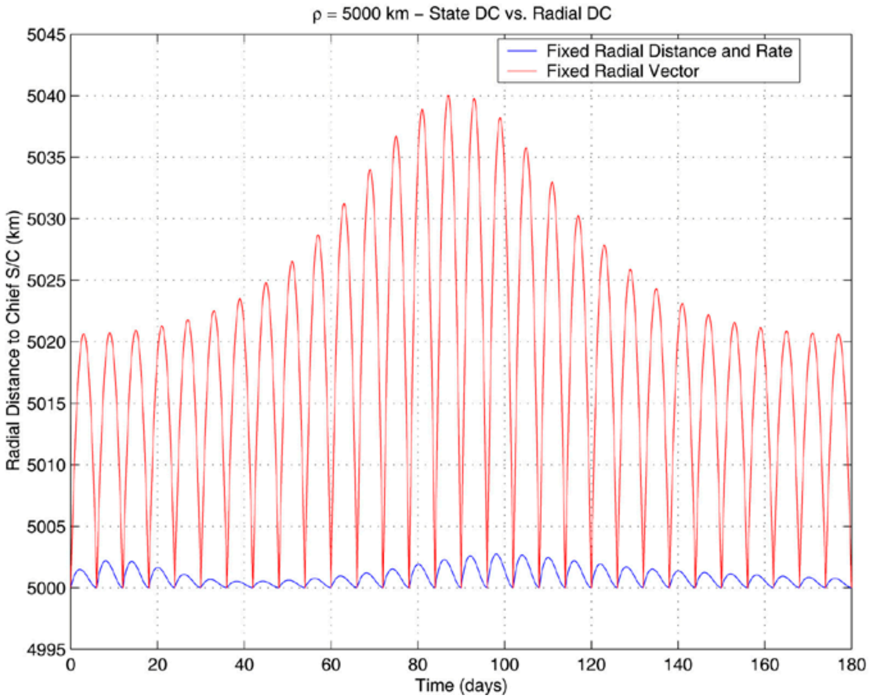


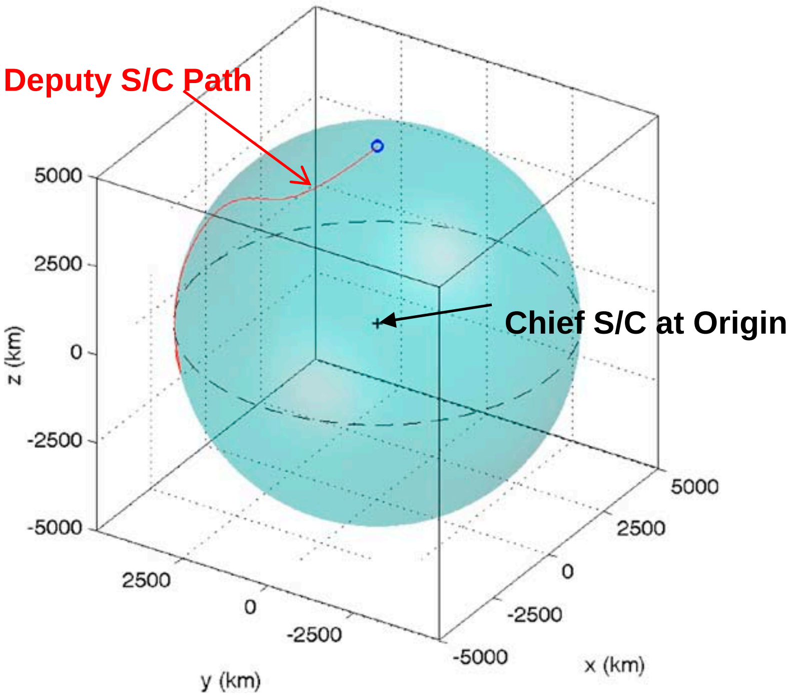
Range + Range Rate Constraint:

$$\bar{g}_f = \begin{bmatrix} r_f \\ \dot{r}_f \end{bmatrix} = \begin{bmatrix} \left(\bar{r}_f^{\,T}\bar{r}_f\right)^{1/2} \\ \dfrac{\bar{r}_f^{\,T}\dot{\bar{r}}_f}{r_f} \end{bmatrix}$$

$$d\bar{g}_f = \underbrace{\begin{bmatrix} \dfrac{\partial r}{\partial \bar{r}} & \dfrac{\partial r}{\partial \dot{\bar{r}}} \\ \dfrac{\partial \dot{r}}{\partial \bar{r}} & \dfrac{\partial \dot{r}}{\partial \dot{\bar{r}}} \end{bmatrix}}_{\text{State Relationship Matrix}} \overbrace{\Phi\left(t,t_0\right)}^{\text{STM}}\delta\bar{x}_0 = \Lambda\left(t\right)\Phi\left(t,t_0\right)\begin{bmatrix} \delta\bar{r}_0 \\ \delta\bar{v}_0^{\,-} + \Delta\bar{V}_0 \end{bmatrix}$$

Comparison of Range and State Targeters

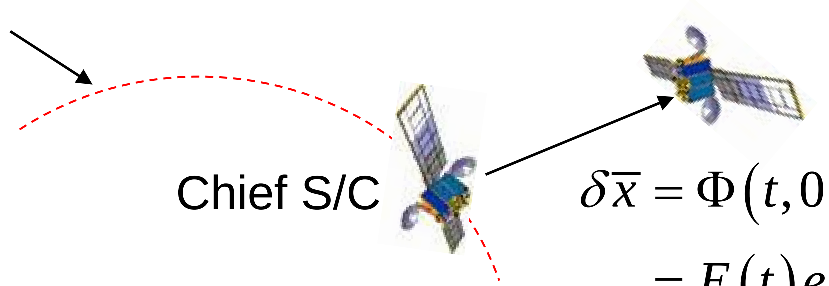




Aeronautics & Astronautics

DESIGN OF NON-NATURAL FORMATIONS USING NATURAL SOLUTION ARCS

CR3BP Analysis of Phase Space Eigenstructure Near Halo Orbit



$$\delta\bar{x} = \Phi(t,0)\,\delta\bar{x}(0)$$
$$= E(t)\,e^{Jt}E^{-1}(0)\,\delta\bar{x}(0)$$

Solution to Variational Eqn. in terms of Floquet Modes:

$$\delta\bar{x}(t) = \sum_{j=1}^{6}\delta\bar{x}_j(t) = \sum_{j=1}^{6} c_j(t)\underbrace{\bar{e}_j(t)}_{\text{Floquet Modes}} = E(t)\,\bar{c}(t)$$

Mode 1 → 1-D Unstable Subspace
Mode 2 → 1-D Stable Subspace
Modes (3,4) and (5,6) → 4-D Center Subspace

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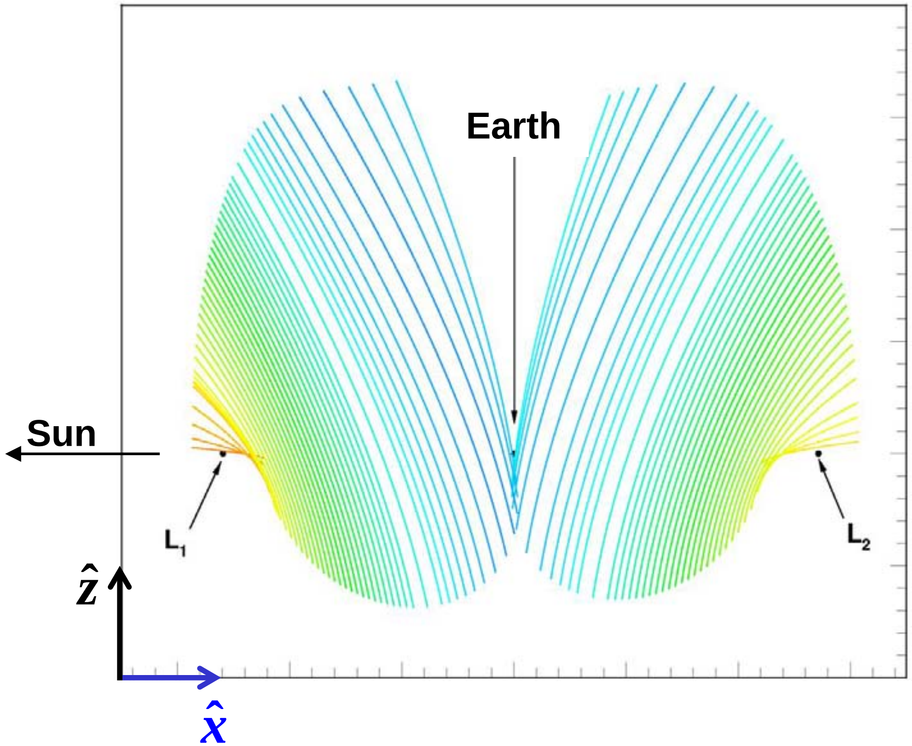
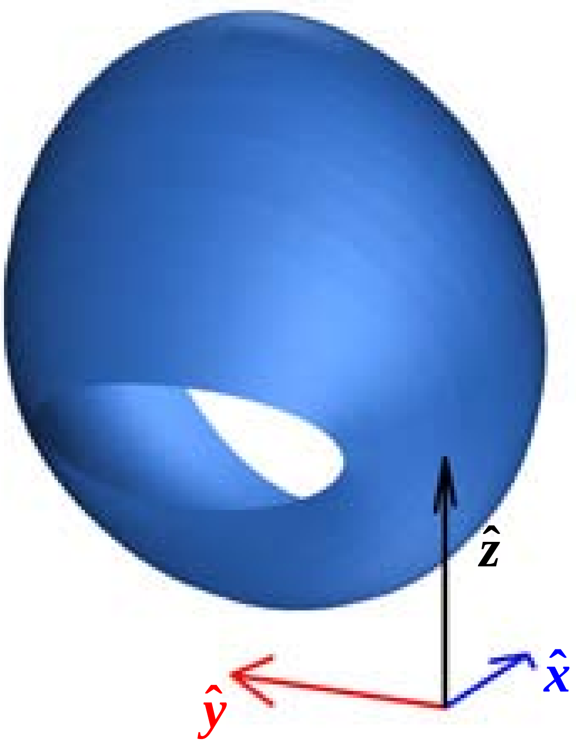
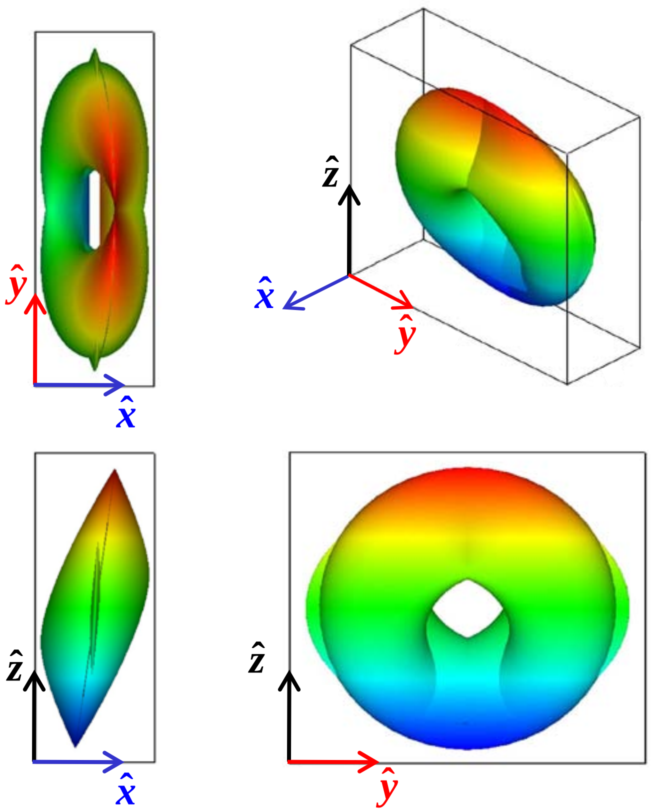
Natural Solutions: Periodic Halo Orbits Near Libration Points

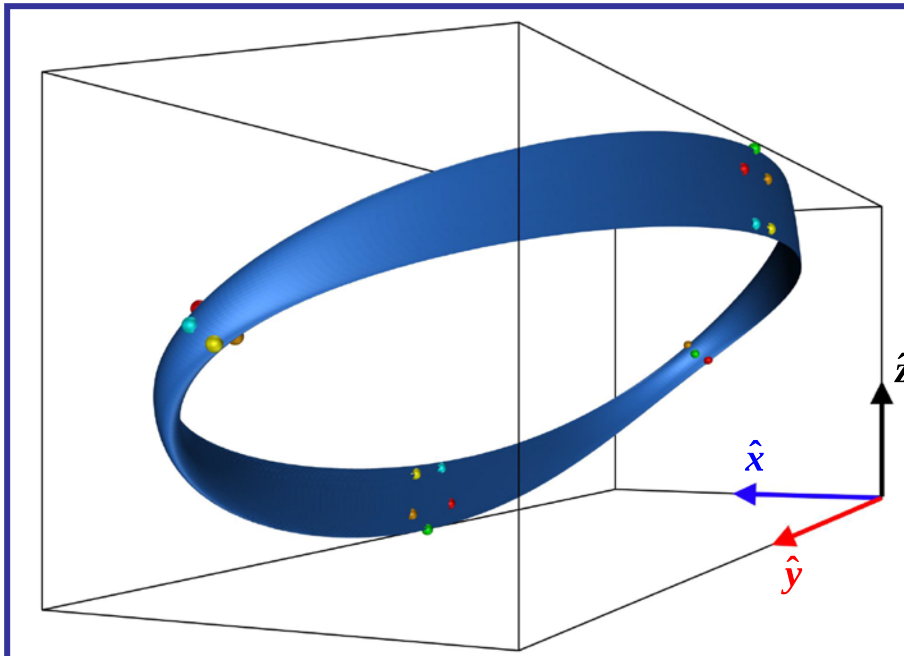




Natural Formations: Quasi-Periodic Relative Orbits → 2-D Torus

Chief S/C Centered View (RLP Frame)





Floquet Controller
(Remove Unstable + 2 Center Modes)

Find $\Delta\overline{V}$ that removes undesired response modes:

$$\sum_{j=1}^{6}\delta\overline{x}_j + \begin{bmatrix} 0_3 \\ I_3 \end{bmatrix}\Delta\overline{V} = \sum_{\substack{j=2,3,4 \\ \text{or} \\ j=2,5,6}} \left(1+\alpha_j\right)\delta\overline{x}_j$$

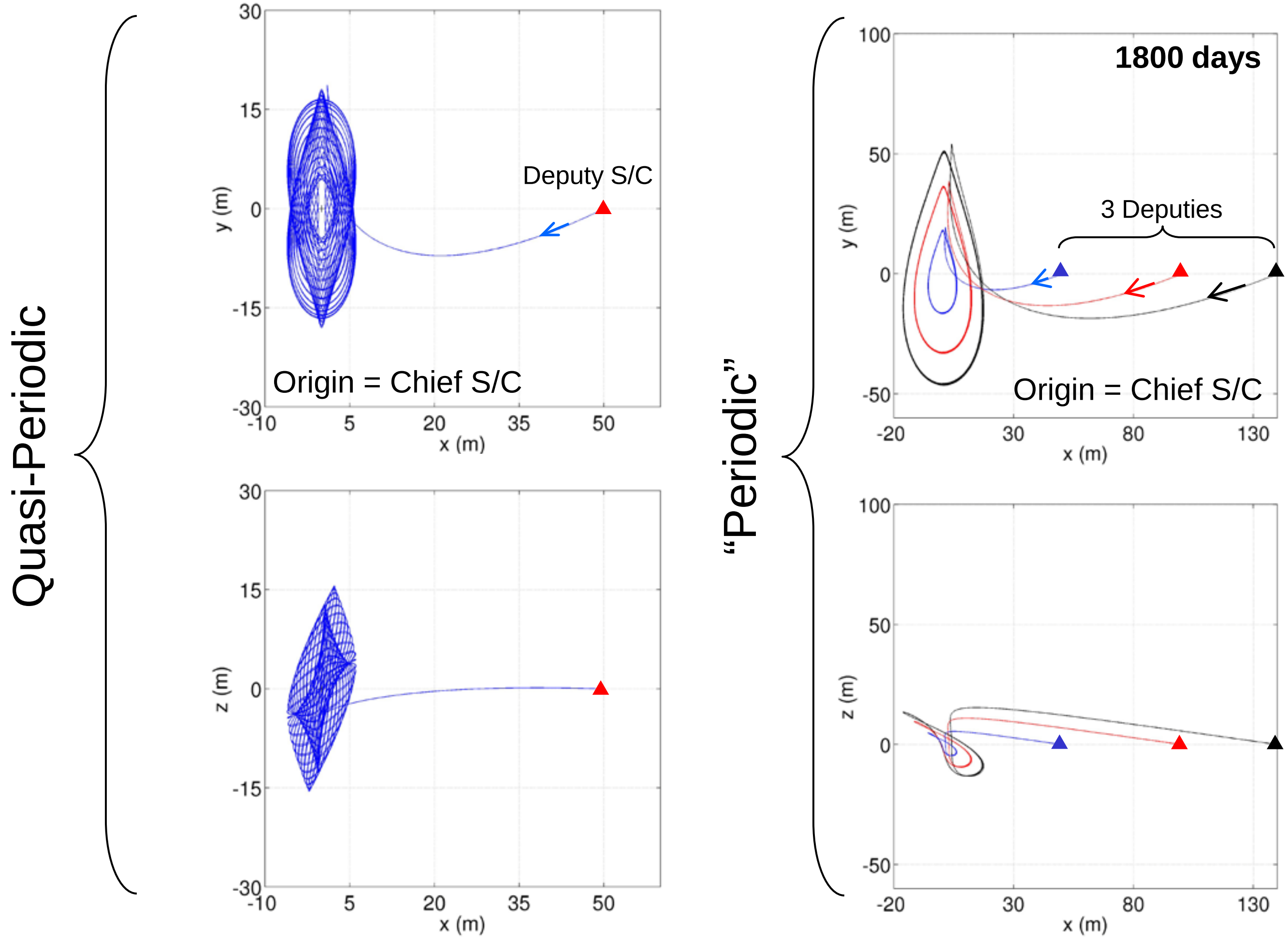
Remove Modes 1, 3, and 4:

$$\begin{bmatrix} \overline{\alpha} \\ \Delta\overline{V} \end{bmatrix} = \begin{bmatrix} \delta\overline{x}_{2\overline{r}} & \delta\overline{x}_{5\overline{r}} & \delta\overline{x}_{6\overline{r}} & 0_3 \\ \delta\overline{x}_{2\overline{v}} & \delta\overline{x}_{5\overline{v}} & \delta\overline{x}_{6\overline{v}} & -I_3 \end{bmatrix}^{-1} \left(\delta\overline{x}_1 + \delta\overline{x}_3 + \delta\overline{x}_4\right)$$

