Geometry of Optimal Coverage for Space-based Targets with Visibility Constraints

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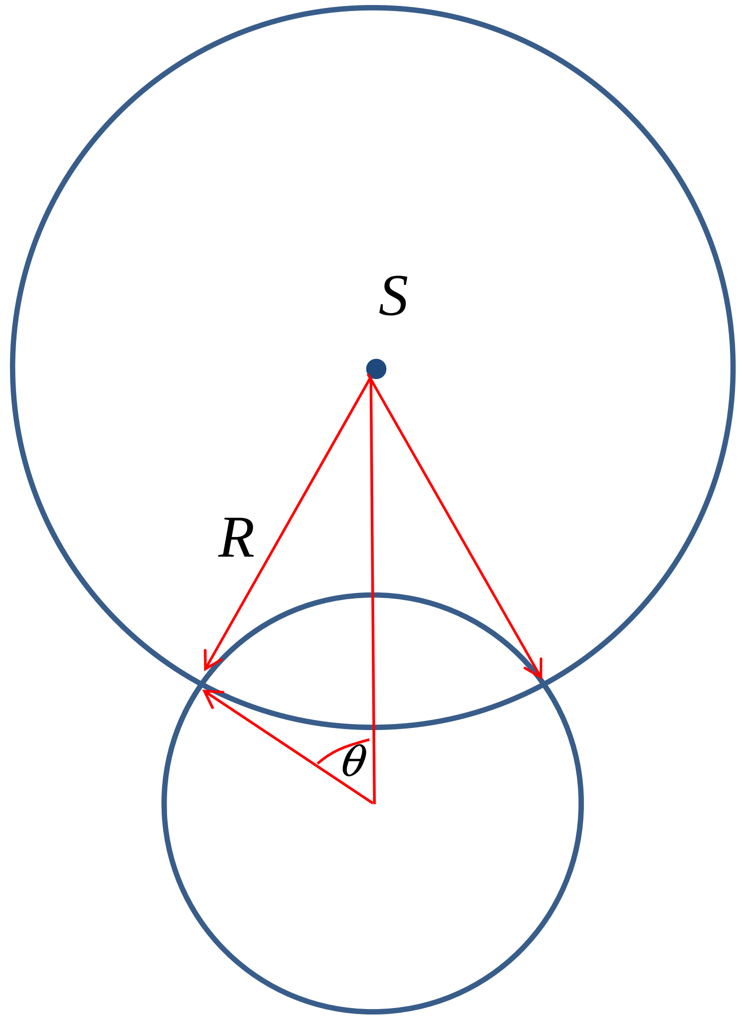
Problem Statement