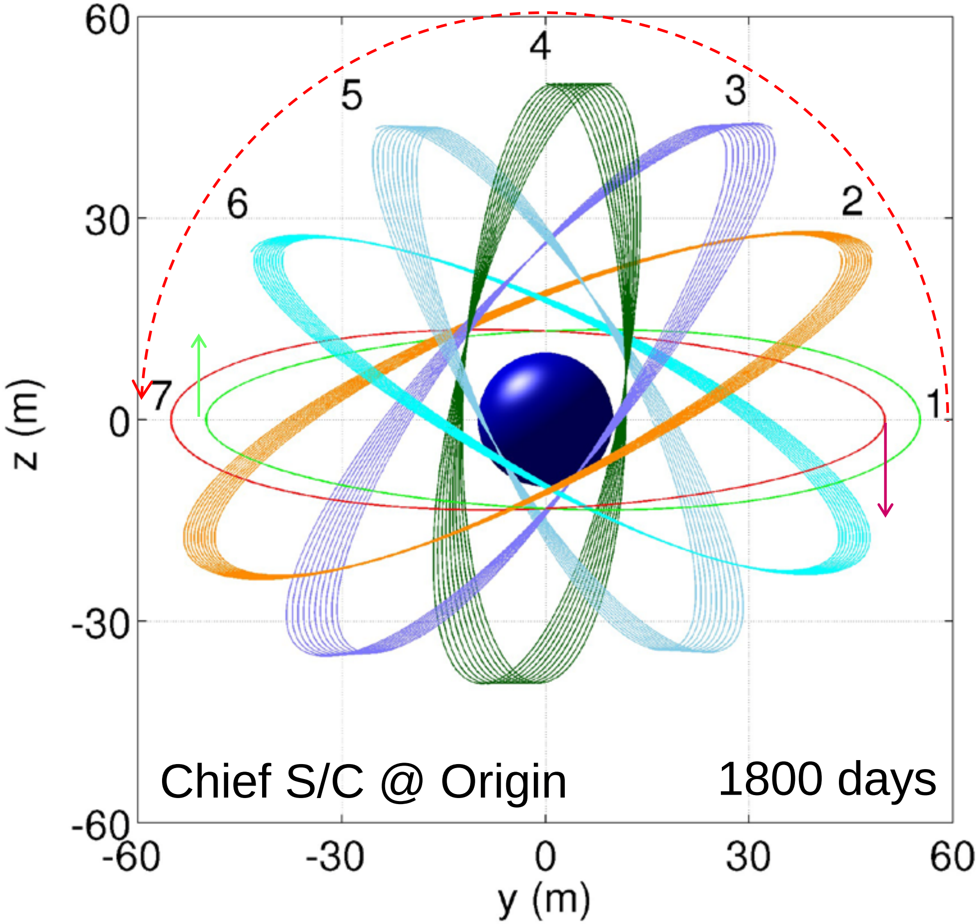
Remove Modes 1, 5, and 6:

$$\begin{bmatrix} \overline{\alpha} \\ \Delta\overline{V} \end{bmatrix} = \begin{bmatrix} \delta\overline{x}_{2\overline{r}} & \delta\overline{x}_{3\overline{r}} & \delta\overline{x}_{4\overline{r}} & 0_3 \\ \delta\overline{x}_{2\overline{v}} & \delta\overline{x}_{3\overline{v}} & \delta\overline{x}_{4\overline{v}} & -I_3 \end{bmatrix}^{-1} \left(\delta\overline{x}_1 + \delta\overline{x}_5 + \delta\overline{x}_6\right)$$

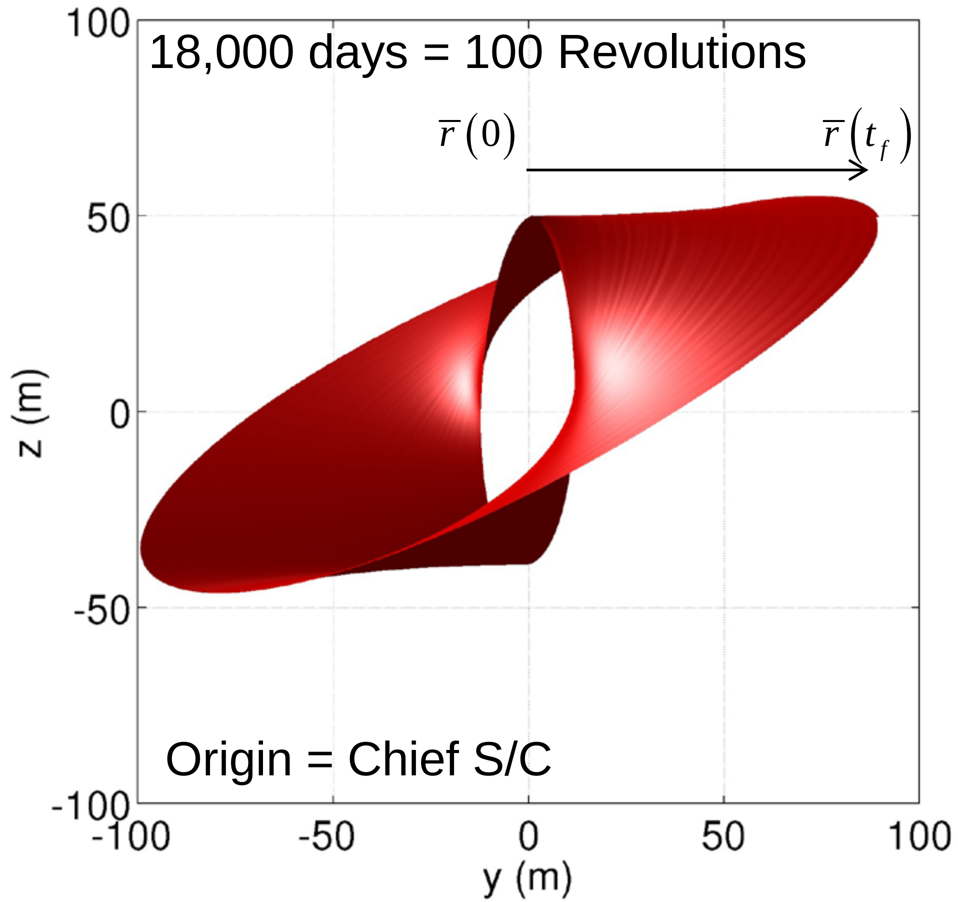
Sample Deployment into Relative Orbits: 1-ΔV at Injection



Natural Formations: Nearly Periodic and Drifting Relative Orbits

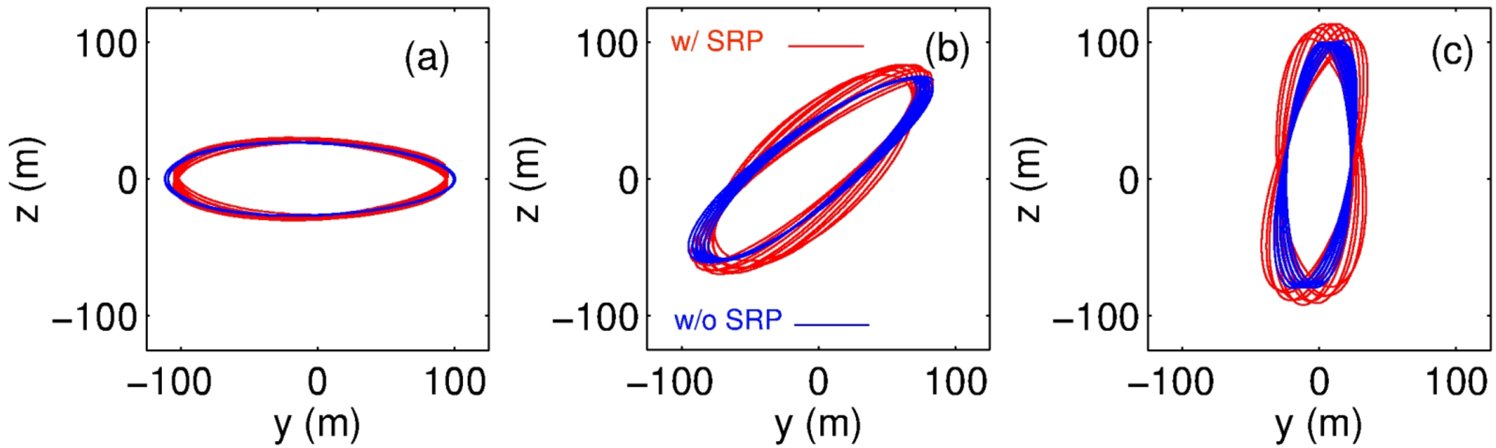


Expansion of Drifting Vertical Orbit



Transitioning Natural Motions into Non-Natural Arcs: Targeter Approach

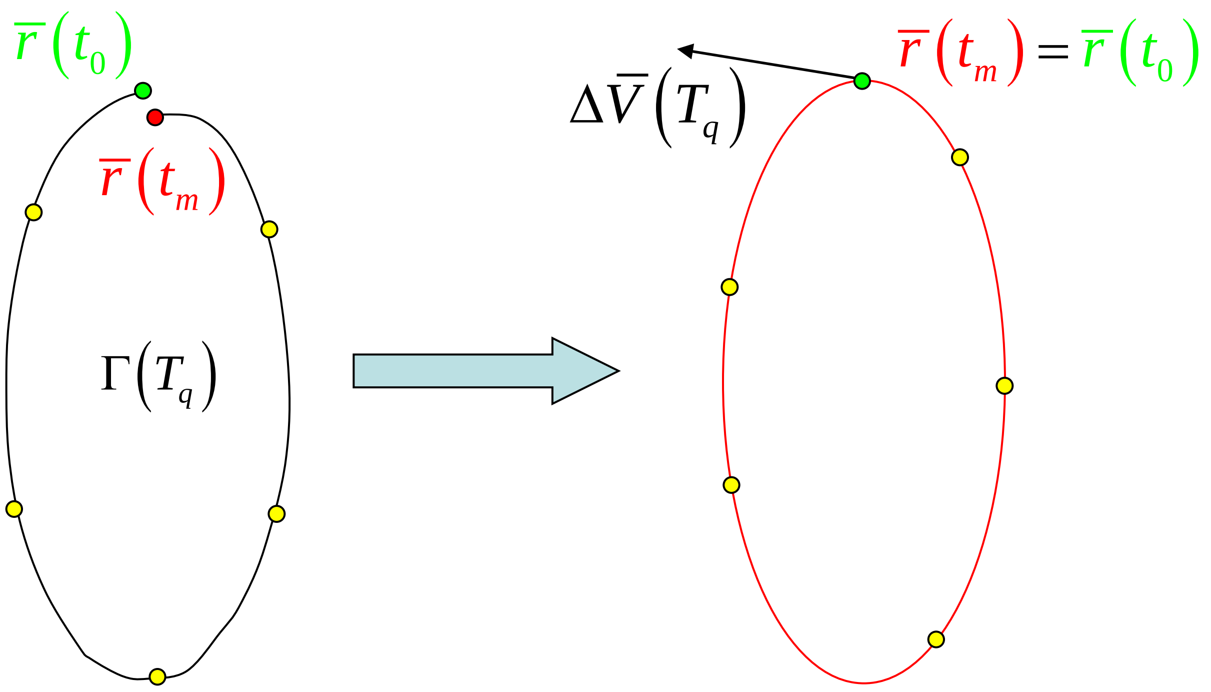
STEP 1: Identify a suitable initial guess



Target → Orbital Drift Control

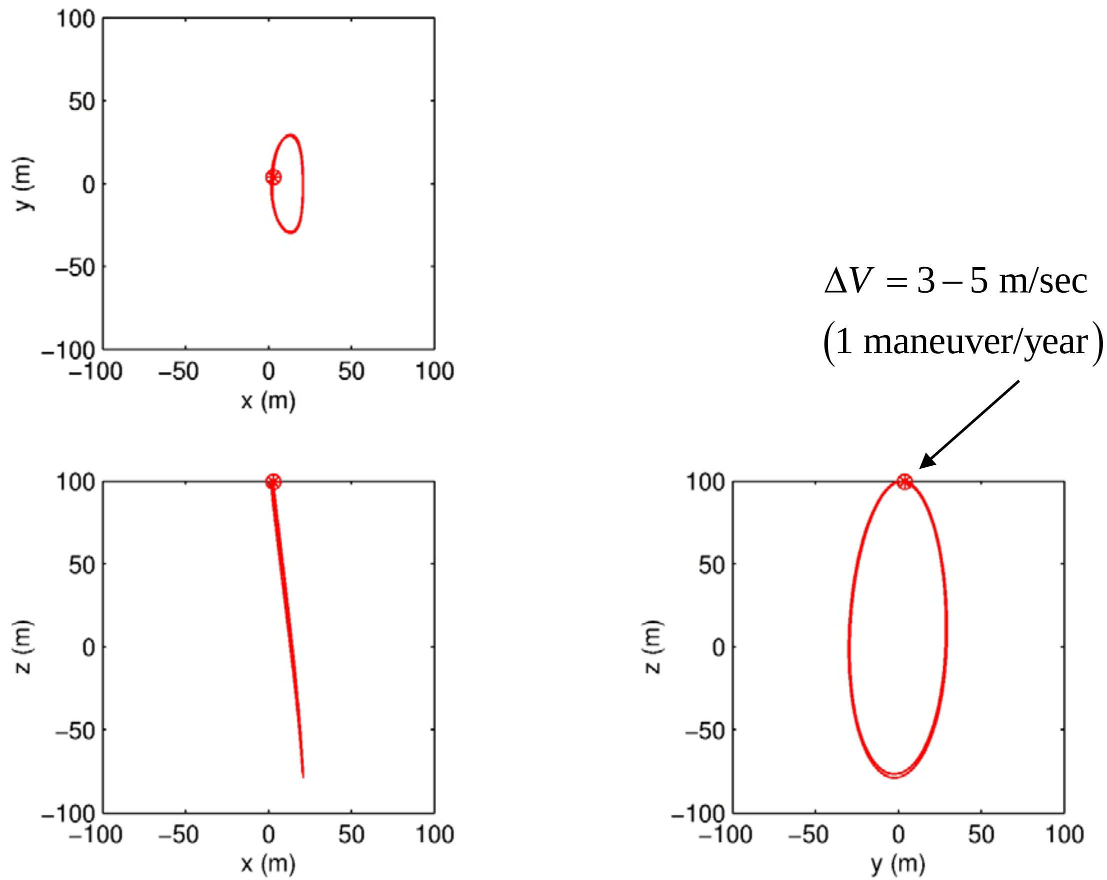
Application of Two Level Corrector

STEP 2: Apply 2-level corrector (Howell and Wilson:1996) w/ end-state constraint



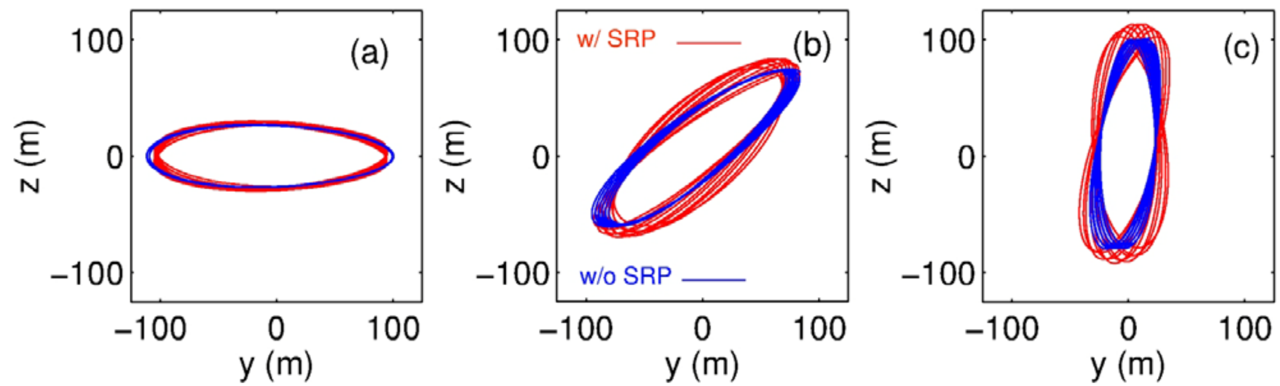
STEP 3: Shift converged patch states forward by 1 period
STEP 4: Reconverge Solution

Drift Controlled Vertical Orbit (6 Revs)

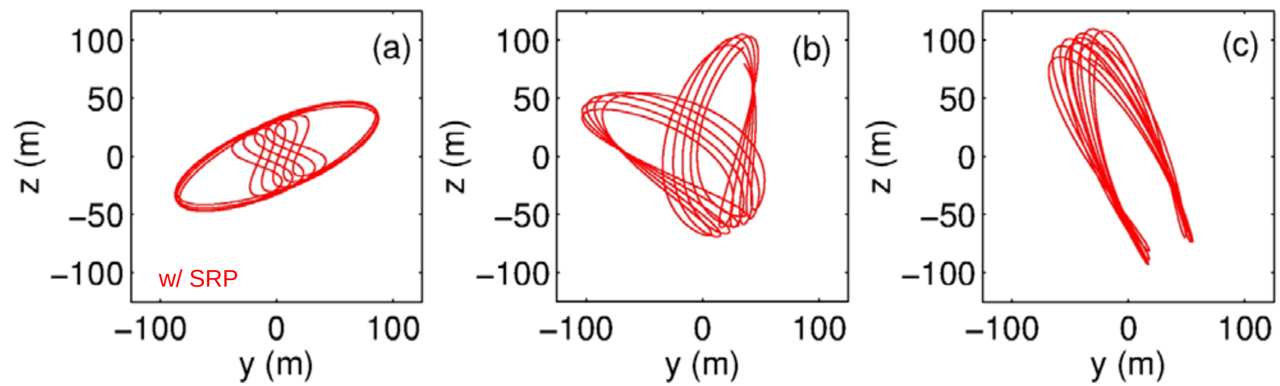


Geometry of Natural Solutions in the Ephemeris Model

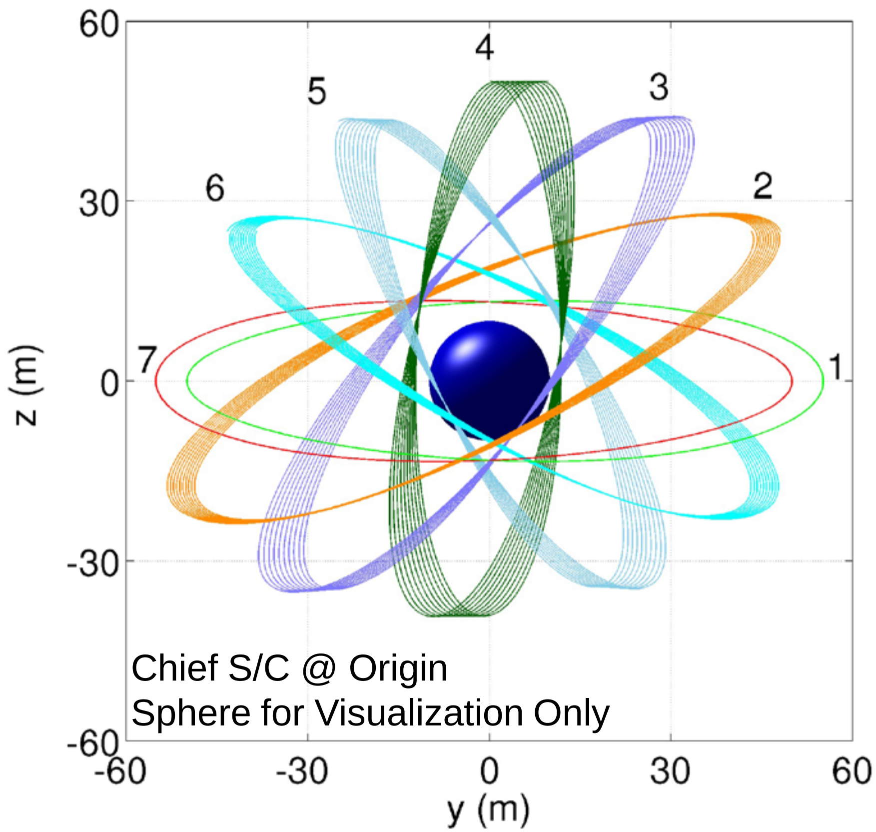
Rotating Frame Perspective:



Inertial Frame Perspective:

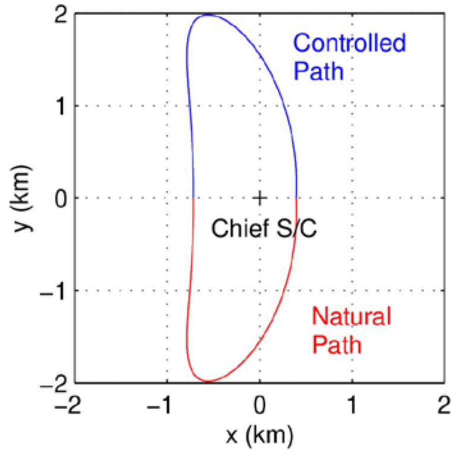


Transitioning Natural Motions into Non-Natural Arcs: IFL Example

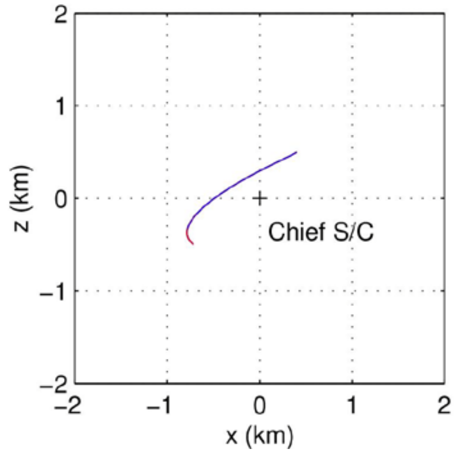


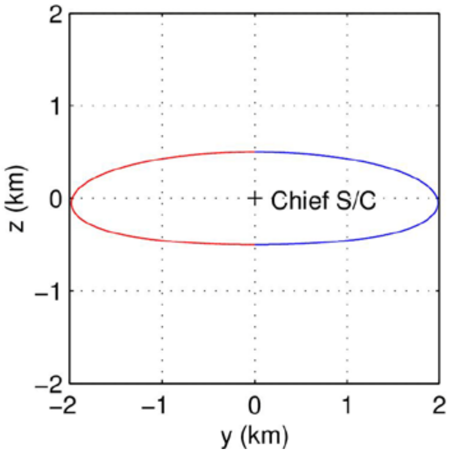
(1) Consider 1st Rev Along Orbit #4 as initial guess to simple targeter.
(2) Choose initial state on xz-plane
(3) Target next plane crossing to be ⊥
(4) Use resulting arc as half of the reference motion.
(5) Numerically mirror solution about xz-plane and store as nominal.
(6) Use IFL control to enforce a closed orbit using stored nominal.

Hybrid Control:
Natural Motions + Continuous Control



½ Period → Natural Arc
½ Period → IFL Control





Concluding Remarks

- Precise Formation Keeping → Continuous Control
  - Is it possible?
    - Depends on hardware capabilities and nominal motion specified
    - Not if thruster On/Off sequences are required & tolerances too high
- Precise Navigation → Natural Formations
  - Targeter Methods
    - Natural motions can be forced to follow non-natural paths
    - Success depends on non-natural motion specified
  - Hybrid Methods (Natural Arcs + Continuous Thrust Arcs)
    - May prove beneficial for non-natural inertial formation design.

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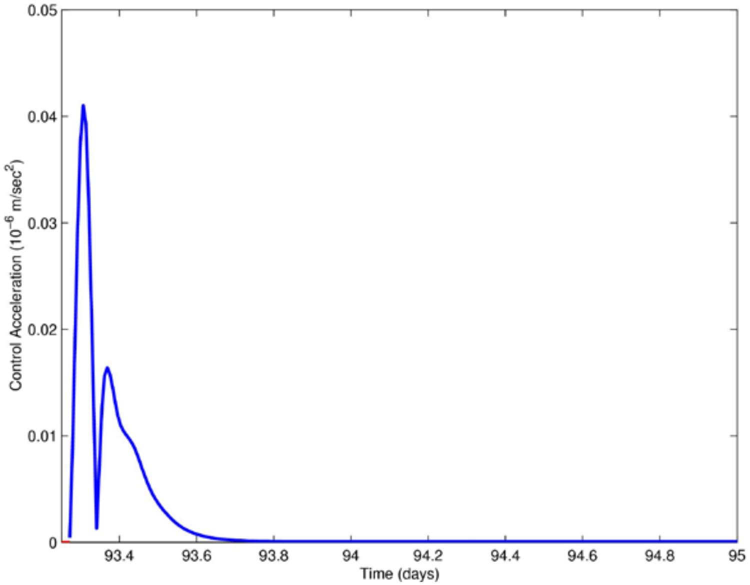
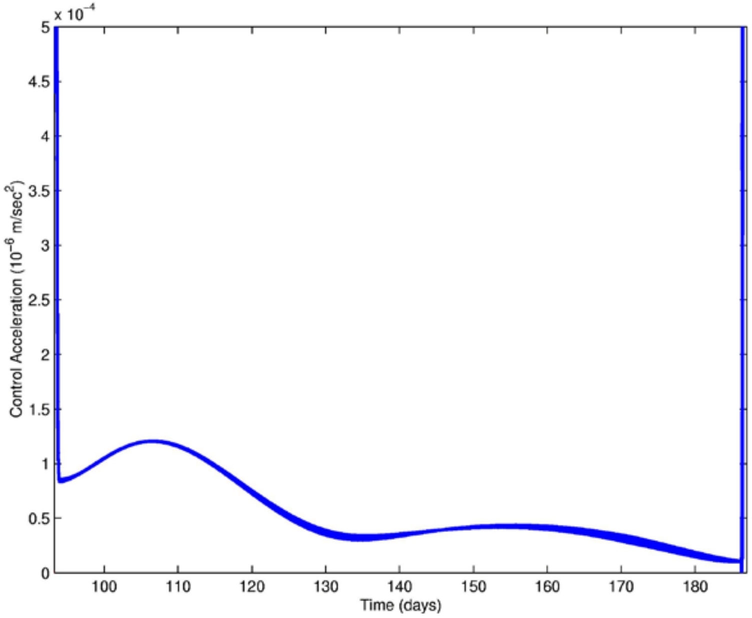
BACKUP SLIDES

Hybrid Control: Accelerations





Radial Targeter: Control Law Formulation

Range + Range Rate Constraint:

$$\bar{g}_f = \begin{bmatrix} r_f \\ \dot{r}_f \end{bmatrix} = \begin{bmatrix} \left(\bar{r}_f^{\,T}\bar{r}_f\right)^{1/2} \\ \dfrac{\bar{r}_f^{\,T}\dot{\bar{r}}_f}{r_f} \end{bmatrix}$$

Desired Range + Range Rate:

$$\bar{g}^*_f = \begin{bmatrix} r^*_f \\ 0 \end{bmatrix}$$

First Order Approximation:

$$\bar{g}^*_f = \bar{g}_f + \frac{\partial \bar{g}}{\partial \bar{x}_0}\delta \bar{x}_0$$

$$d\bar{g} = \left(\bar{g}_f^{\,*} - \bar{g}_f\right) = \frac{\partial \bar{g}}{\partial \bar{x}}\frac{\partial \bar{x}}{\partial \bar{x}_0}\delta \bar{x}_0 = \underbrace{\begin{bmatrix} \dfrac{\partial r}{\partial \bar{r}} & \dfrac{\partial r}{\partial \dot{\bar{r}}} \\ \dfrac{\partial \dot{r}}{\partial \bar{r}} & \dfrac{\partial \dot{r}}{\partial \dot{\bar{r}}} \end{bmatrix}}_{\text{State Relationship Matrix}} \overbrace{\Phi\left(t,t_0\right)}^{\text{STM}}\delta \bar{x}_0 = \Lambda\left(t\right)\Phi\left(t,t_0\right)\delta \bar{x}_0$$

Dynamical Model

Generalized Dynamical Model for Each S/C:

$$ {}^{I}\ddot{\bar{r}}^{P_2 P_s}(t) = \underbrace{-\frac{\mu_{P_2}}{r^{P_2 P_s}} + \sum_{j=1,\, j\neq 2,s}^{N} \mu_{P_j} \left(\frac{\bar{r}^{P_s P_j}}{\left(r^{P_s P_j}\right)^3} - \frac{\bar{r}^{P_2 P_j}}{\left(r^{P_2 P_j}\right)^3} \right)}_{\text{Gravity Terms}} + \underbrace{\bar{f}_{srp}^{(P_s)}}_{\text{Solar Radiation Pressure}} + \underbrace{\bar{u}(t)}_{\text{Control Input}} $$

Control Input

Gravity Terms

Solar Radiation Pressure

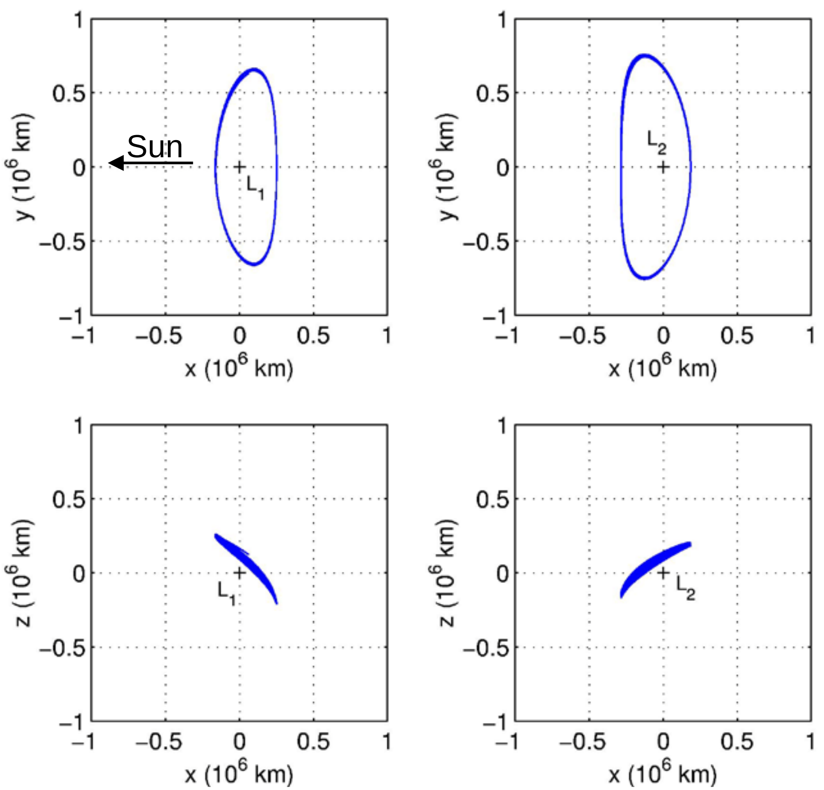
Assumptions:

Chief S/C $\rightarrow$ Evolves Along Natural Solution $\therefore\ \bar{u}_c(t) = \bar{0}$ (Nominal)

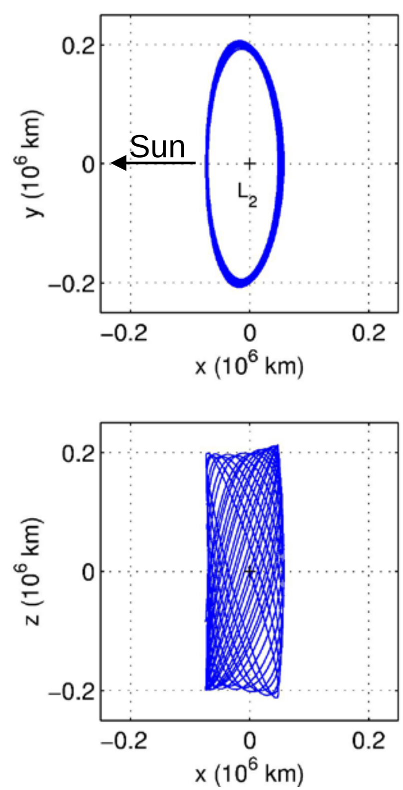
Deputy S/C $\rightarrow$ Evolves Along Non-Natural Solution $\therefore\ \bar{u}_d(t) \neq \bar{0}$

Chief S/C Motion: Natural Solutions Near L_1 and L_2

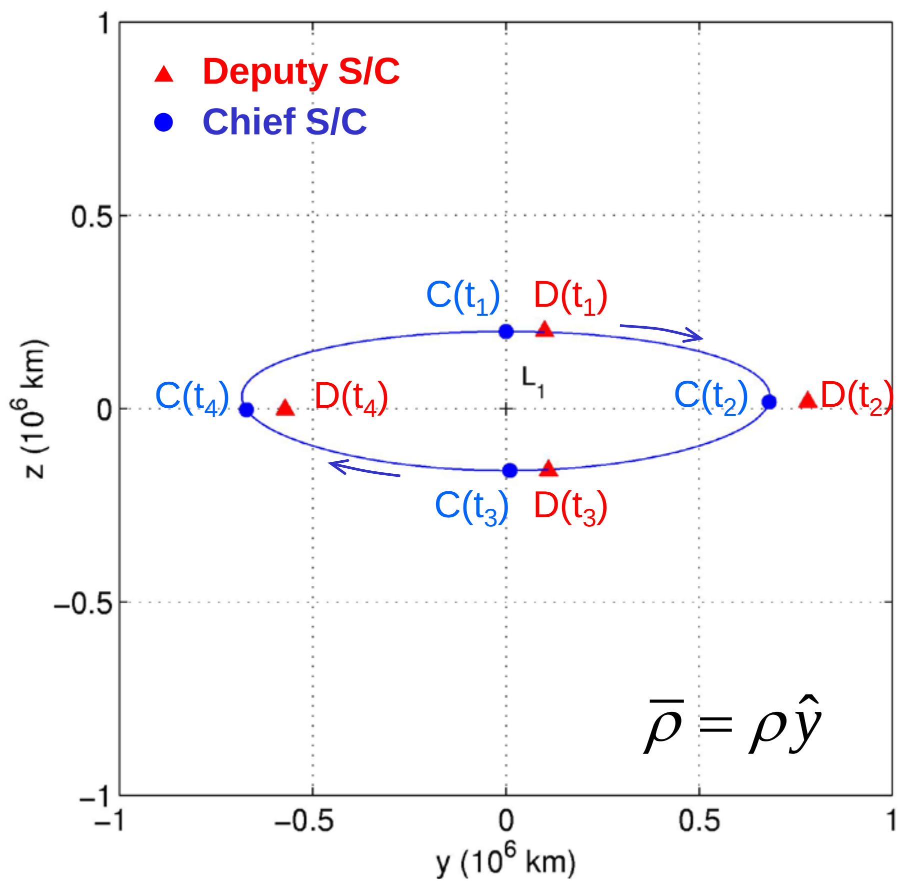
"Halo" Orbits Near L_i



Lissajous Trajectories Near L_i

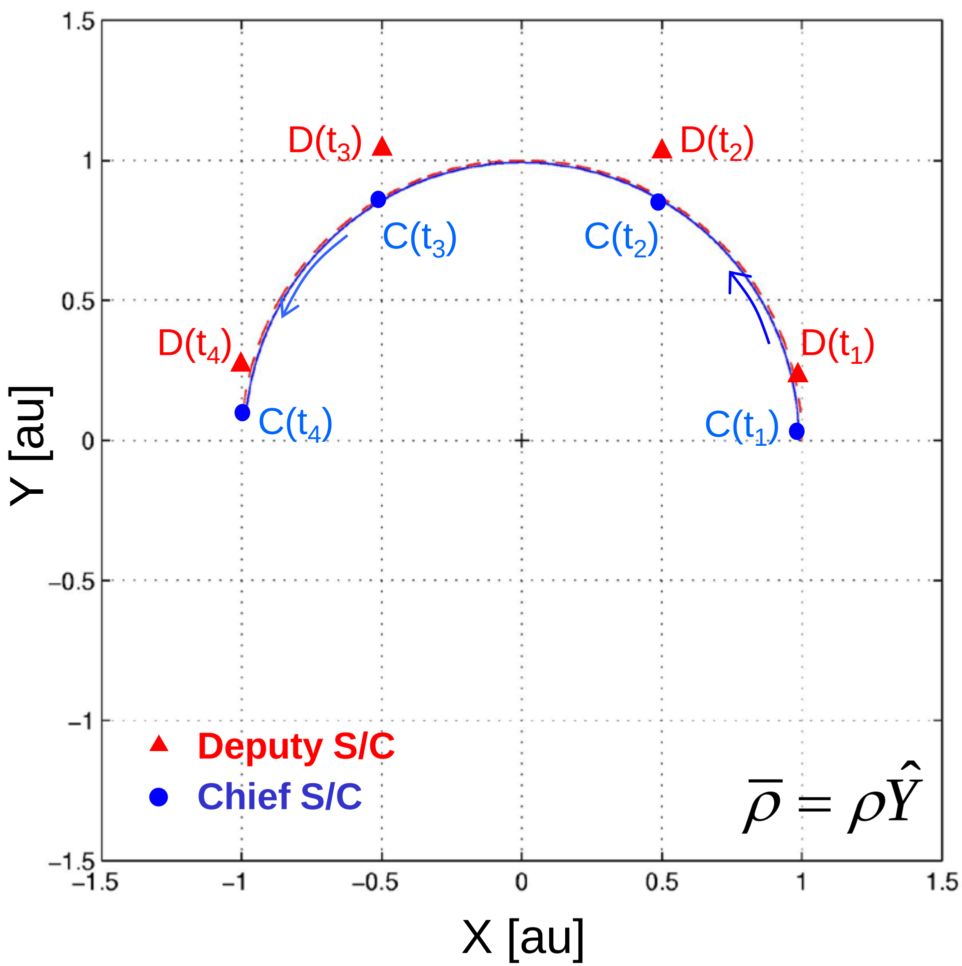


Controlled Deputy S/C Motion (Example 1): Formation Fixed in the Rotating Frame



$$\bar{\rho} = \rho \hat{y}$$

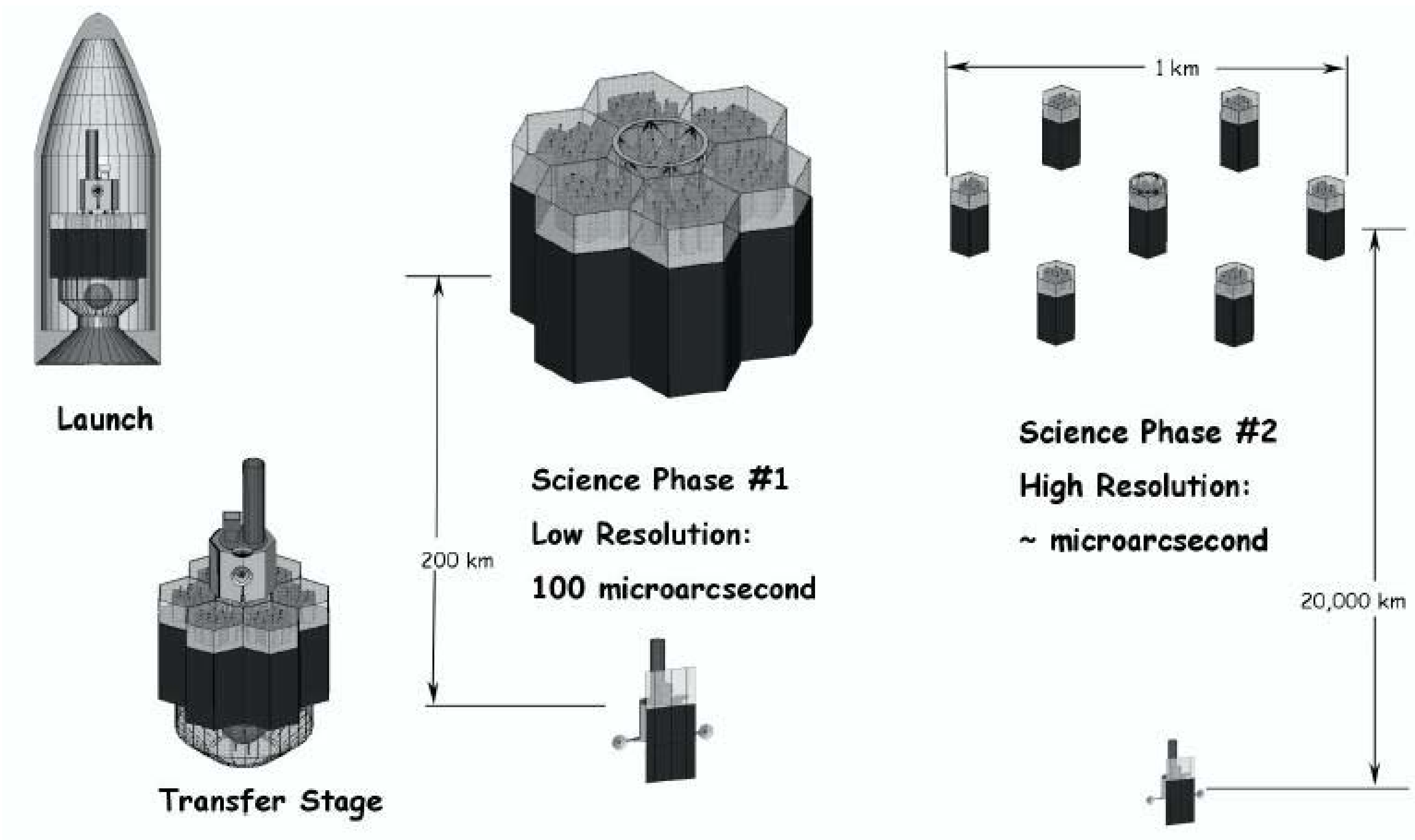
Controlled Deputy S/C Motion (Example 2): Formation Fixed in the Inertial Frame