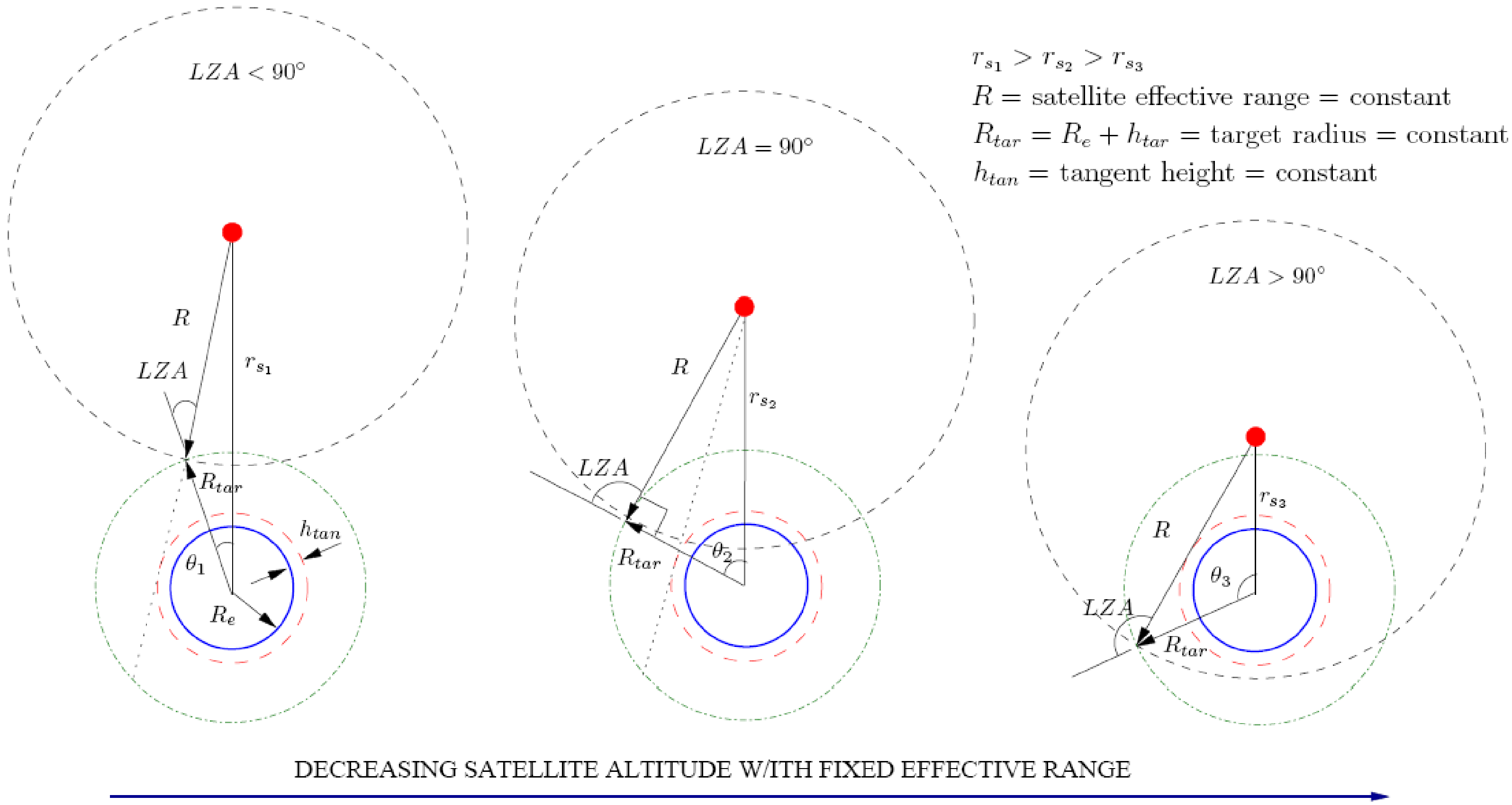
- Define algorithm for maximizing coverage of space based targets within the bounds of a pre-specified altitude band.
- Assumptions:
  - No visibility below given altitude (tangent height)
  - Focus only on space based targets
  - Sensor range pre-defined

BTH Coverage Problem



MAX BTH COVERAGE = MAX COVERAGE ANGLE

Optimal Satellite Height for Maximum ATH Coverage



$$h_s = \sqrt{(R_e + h_{low})^2 + R^2 - 2R(R_e + h_{low})\cos\phi} - R_e$$

$$\phi = \sin^{-1}\left(\frac{R_e + h_t}{R_e + h_{low}}\right)$$

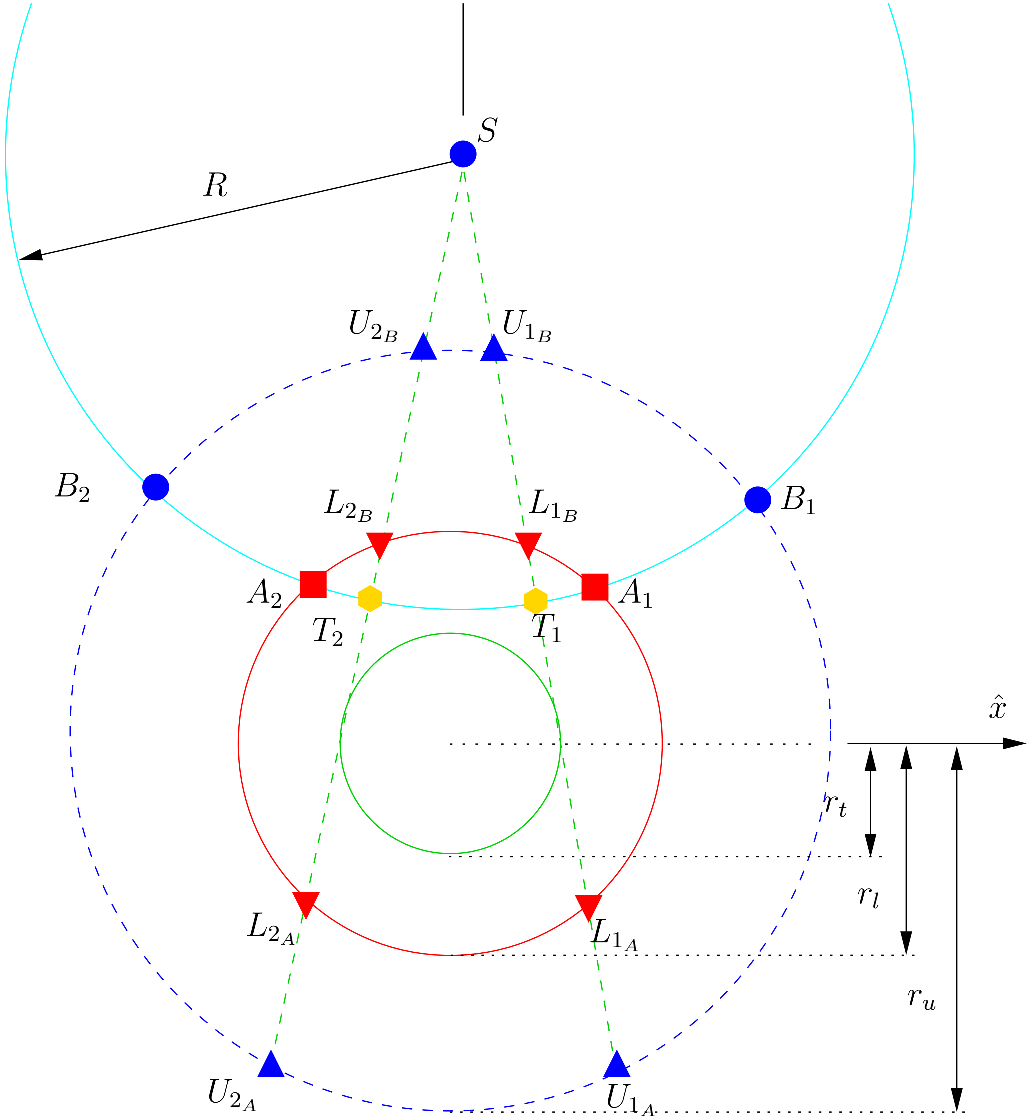
Maximizing Visibility within a Bounded Altitude Range