MAXIM:
APPLICATIONS OF IFL AND OFL

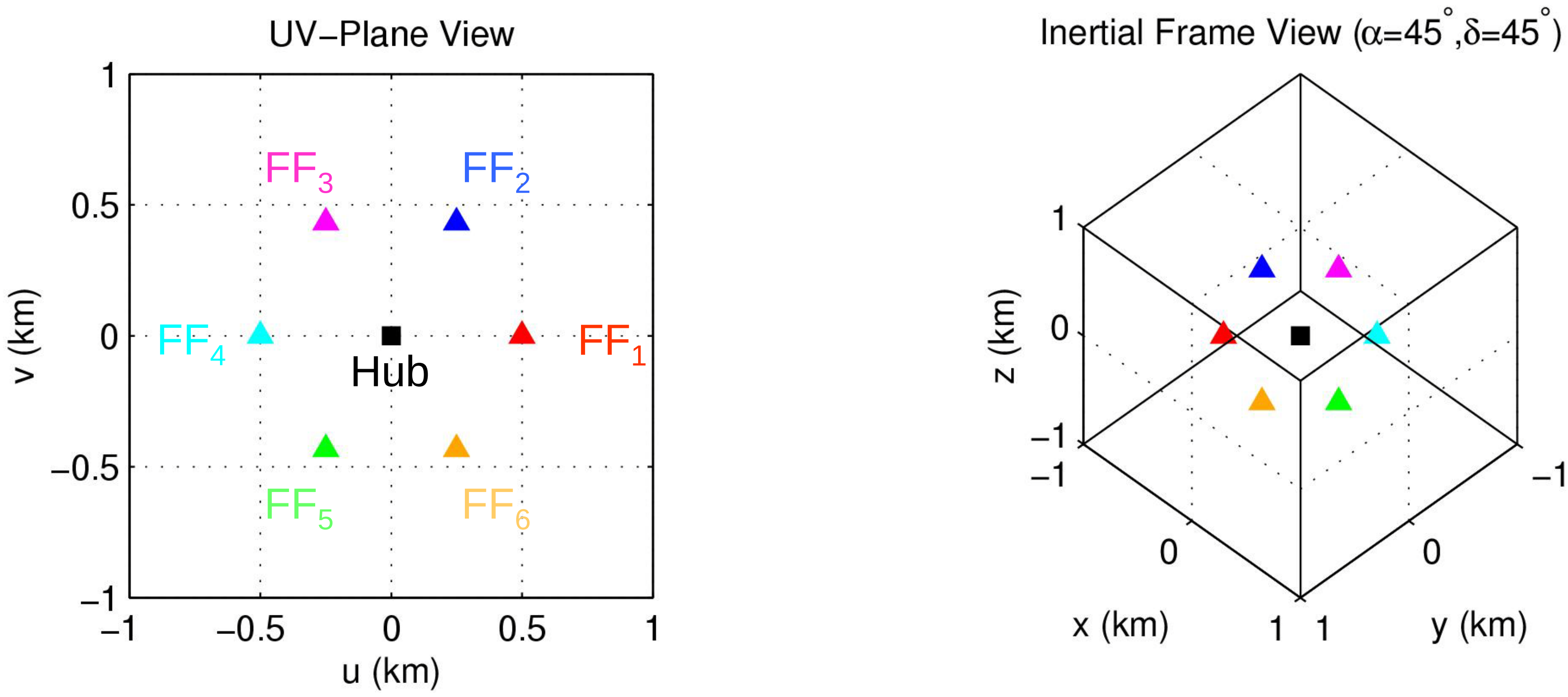
Purdue University

MAXIM Mission Sequence



MAXIM:
THRUSTER ON-OFF SEQUENCE

Free Flyer Configuration



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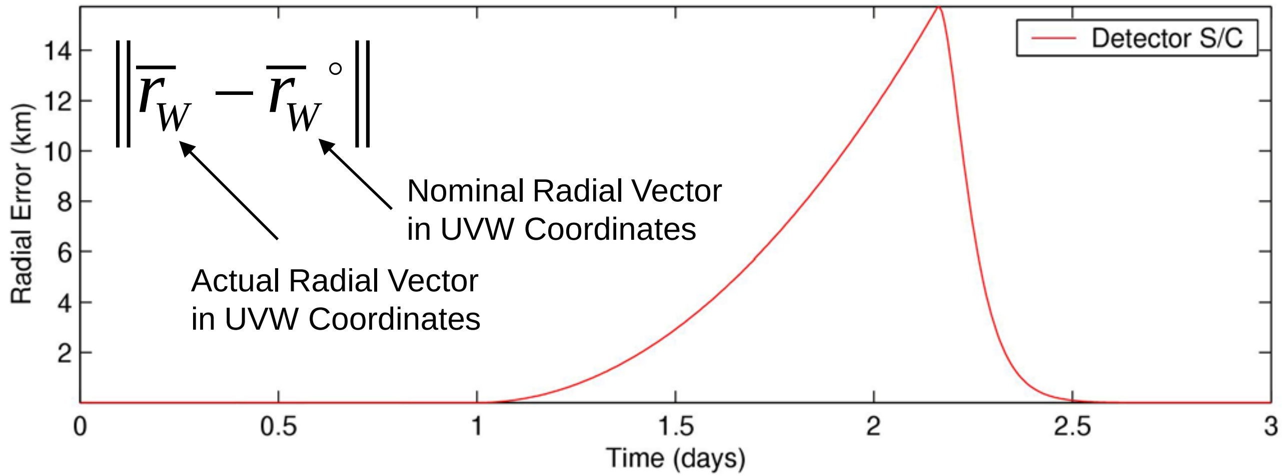
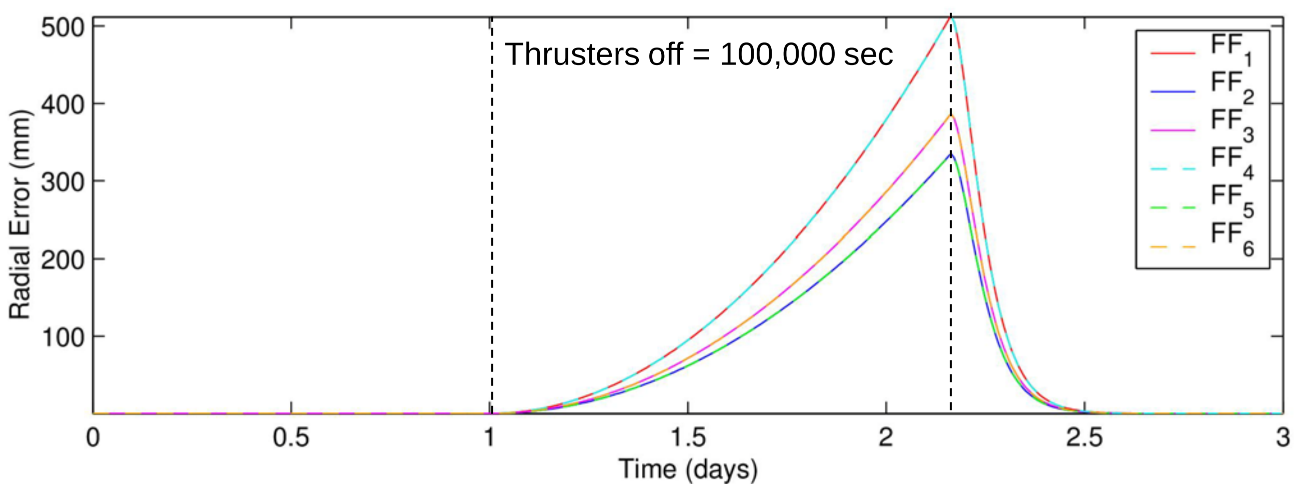
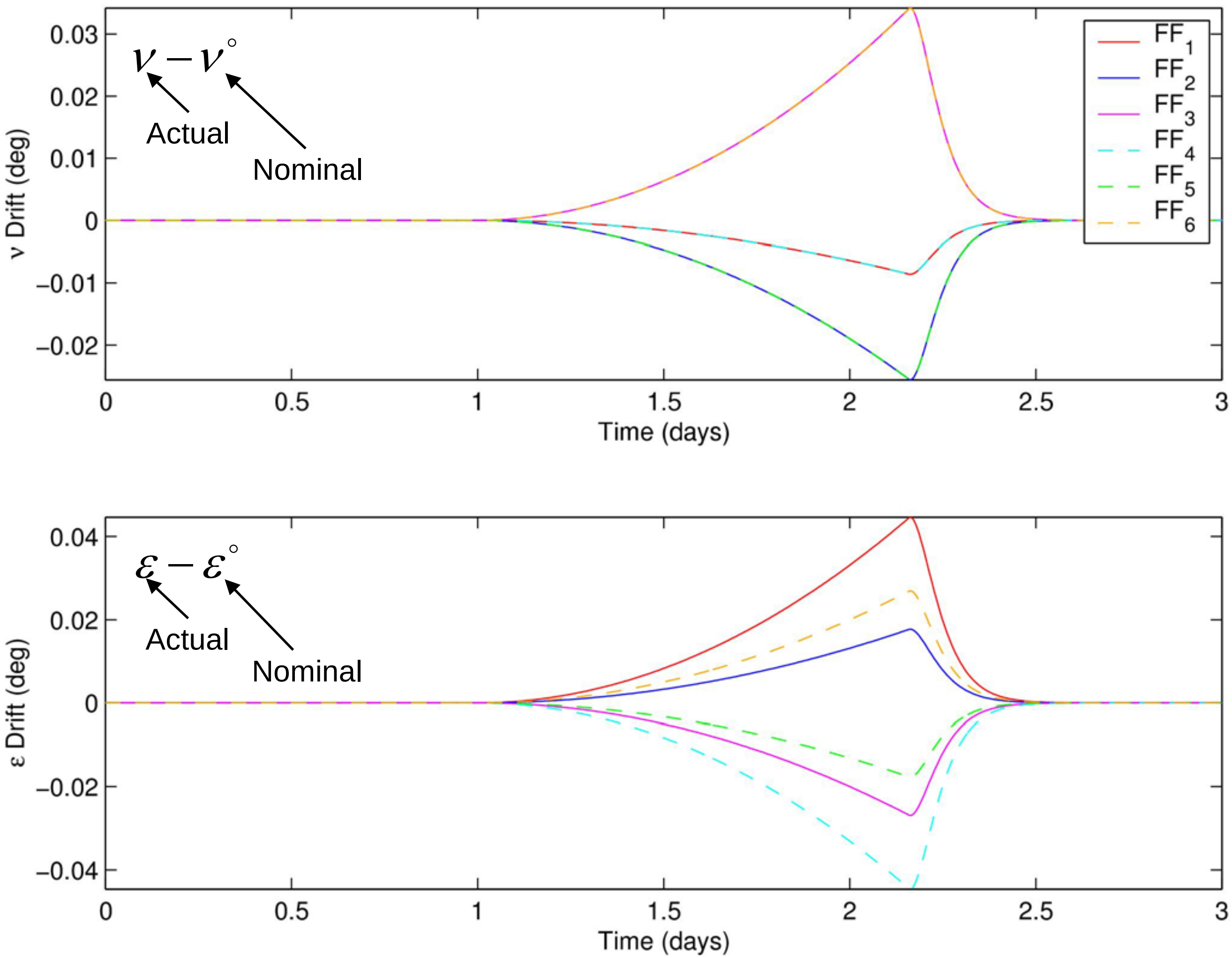
Radial Error wrt. Hub S/C



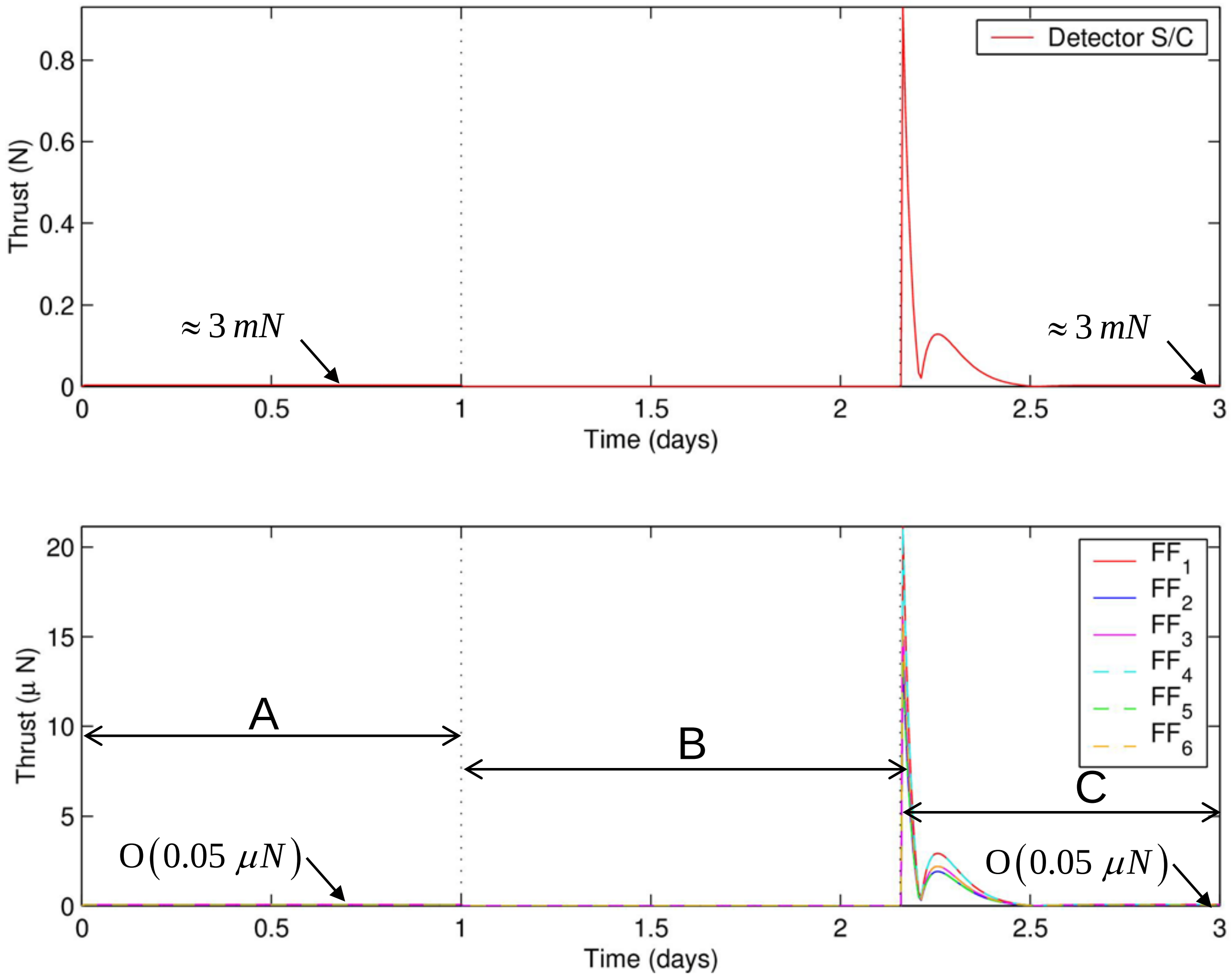


Free Flyers
UV-Plane Angular Drift (DEG)



Thrust Profile

Thrusters Off Between t_1 = 1 day & $t_2 = t_1$ + 100,000 sec.



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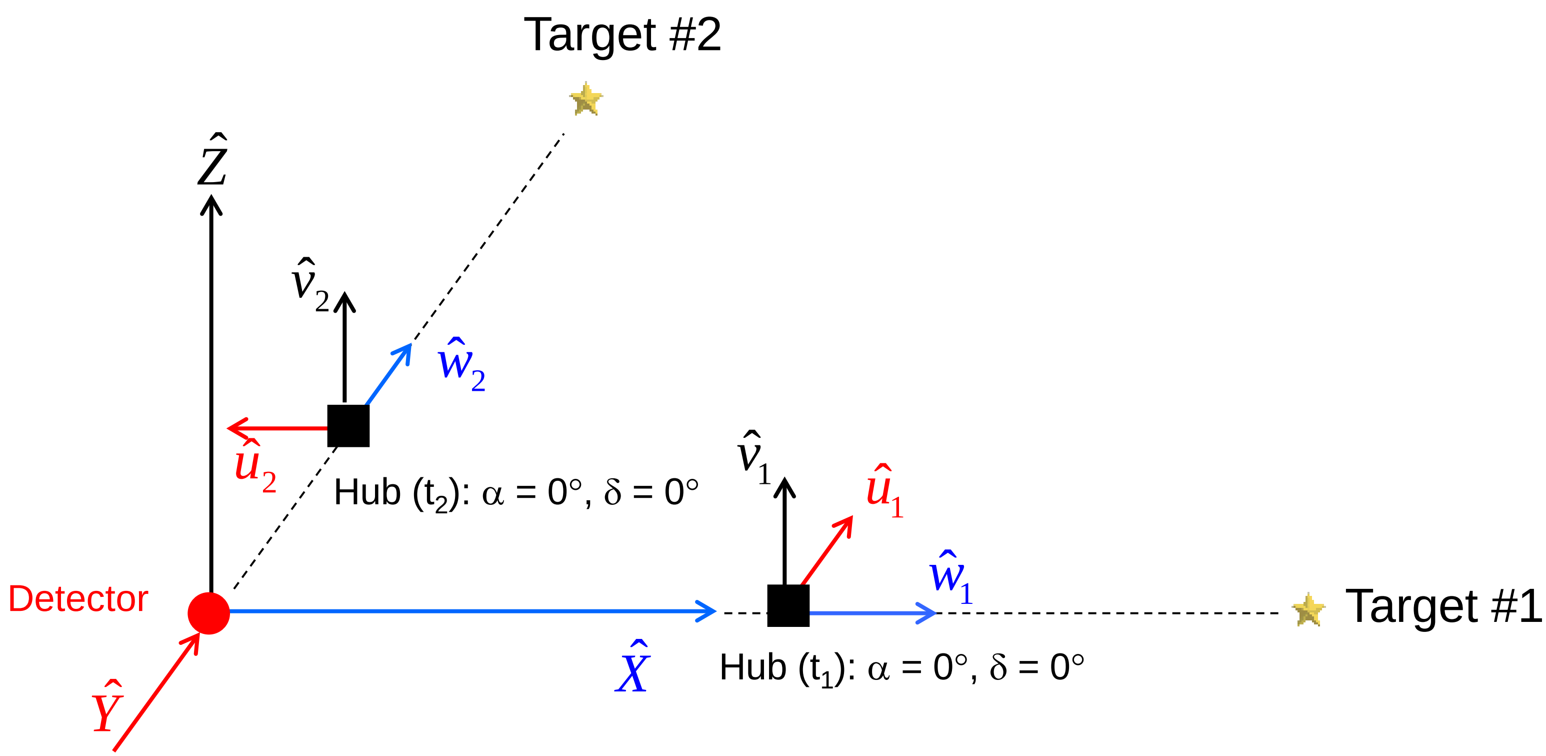
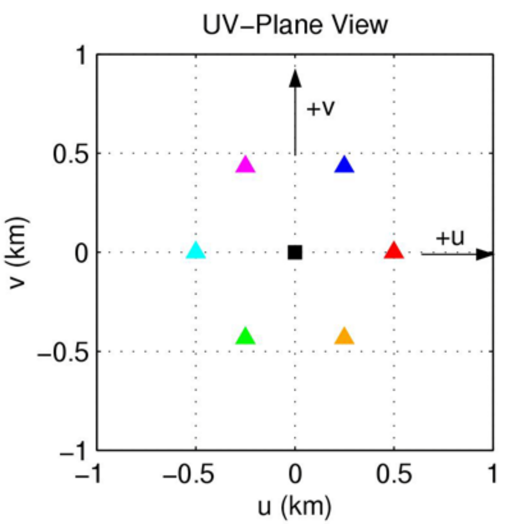
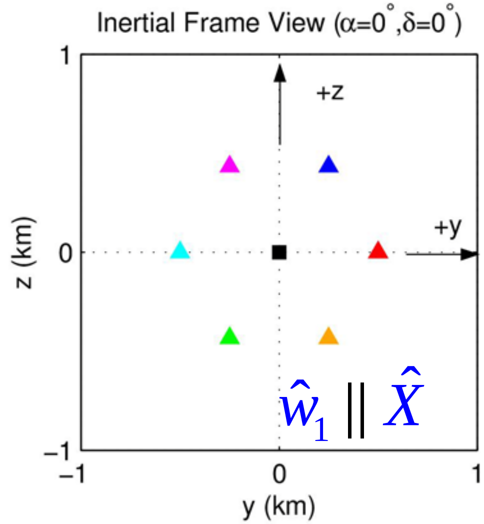
MAXIM:
FORMATION RECONFIGURATION

Target Reconfiguration

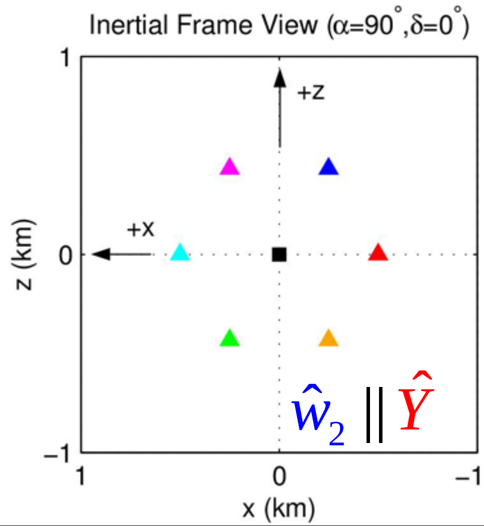


Graphical Representation of Reconfiguration for Free Flyers





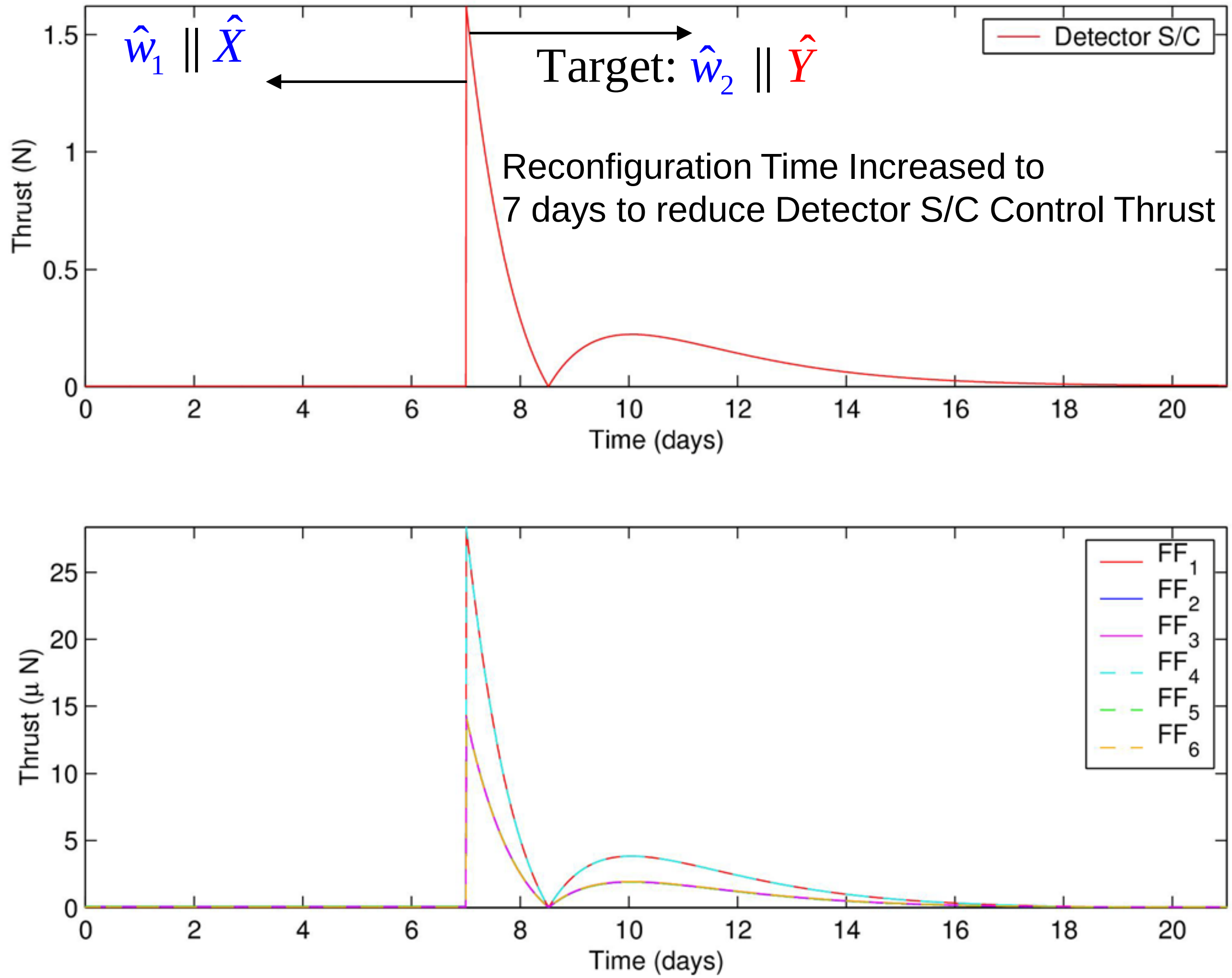
INITIAL ORIENTATION OF UV-PLANE



FINAL ORIENTATION OF UV-PLANE

Thrust to Reconfigure

From $\alpha = 0^o$ to $\alpha = 90^o$ with $\delta = 0^o$



Mission Specifications

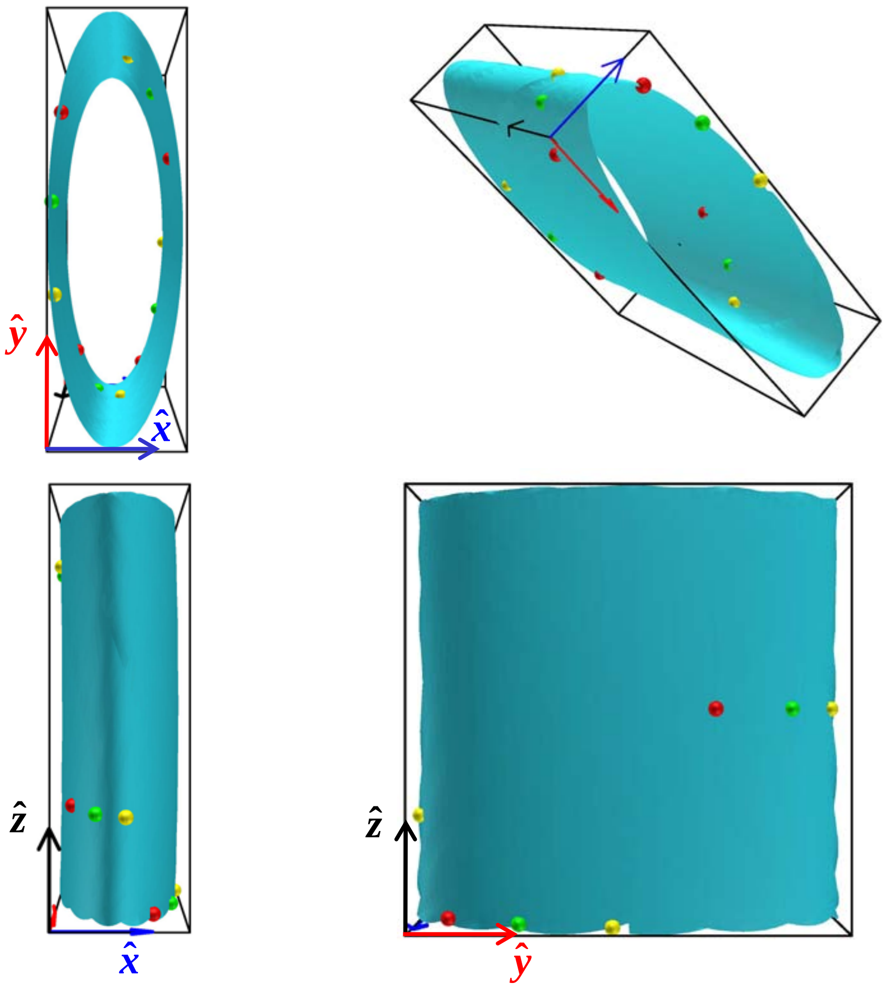
- Hold periscope positions to within 15-μm
- Detector pointing accuracy – arcminutes
- ∠ Periscope-Detector-Target alignment – μas
- Phase 1 → 1 Target /week
- Phase 2 → 1 Target/ 3 weeks

} Frequent Reconfigurations

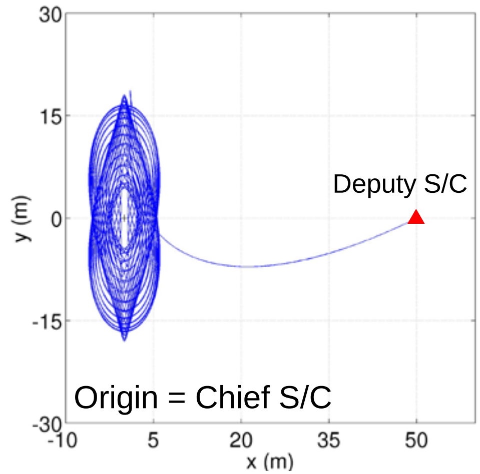
- Hub → inter. comm. port between detector & freeflyers
- Reconfiguration (Formation Slewing) Times:
  - 1 Day for Phase 1
  - 1 Week for Phase 2
- Propulsion
  - Formation Slewing → 0.02 N (Hydrazine)
  - Formation keeping → 0.03 mN (PPTs)

NATURAL FORMATIONS

Natural Formations:
String of Pearls

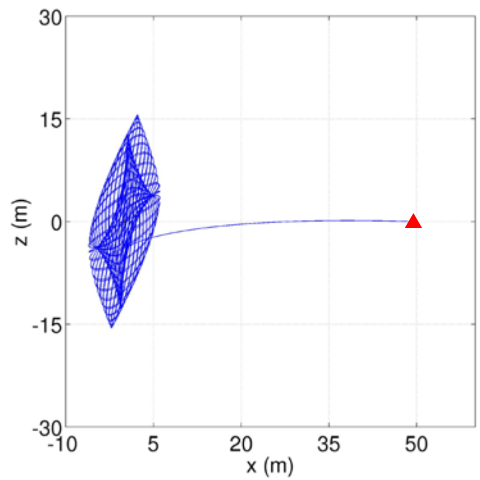


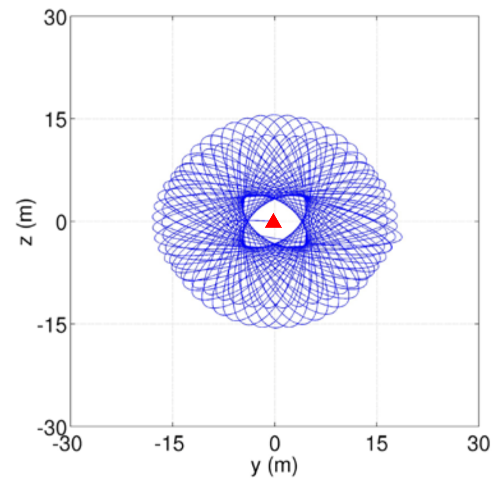
Deployment into Torus (Remove Modes 1, 5, and 6)



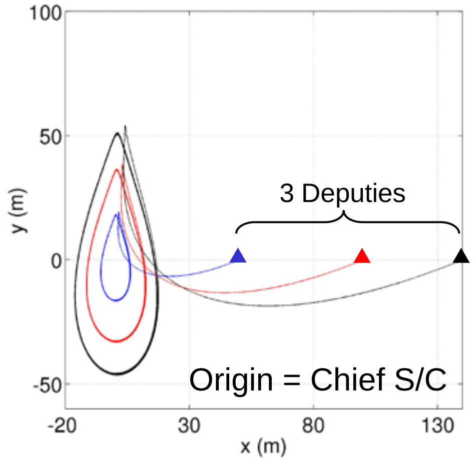
$$\bar{r}(0) = [5 \quad 00 \quad 0]\ \mathrm{m}$$

$$\dot{\bar{r}}(0) = [1 \quad -1 \quad 1]\ \mathrm{m/sec}$$



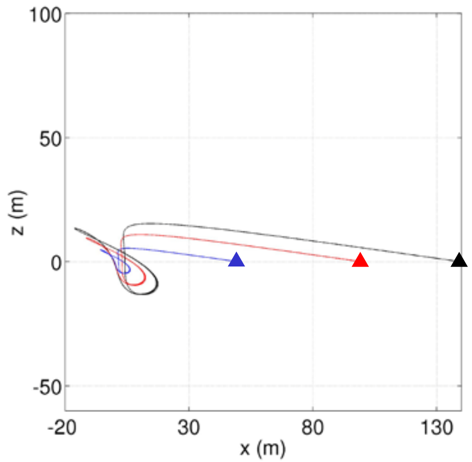


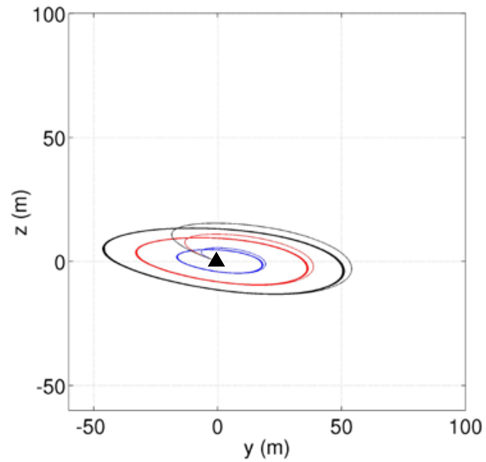
Deployment into Natural Orbits
(Remove Modes 1, 3, and 4)



$$\bar{r}(0) = [r_0 \quad 0 \quad 0] \text{ m}$$

$$\dot{\bar{r}}(0) = [1 \quad -1 \quad 1] \text{ m/sec}$$





SPHERICAL FORMATIONS

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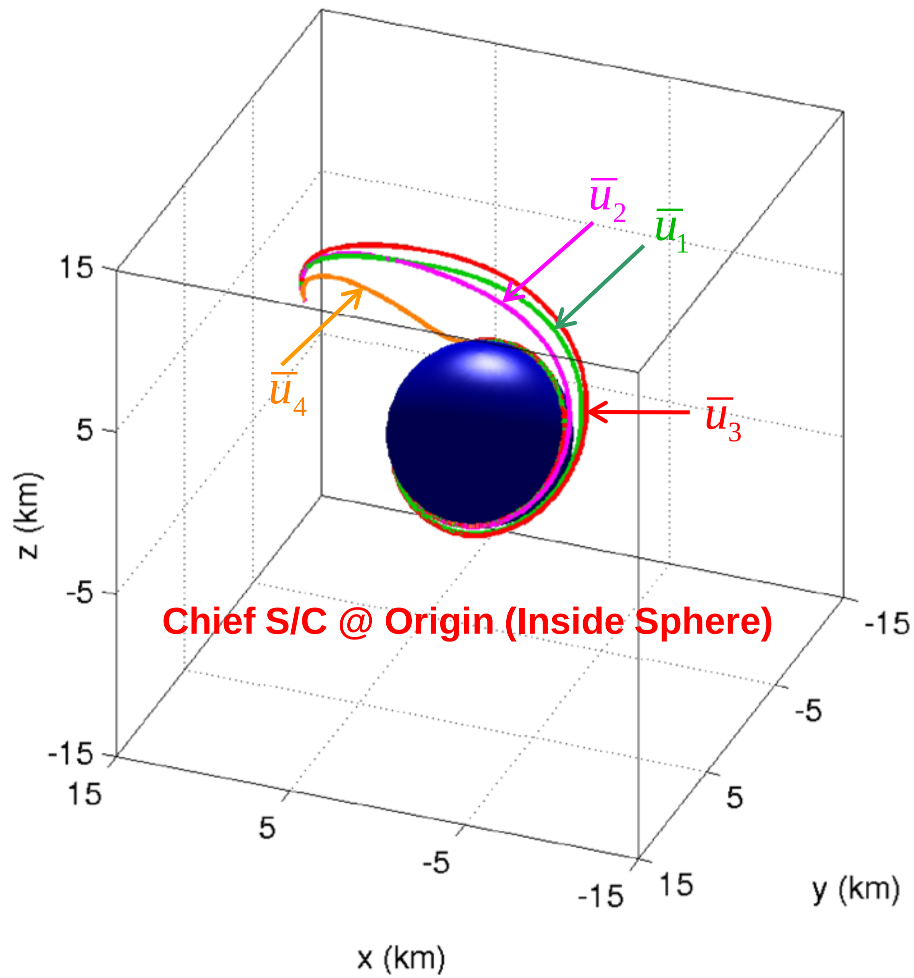
OFL Controlled Response of Deputy S/C

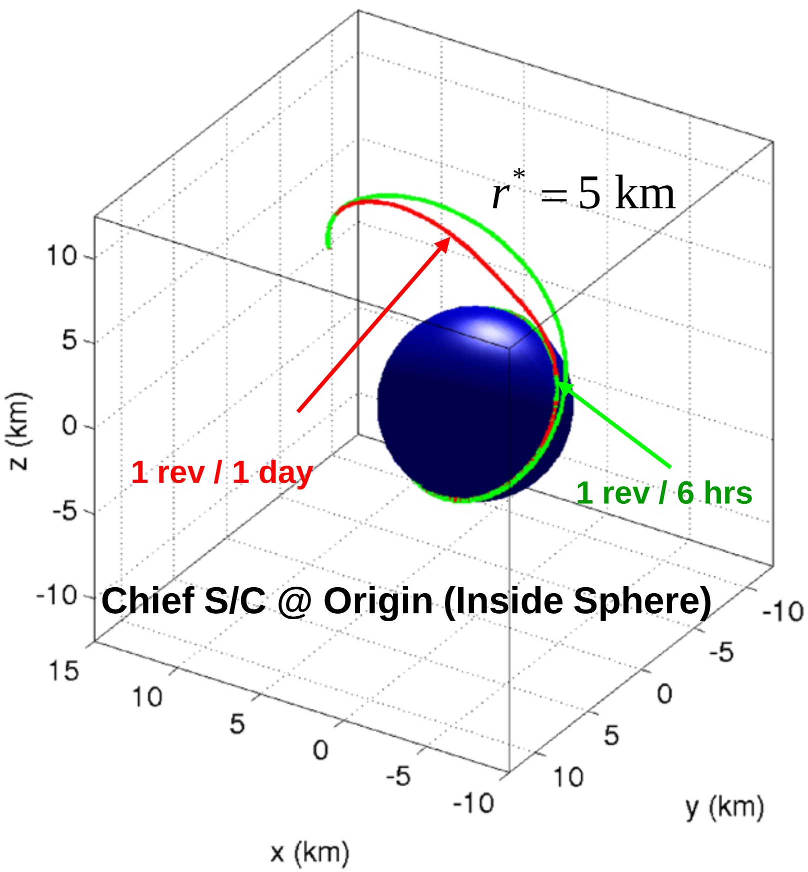
Radial Distance Tracking

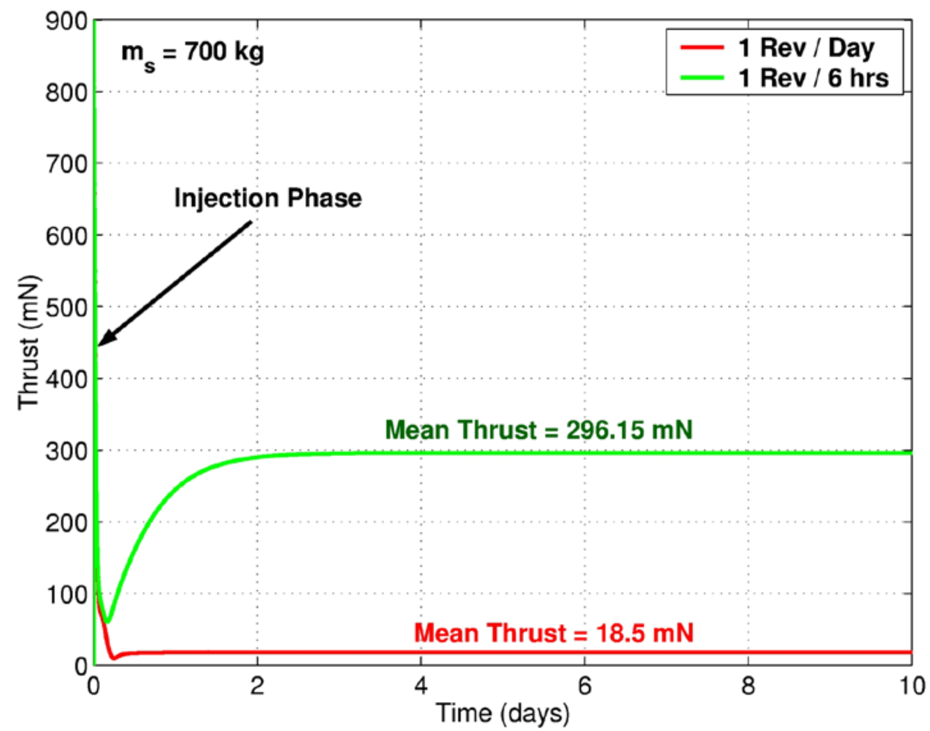
	Control Law	
1	$\bar{u}(t) = \frac{H(\bar{r},\dot{\bar{r}})}{r}\hat{r}$	Geometric Approach: Radial inputs only
2	$\bar{u}(t) = \left\{\frac{g(\bar{r},\dot{\bar{r}})}{r} - \frac{\dot{\bar{r}}^T\dot{\bar{r}}}{r^2}\right\}\bar{r} + \left(\frac{\dot{r}}{r}\right)\dot{\bar{r}} - \Delta\bar{f}(\bar{r})$	
3	$\bar{u}(t) = \left\{\frac{1}{2}\frac{g(\bar{r},\dot{\bar{r}})}{r^2} - \frac{\dot{\bar{r}}^T\dot{\bar{r}}}{r^2}\right\}\bar{r} - \Delta\bar{f}(\bar{r})$	
4	$\bar{u}(t) = \left\{-rg(\bar{r},\dot{\bar{r}}) - \frac{\dot{\bar{r}}^T\dot{\bar{r}}}{r^2}\right\}\bar{r} + 3\left(\frac{\dot{r}}{r}\right)\dot{\bar{r}} - \Delta\bar{f}(\bar{r})$	