- Goal: To maximize the area of intersection between the following curves (2D) or surfaces (3D)
  - **UTAS** → **U**pper **T**arget **A**ltitude **S**hell
  - **LTAS** → **L**ower **T**arget **A**ltitude **S**hell
  - **RS** → **R**ange **S**hell
  - **TL** → **T**angent **L**ine

Factors Influencing Area Calculation

- Satellite Altitude
- Separation of UTAS and LTAS
- Size of RS and where it intersects the TL
- Intersections of TL with UTAS and LTAS

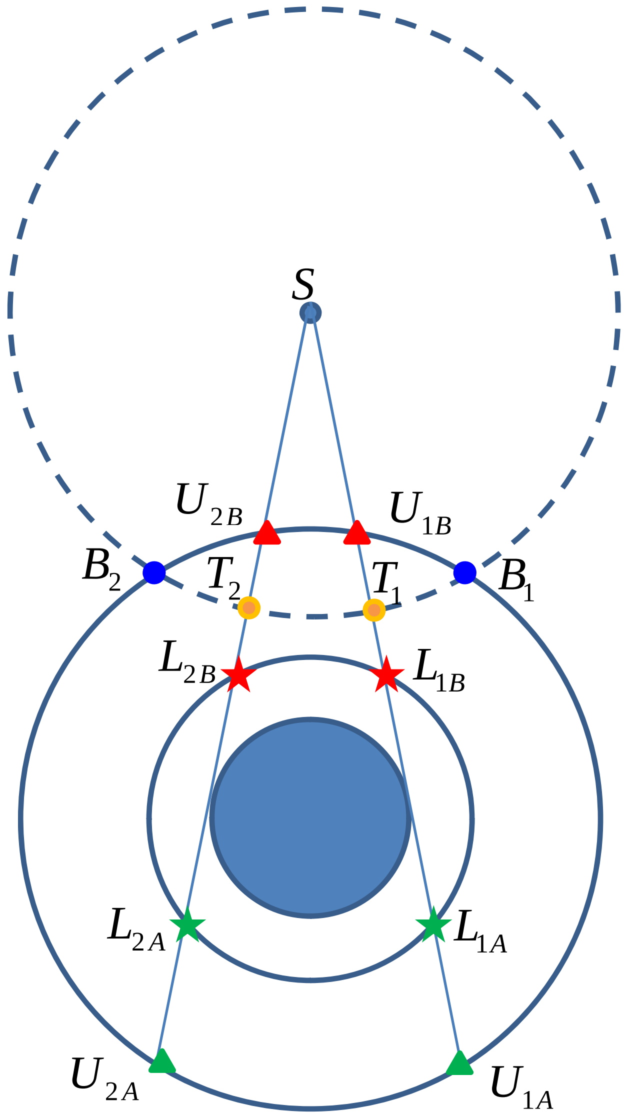
Step 1: Formulate Conditions & Eqns. For Curve Intersections