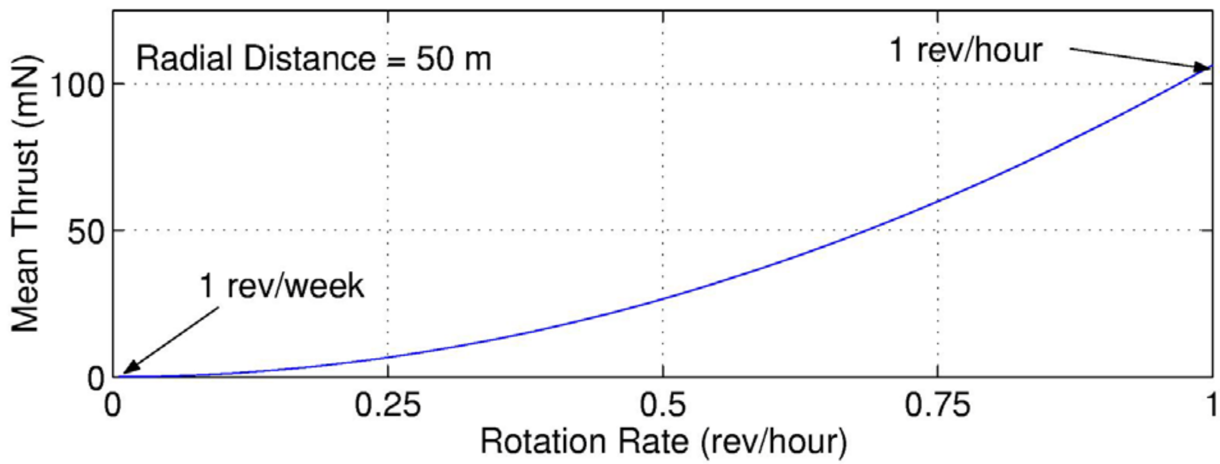
OFL Controlled Response of Deputy S/C

Radial Distance + Rotation Rate Tracking





Impact Commanded Rotation Rate on Cost

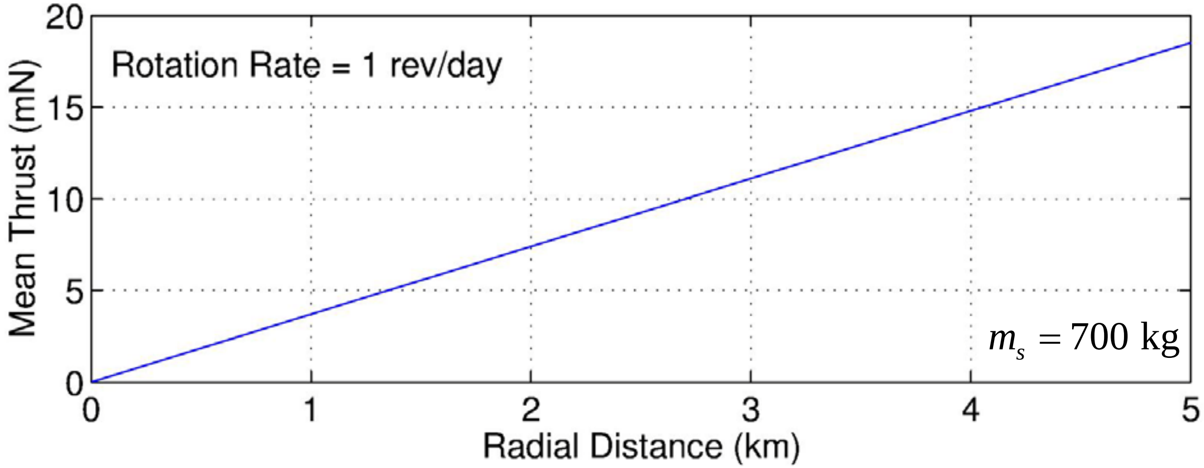


1 rev /24 hrs $\rightarrow$ 0.19 mN

1 rev /12 hrs $\rightarrow$ 0.76 mN

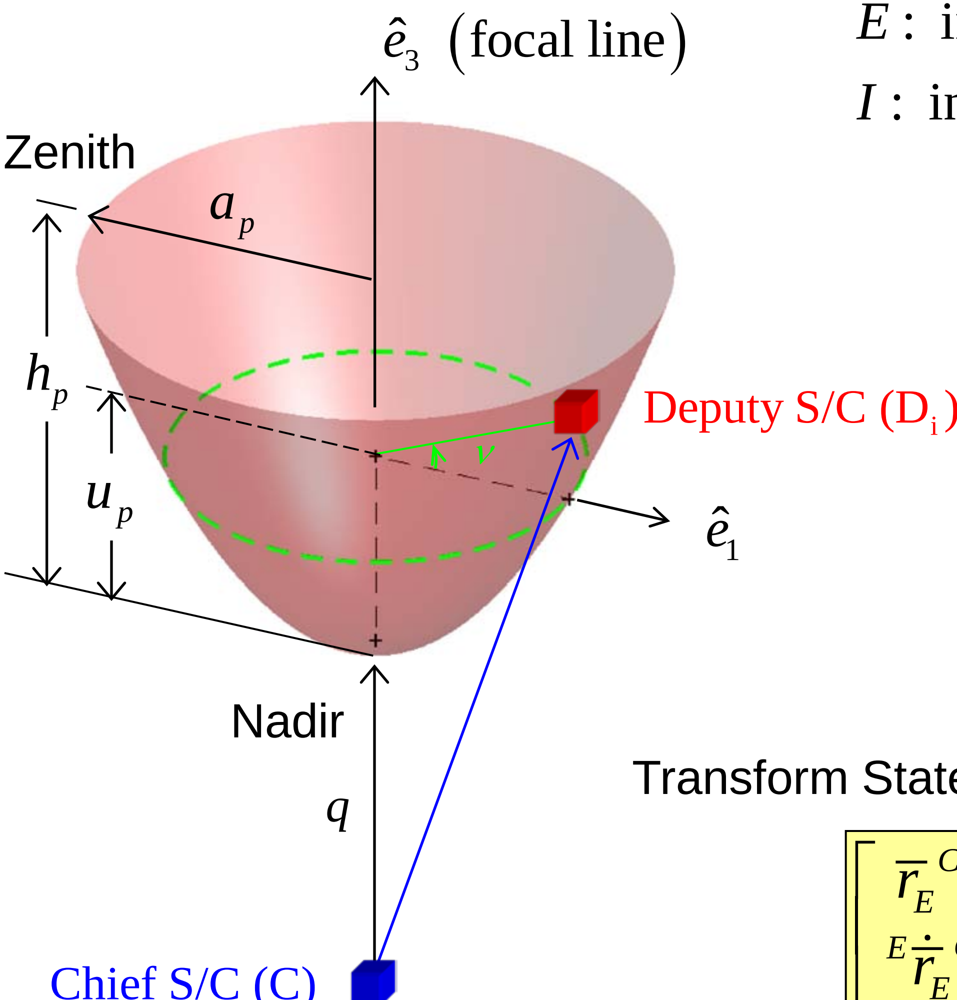
1 rev / 6 hrs $\rightarrow$ 6.40 mN

1 rev / 1 hrs $\rightarrow$ 106.50 mN



ASPHERICAL FORMATIONS

Parameterization of Parabolic Formation



E : inertially fixed focal frame

I : inertially fixed ephemeris frame

$$\bar{r}_E^{CD_i} = \tilde{x}\hat{e}_1 + \tilde{y}\hat{e}_2 + \tilde{z}\hat{e}_3$$

$$\tilde{x} = a_p\sqrt{u_p / h_p}\cos\nu$$

$$\tilde{y} = a_p\sqrt{u_p / h_p}\sin\nu$$

$$\tilde{z} = u_p + q$$

Transform State from Focal to Ephemeris Frame

$$\begin{bmatrix} \bar{r}_E^{CD_i} \\ {}^E\dot{\bar{r}}_E^{CD_i} \end{bmatrix} = \left\{ {}^I C^E \right\}^T \begin{bmatrix} \bar{r}_I^{CD_i} \\ {}^I\dot{\bar{r}}_I^{CD_i} \end{bmatrix}$$

Controller Development

Desired Response for $u,\ q,$ and $\dot{v}$:

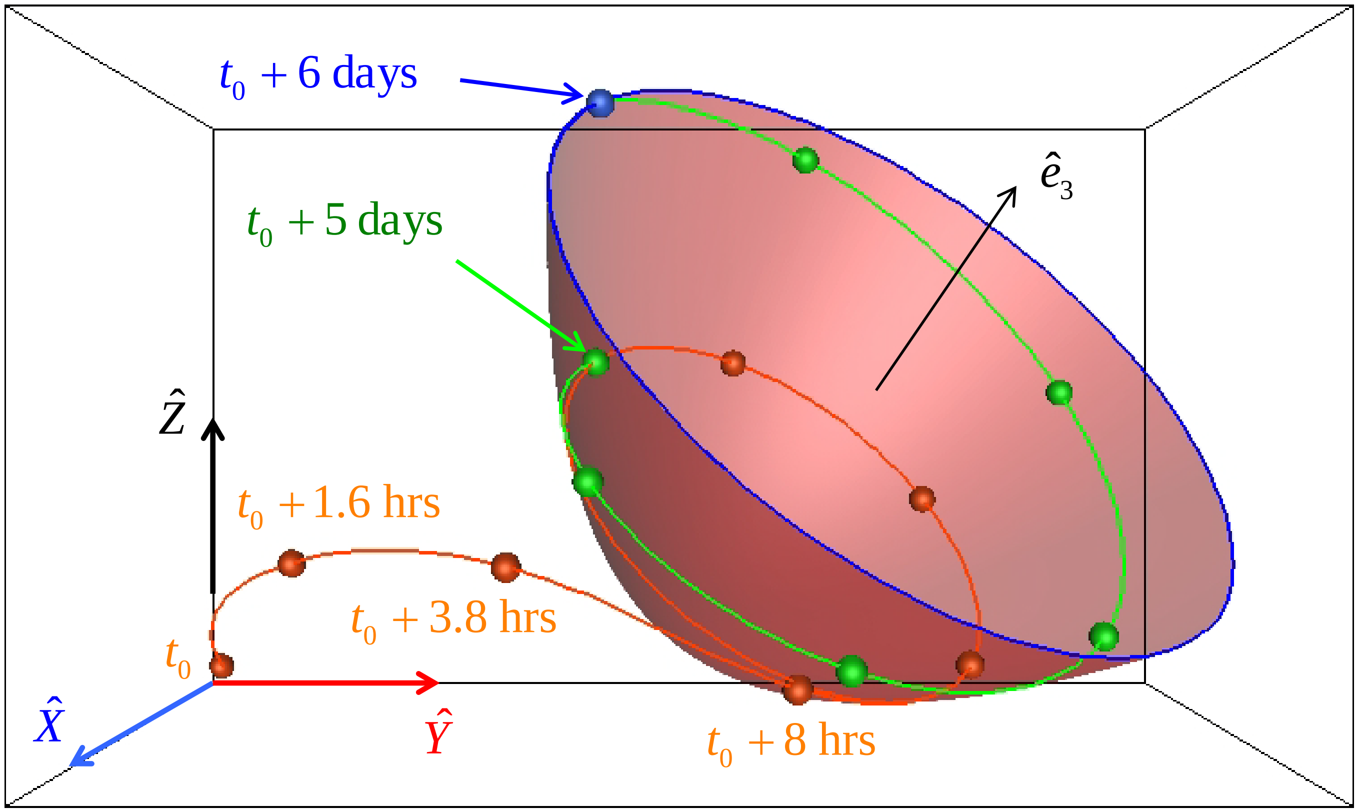
$$
\left.\begin{aligned}
g_u\left(u_p,\dot{u}_p\right) &= \ddot{u}_p^* - 2\omega_n\left(\dot{u}_p - \dot{u}_p^*\right) - \omega_n^{\,2}\left(u_p - u_p^*\right)\\
g_q\left(u_p,\dot{u}_p\right) &= \ddot{q}^* - 2\omega_n\left(\dot{q} - \dot{q}^*\right) - \omega_n^{\,2}\left(q - q^*\right)
\end{aligned}\right\}\ \delta u, \delta q \rightarrow \text{critically damped}
$$

$$
g_v\left(\dot{v}\right) = \ddot{v}^* - k\omega_n\left(\dot{v} - \dot{v}^*\right) \quad \}\ \delta\dot{\theta} \rightarrow \text{exponential decay}
$$

Solve for Control Law:

$$
\begin{bmatrix}
\frac{2h}{a^2}\tilde{x} & \frac{2h}{a^2}\tilde{y} & 0\\
-\frac{2h}{a^2}\tilde{x} & -\frac{2h}{a^2}\tilde{y} & 1\\
\frac{\tilde{x}}{\left(\tilde{x}^2+\tilde{y}^2\right)} & -\frac{\tilde{y}}{\left(\tilde{x}^2+\tilde{y}^2\right)} & 0
\end{bmatrix}
\begin{bmatrix}\tilde{u}_x\\ \tilde{u}_y\\ \tilde{u}_z\end{bmatrix}
=
\begin{bmatrix}
g_u\left(u,\dot{u}\right) - \frac{2h}{a^2}\left(\dot{\tilde{x}}^2 + \dot{\tilde{y}}^2 + \tilde{x}\Delta\tilde{f}_x + \tilde{y}\Delta\tilde{f}_y\right)\\
g_q\left(q,\dot{q}\right) + \frac{2h}{a^2}\left(\dot{\tilde{x}}^2 + \dot{\tilde{y}}^2 + \tilde{x}\Delta\tilde{f}_x + \tilde{y}\Delta\tilde{f}_y\right) - \Delta\tilde{f}_z\\
g_{\dot{v}}\left(\dot{v}\right) + 2\frac{\left(\tilde{x}\dot{\tilde{x}} + \tilde{y}\dot{\tilde{y}}\right)\left(\tilde{x}\dot{\tilde{y}} - \tilde{y}\dot{\tilde{x}}\right)}{\left(\tilde{x}^2+\tilde{y}^2\right)^2} + \frac{\left(\tilde{y}\Delta\tilde{f}_x - \tilde{x}\Delta\tilde{f}_y\right)}{\left(\tilde{x}^2+\tilde{y}^2\right)}
\end{bmatrix}
$$

OFL Controlled Parabolic Formation



$q = 10$ km

$\dot{\nu} = 1$ rev/day

$h_p = 500$ m

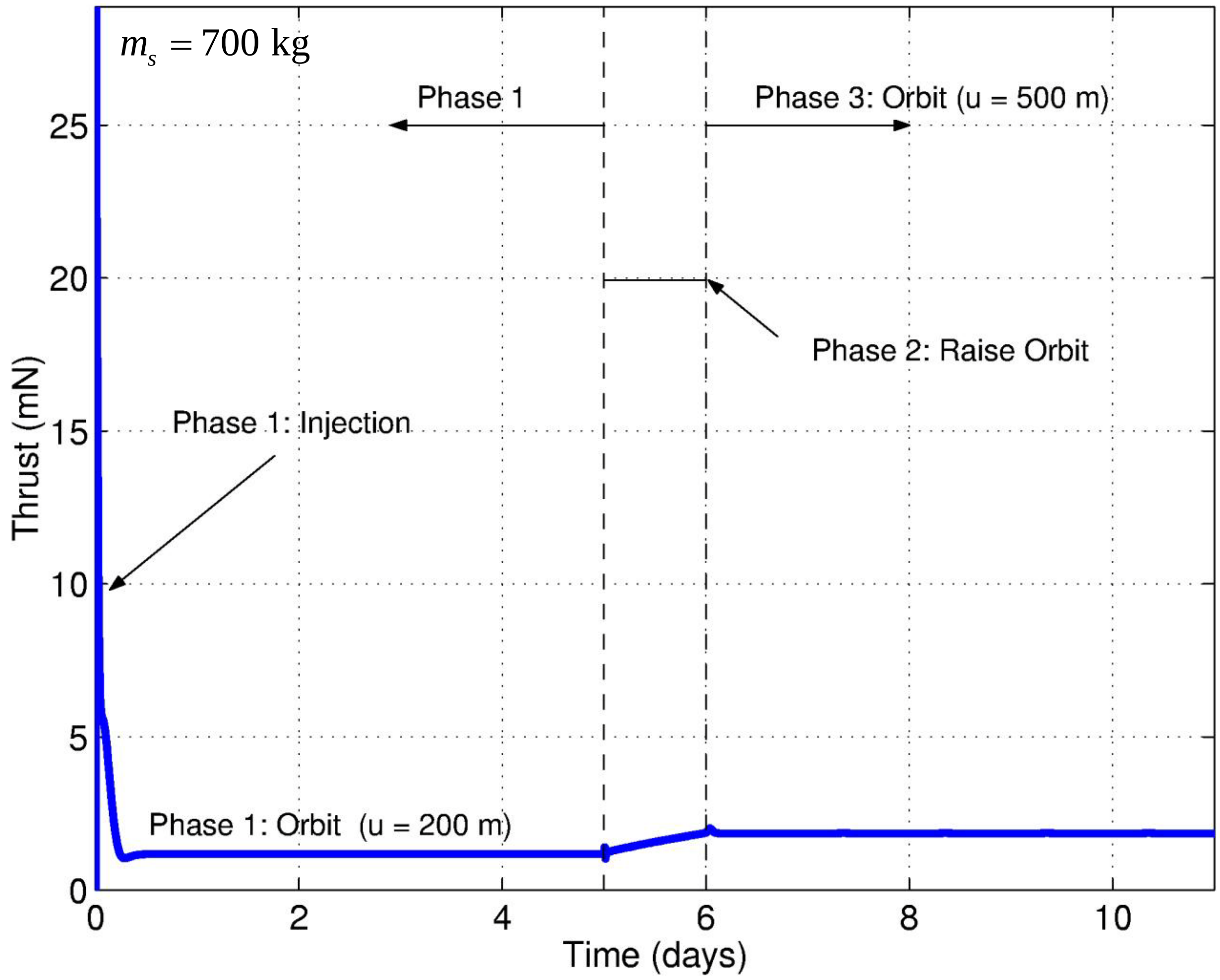
$a_p = 500$ m

Phase I: $u_p = 200$ m

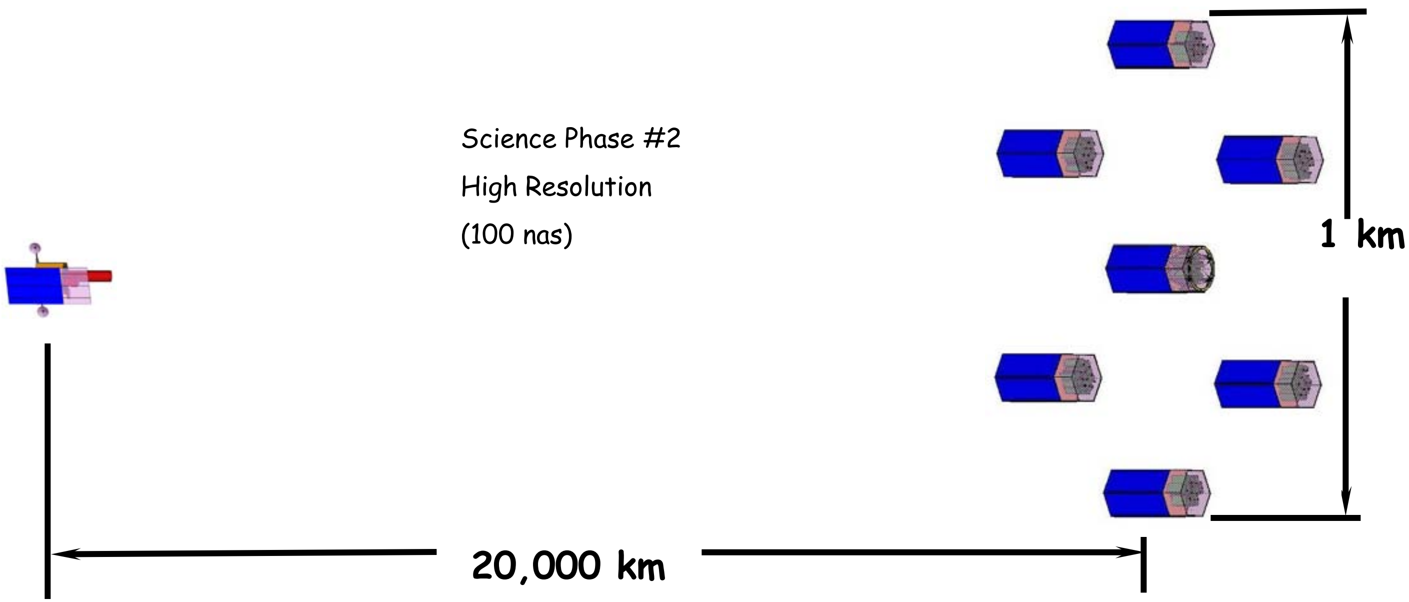
Phase II: $\dot{u}_p = 300$ m / 1 day

Phase III: $u_p = 500$ m

OFL Thrust Profile

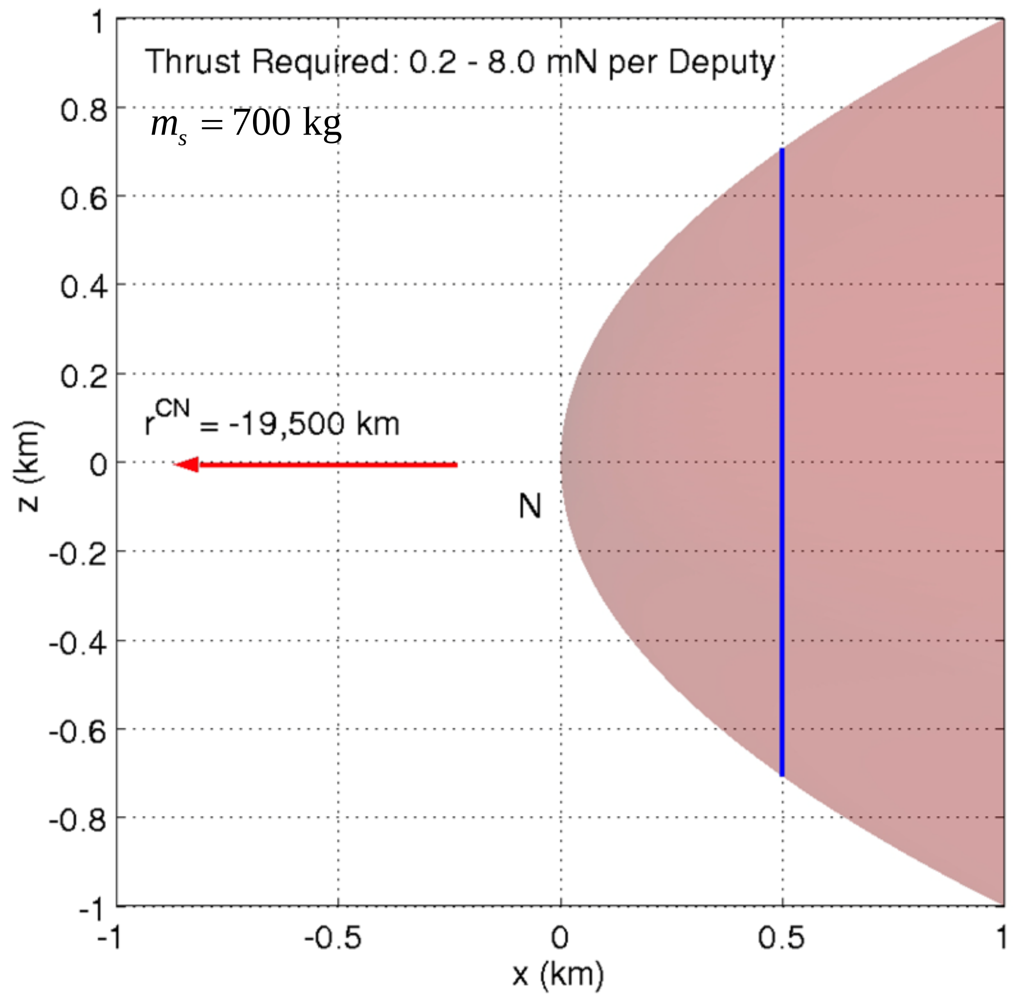


New Maxim Pathfinder



http://maxim.gsfc.nasa.gov/documents/SPIE-2002/spie2002.ppt

Maxim Configuration Example



$q = 19,500\text{km}$

$\dot{v} = 1 \text{ rev/day}$

$u_p = 500 \text{ m}$

$h_p = 1 \text{ km}$

$a_p = 1 \text{ km}$

NONLINEAR OPTIMAL CONTROL

Formulation

$$\min J = \phi\left(\bar{x}_N\right) + \sum_{i=0}^{N-1} L\left(t_i, \bar{x}_i, \bar{u}_i\right) = \phi\left(\bar{x}_N\right) + \sum_{i=0}^{N-1} \int_{t_i}^{t_{i+1}} \tilde{L}\left(t, \bar{x}, \bar{u}\right) d$$

Subject to:

$$\bar{x}_{i+1} = \bar{F}\left(t_i, \bar{x}_i, \bar{u}_i\right); \quad \text{Subject to } \bar{x}\left(0\right) = \bar{x}_0$$

Equivalent Representation as Augmented Nonlinear System:

$$\min \tilde{J} = \phi\left(\bar{x}_N\right) + x_{n+1}\left(t_N\right) = \tilde{\phi}\left(\tilde{x}_N\right)$$

$$\tilde{x}_{i+1} = \begin{bmatrix} \bar{x}_{i+1} \\ x_{n+1}\left(t_{i+1}\right) \end{bmatrix} = \begin{bmatrix} \bar{F}\left(t_i, \bar{x}_i, \bar{u}_i\right) \\ x_{n+1}\left(t_i\right) + L\left(t_i, \bar{x}_i, \bar{u}_i\right) \end{bmatrix} = \tilde{F}\left(t_i, \tilde{x}_i, \bar{u}_i\right);$$

$$\text{Subject to } \tilde{x}_0 = \begin{bmatrix} \bar{x}_0 \\ 0 \end{bmatrix}$$

Euler-Lagrange Optimality Conditions (Based on Calculus of Variations)

$$H_i = \tilde{\lambda}_{i+1}^T \tilde{F}\left(t_i, \overline{x}_i, \overline{u}_i\right)$$

Condition #1: $\tilde{\lambda}_i^T = \dfrac{\partial H_i}{\partial \tilde{x}_i} = \tilde{\lambda}_{i+1}^T \dfrac{\partial \tilde{F}_i}{\partial \tilde{x}_i} \quad \rightarrow \quad \tilde{\lambda}_N^T = \begin{bmatrix} \dfrac{\partial \phi\left(\overline{x}_N\right)}{\partial \overline{x}_N} & 1 \end{bmatrix}$

Condition #2: $\overline{0} = \dfrac{\partial H_i}{\partial \overline{u}_i} = \tilde{\lambda}_{i+1}^T \dfrac{\partial \tilde{F}_i}{\partial \overline{u}_i}; \quad i = 0, \ldots, N-1$

Identify $\dfrac{\partial \tilde{F}_i}{\partial \tilde{x}_i}$ and $\dfrac{\partial \tilde{F}_i}{\partial \overline{u}_i}$ from augmented linear system.

Identification of Gradients From the Linearized Model

Augmented Nonlinear System:

$$\begin{bmatrix} \dot{\overline{x}} \\ \dot{x}_{n+1} \end{bmatrix} = \begin{bmatrix} \overline{f}\left(t,\overline{x},\overline{u}\right) \\ \tilde{L}\left(t,\overline{x},\overline{u}\right) \end{bmatrix}; \quad \begin{bmatrix} \overline{x}\left(0\right) \\ x_{n+1}\left(0\right) \end{bmatrix} = \begin{bmatrix} \overline{x}_0 \\ 0 \end{bmatrix}$$

Augmented Linear System:

$$\delta\dot{\tilde{x}}\left(t\right) = \tilde{A}\left(t\right)\delta\tilde{x}\left(t\right) + \tilde{B}\left(t\right)\delta\overline{u}\left(t\right)$$

$$\tilde{A}\left(t\right) = \begin{bmatrix} A\left(t\right) & \overline{0} \\ \dfrac{\partial\tilde{L}}{\partial\overline{x}} & \overline{0} \end{bmatrix} \qquad \tilde{B}\left(t\right) = \begin{bmatrix} 0_3 \\ I_3 \\ \overline{0}^T \end{bmatrix}$$

Solution to Linearized Equations

$$\delta\tilde{x}(t) = \tilde{\Phi}(t,t_0)\,\delta\tilde{x}_0 + \int_{t_0}^{t} \Phi(t,\tau)\,B(\tau)\,\delta\overline{u}(\tau)\,d\tau$$

$$\dot{\tilde{\Phi}}(t,t_0) = \tilde{A}(t)\,\tilde{\Phi}(t,t_0); \quad \tilde{\Phi}(t_0,t_0) = I_7$$

Relation to Gradients in E-L Optimality Conditions:

$$\delta\tilde{x}_{i+1} = \underbrace{\tilde{\Phi}(t_{i+1},t_i)}_{\frac{\partial\tilde{F}}{\partial\tilde{x}_i}}\,\delta\tilde{x}_i + \int_{t_i}^{t_{i+1}} \Phi(t_{i+1},\tau)\,B(\tau)\,\delta\overline{u}(\tau)\,d\tau$$

Control Gradient for Impulsive Control

$$\begin{aligned}\delta\tilde{x}_{i+1}^{-} &= \tilde{\Phi}\left(t_{i+1},t_i\right)\delta\tilde{x}_i^{+} \\ &= \tilde{\Phi}\left(t_{i+1},t_i\right)\left(\delta\tilde{x}_i^{-}+\tilde{B}\Delta\overline{V}_i\right) \\ &= \tilde{\Phi}\left(t_{i+1},t_i\right)\delta\tilde{x}_i^{-}+\underbrace{\tilde{\Phi}\left(t_{i+1},t_i\right)}_{\dfrac{\partial\tilde{F}}{\partial\overline{u}_i}}\tilde{B}\Delta\overline{V}_i\end{aligned}$$

$$\frac{\partial\tilde{F}}{\partial\overline{u}_i}=\Phi\left(t_{i+1},t_i\right)\tilde{B}$$

Control Gradient for Constant Thrust Arcs

$$\delta \tilde{x}_{i+1} = \tilde{\Phi}(t_{i+1}, t_i)\,\delta \tilde{x}_i + \underbrace{\left[\int_{t_i}^{t_{i+1}} \Phi(t_{i+1}, \tau) B(\tau)\, d\tau\right]}_{\dfrac{\partial \tilde{F}}{\partial \overline{u}_i}} \delta \overline{u}_i$$

$$\frac{\partial \tilde{F}}{\partial \overline{u}_i} = \Phi(t_{i+1}, t_i)\left[\int_{t_i}^{t_{i+1}} \Phi(\tau, t_i)^{-1} B(\tau)\, d\tau\right]$$

Equations to Integrate Numerically

$$\begin{bmatrix} \dot{x} \\ \dot{x}_{n+1} \\ \dot{\tilde{\Phi}}(t, t_i) \\ \dot{\Phi}^*(t, t_i) \end{bmatrix} = \begin{bmatrix} \overline{f}(t, \overline{x}, \overline{u}) \\ L(t, \overline{x}, \overline{u}) \\ \tilde{A}(t)\tilde{\Phi}(t, t_i) \\ \tilde{\Phi}(t, t_i)^{-1} \tilde{B} \end{bmatrix}$$

Numerical Solution Process: Global Approach

(1) Input $\tilde{x}_0, t_N$, and initial guess for $\overline{u}_i$; $(i = 0,1,\ldots,N-1)$

(2) 1-Scalar Equation to Optimize in $3(N-1)$ Control Variables

Use optimizer to identify optimal $\overline{u}_i$ given $\dfrac{\partial H_i}{\partial \overline{u}_i}$.

During each iteration of the optimizer, the following steps are followed:

(a) Sequence $\overline{x}_i$ forward and store; $i = 1,\ldots,N-1$

(b) Evaluate cost functional, $J = \tilde{\phi}(\tilde{x}_N)$

(c) Evaluate $\tilde{\lambda}_N^T = \dfrac{\partial \tilde{\phi}(\tilde{x}_N)}{\partial \tilde{x}_N} = \begin{bmatrix} \dfrac{\partial \phi_N}{\partial \overline{x}_N} & 1 \end{bmatrix}$

(d) Sequence $\tilde{\lambda}_i$ backward and compute $\dfrac{\partial H_i}{\partial \overline{u}_i}$; $i = N-1,\ldots,1$

(e) J and $\dfrac{\partial H_i}{\partial \overline{u}_i}$ used in next update of control input.

Formulation

$$\min J = \phi\left(\overline{x}_N\right) + \sum_{i=0}^{N-1} L\left(t_i, \overline{x}_i, \overline{u}_i\right) = \phi\left(\overline{x}_N\right) + \sum_{i=0}^{N-1} \int_{t_i}^{t_{i+1}} \tilde{L}\left(t, \overline{x}, \overline{u}\right) d$$

Subject to:

$$\overline{x}_{i+1} = \overline{F}\left(t_i, \overline{x}_i, \overline{u}_i\right); \quad \text{Subject to } \overline{x}(0) = \overline{x}_0$$

Equivalent Representation as Augmented Nonlinear System:

$$\min \tilde{J} = \phi\left(\overline{x}_N\right) + x_{n+1}\left(t_N\right) = \tilde{\phi}\left(\tilde{x}_N\right)$$

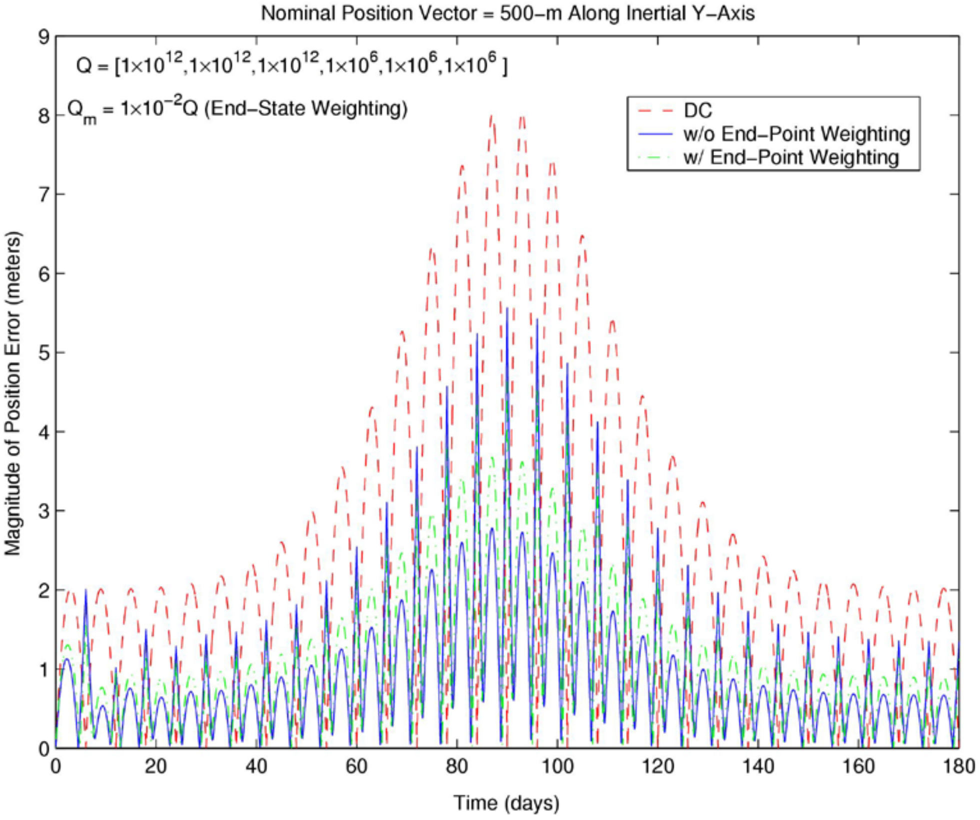
$$\tilde{x}_{i+1} = \begin{bmatrix} \overline{x}_{i+1} \\ x_{n+1}\left(t_{i+1}\right) \end{bmatrix} = \begin{bmatrix} \overline{F}\left(t_i, \overline{x}_i, \overline{u}_i\right) \\ x_{n+1}\left(t_i\right) + L\left(t_i, \overline{x}_i, \overline{u}_i\right) \end{bmatrix} = \tilde{F}\left(t_i, \tilde{x}_i, \overline{u}_i\right);$$

$$\text{Subject to } \tilde{x}_0 = \begin{bmatrix} \overline{x}_0 \\ 0 \end{bmatrix}$$

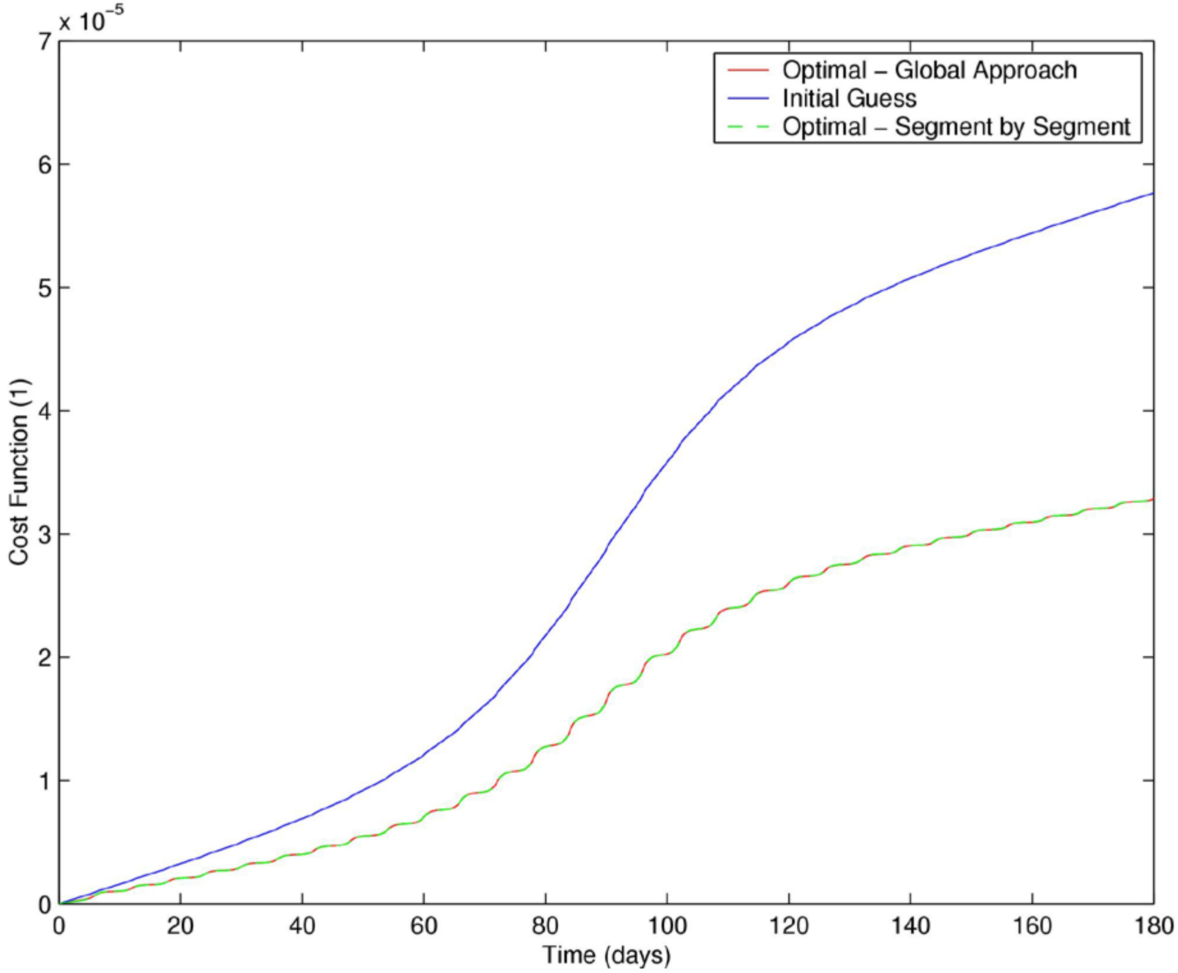
Impulsive Optimal Control

Minimize State Error with End-State Weighting

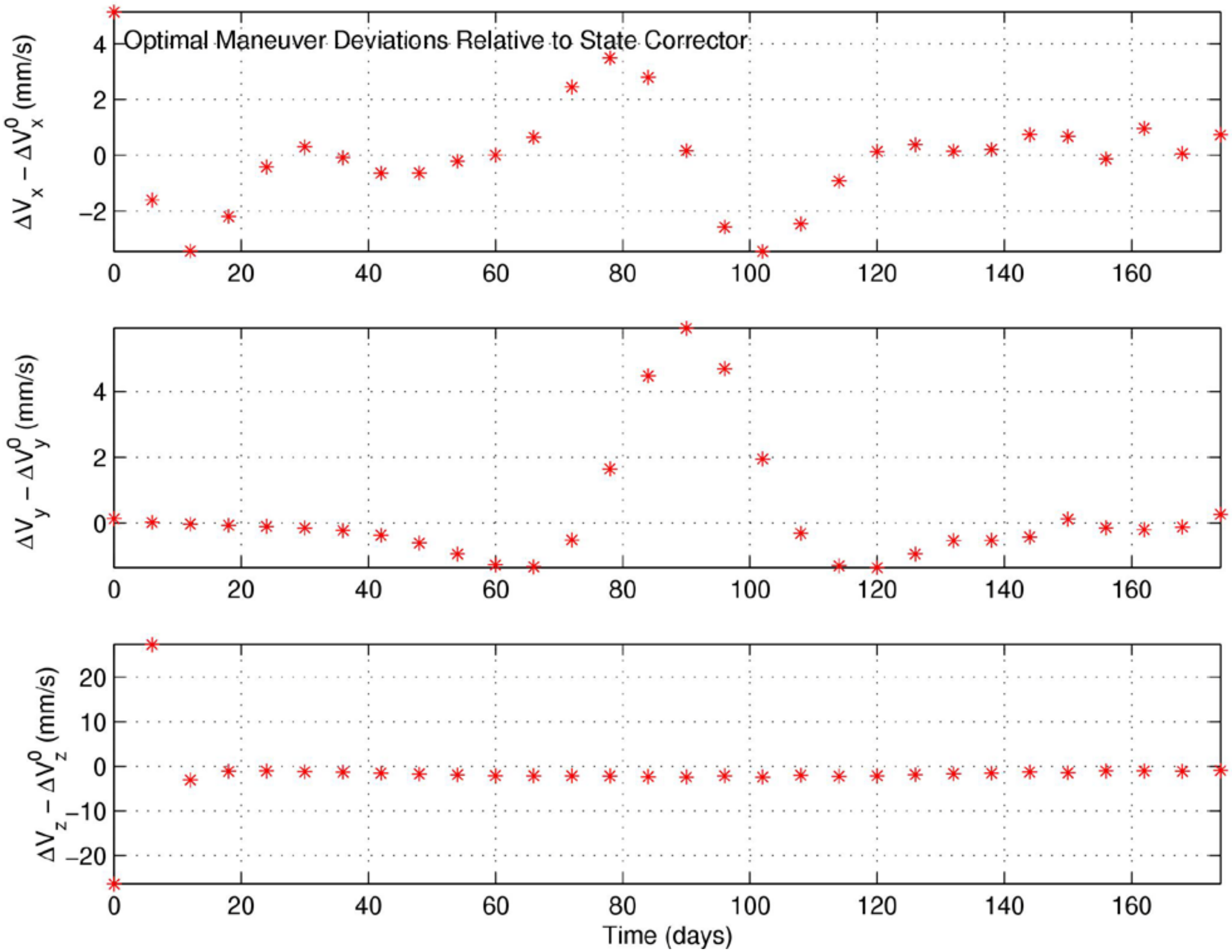
$$\min J = \frac{1}{2}\left(\overline{x}_N - \overline{x}_N^{\circ}\right)^T W \left(\overline{x}_N - \overline{x}_N^{\circ}\right) + \sum_{i=0}^{N-1} \int_{t_i}^{t_{i+1}} \frac{1}{2}\left(\overline{x} - \overline{x}^{\circ}\right)^T Q \left(\overline{x} - \overline{x}^{\circ}\right) d$$



State Corrector vs. Nonlinear Optimal Control: Cost Function

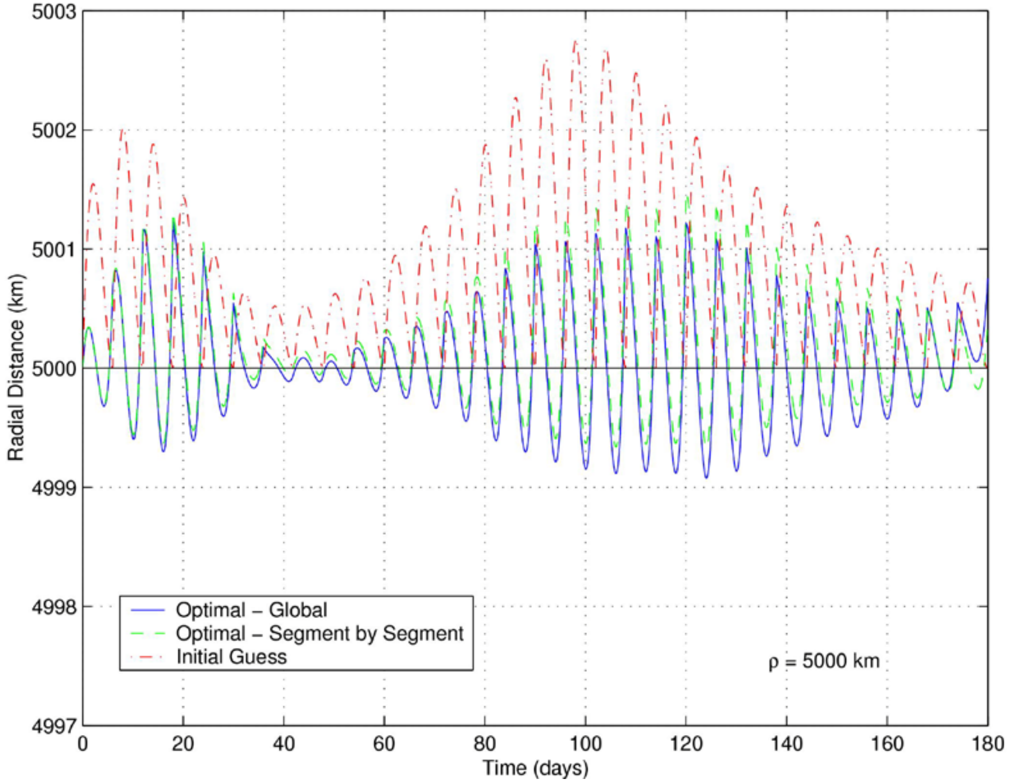


State Corrector vs. Nonlinear Optimal Control: Impulsive Maneuver Differences



Impulsive Radial Optimal Control

$$\min J = \sum_{i=0}^{N-1} \int_{t_i}^{t_{i+1}} \frac{1}{2} q \left(r - r^\circ \right)^2 dt$$



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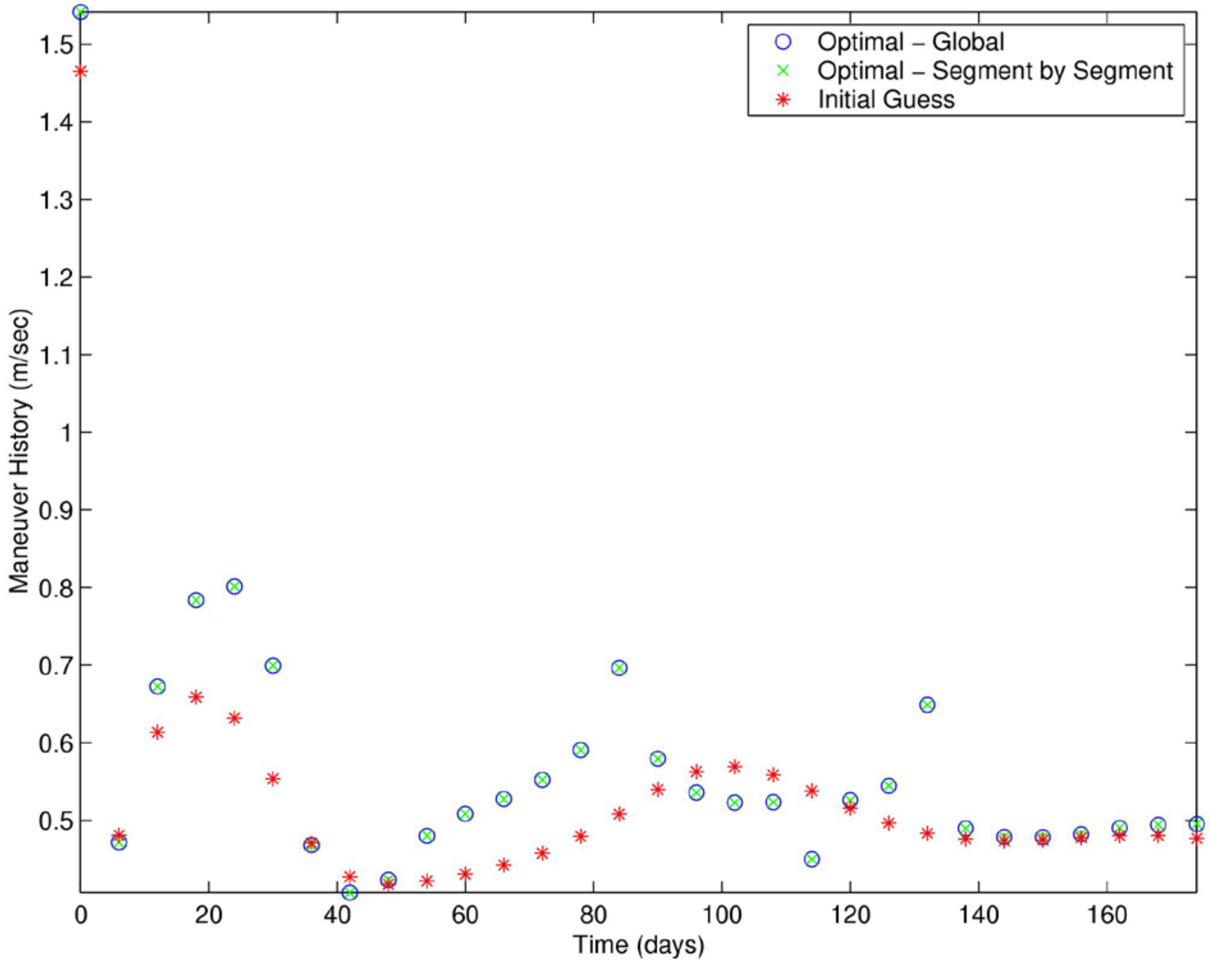
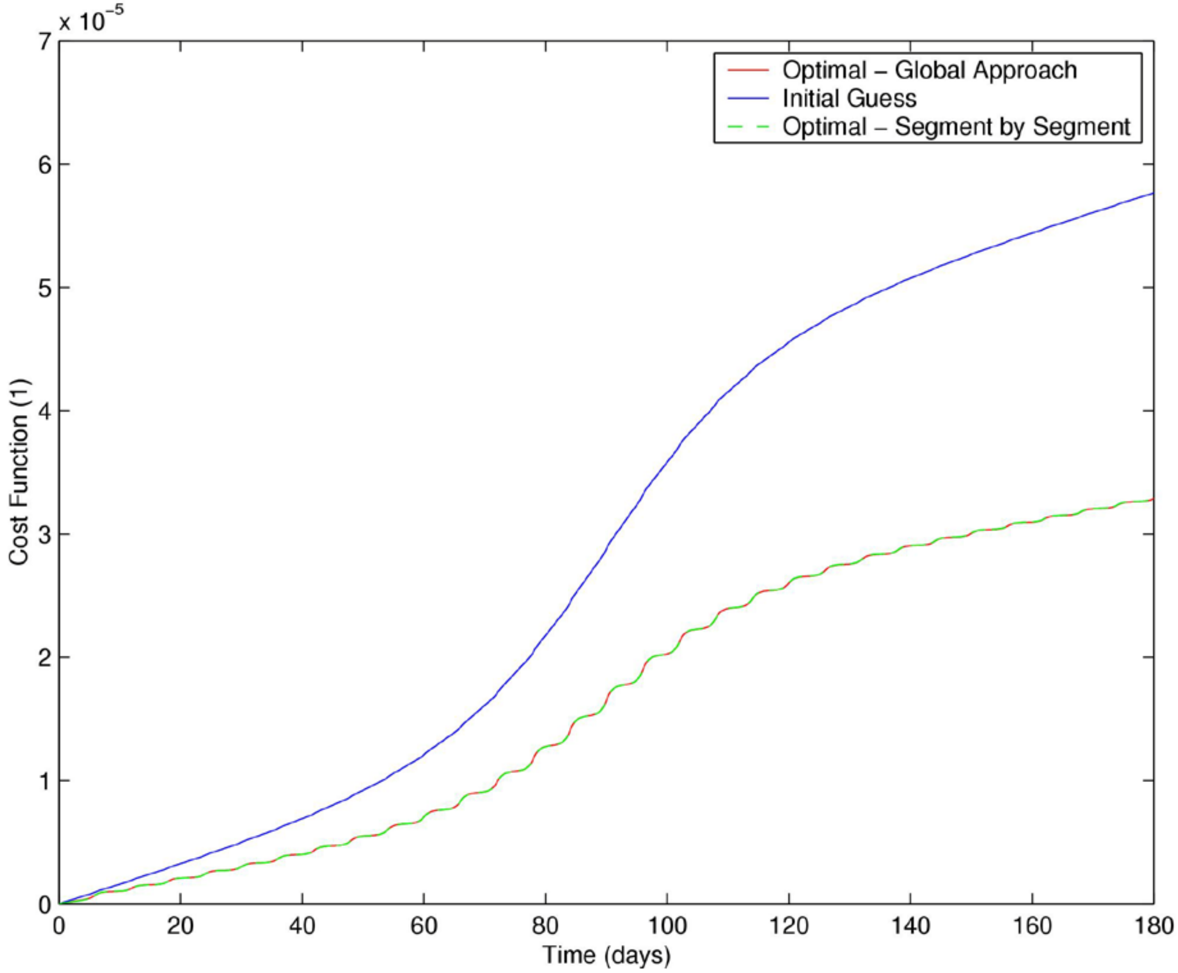
Radial Optimal Control: Maneuver History

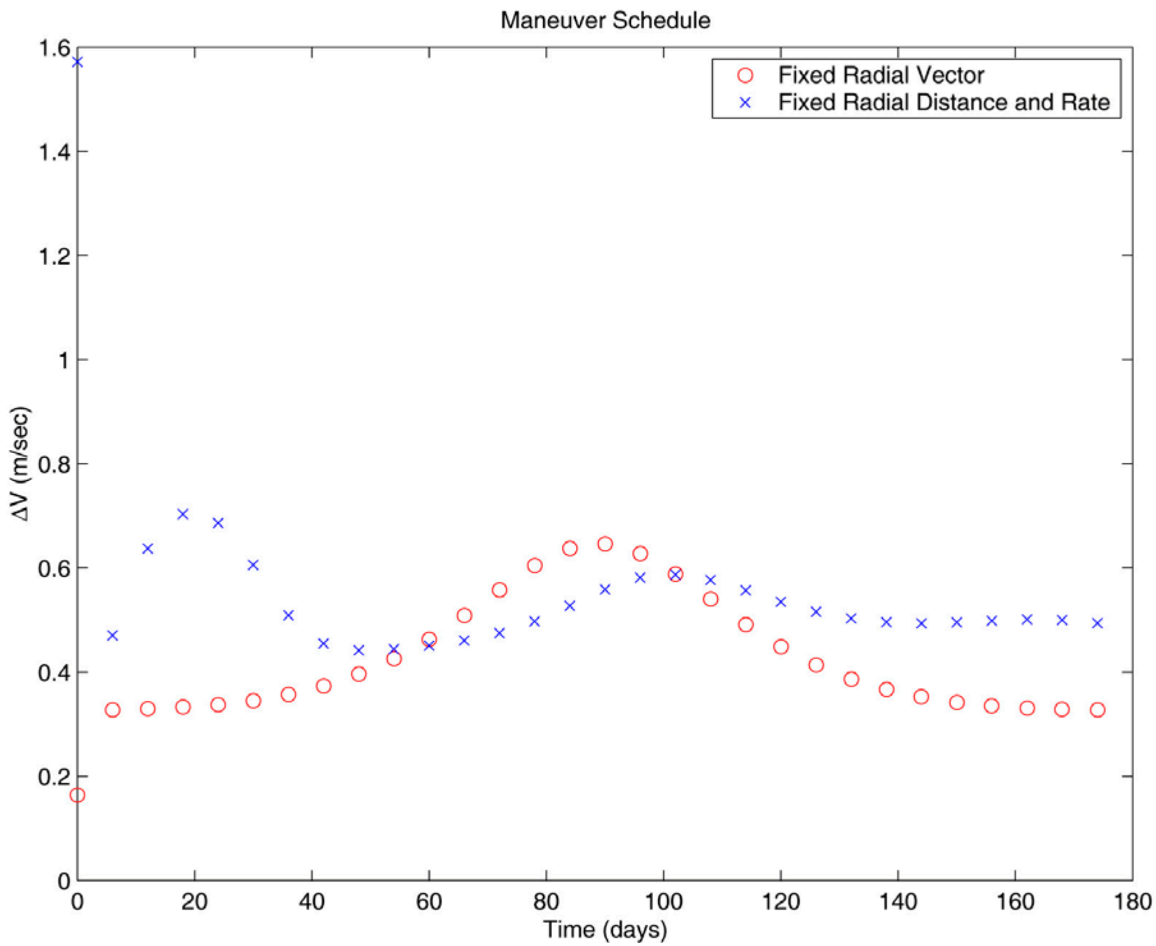


State Corrector vs. Nonlinear Optimal Control: Cost Function



RANGE + RANGE RATE TARGETER

Comparison of Range and State Targeters



Range Targeter: Spatial Behavior of Corrected Solution