Step 2: Computing the Coverage Area as a function of Satellite Altitude

- Initially, if coverage exists at all, the area of coverage can be thought of as πR^2 – (area outside UTAS) – (area below LTAS) – (area below TL)
- Each of these three terms depend on the size of the RS and the separation between the UTAS and the LTAS
- There is no single equation that generally describes the coverage area. Thus, all special cases must be identified a priori
- The area may be represented as a piecewise continuous function, but it is a highly nonlinear function.
- Identifying the optimal height is best accomplished by understanding the geometrical structure of the problem and through adequate numerical analysis.

Step 3: Identify Special Cases Depending on Location of Critical Intersections

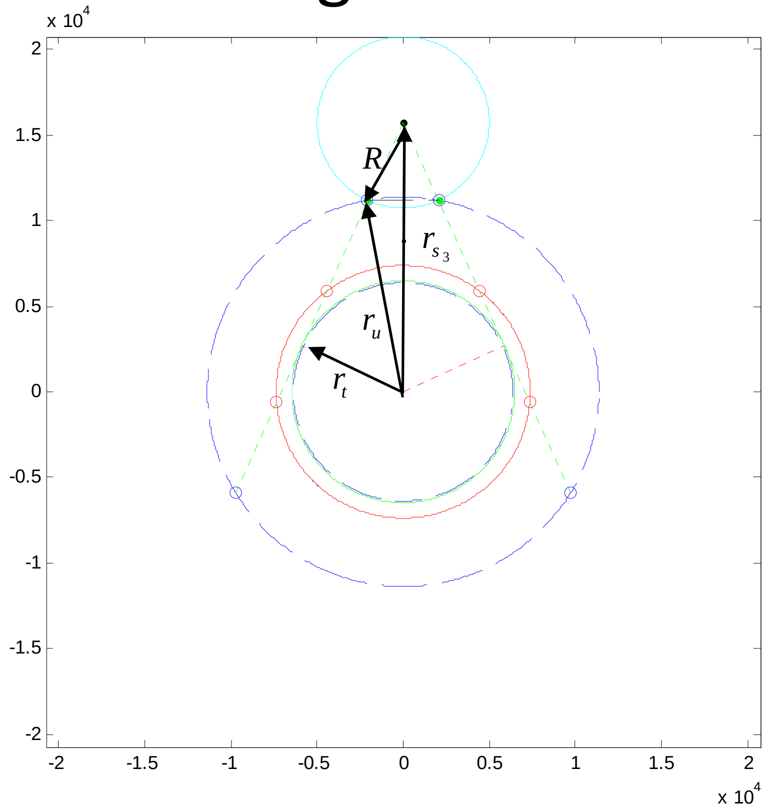


Step 4: Identify Simplest Form of Area Equation for each Possible Case

- There are multiple ways of formulating the same area equation, some more difficult than others.
- Divide area calculation into basic shapes
  - Triangles
  - Arc segments
  - Circular Sectors
- Computation depends only on Cartesian coordinates of Primary and Secondary Intersections
- Composite area equation depends only on elementary components

Constrained Search Space

- Satellite MUST be located:
  - Above the THS
  - Below no-coverage altitude:



$$r_{s_3} = \sqrt{\left(R + \sqrt{r_u^{\,2} - r_t^{\,2}}\right)^2 + r_t^{\,2}}$$

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Intersections of RS with the U/LTAS

$$\left.\begin{aligned} x_{B_1}{}^2 + \left(y_{B_1} - y_s\right)^2 = R^2 \\ x_{B_1}{}^2 + y_{B_1}{}^2 = r_u{}^2 \end{aligned}\right\} \quad \begin{aligned} y_{B_1} &= y_{B_2} = \frac{\left(r_u{}^2 + r_s{}^2 - R^2\right)}{2r_s} \\ x_{B_1} &= -x_{B_2} = \sqrt{r_u{}^2 - y_{B_2}{}^2} \end{aligned}$$

$$\left.\begin{aligned} x_{A_1}{}^2 + \left(y_{A_1} - y_s\right)^2 = R^2 \\ x_{A_1}{}^2 + y_{A_1}{}^2 = r_l{}^2 \end{aligned}\right\} \quad \begin{aligned} y_{A_1} &= y_{A_2} = \frac{\left(r_l{}^2 + r_s{}^2 - R^2\right)}{2r_s} \\ x_{A_1} &= -x_{A_2} = \sqrt{r_l{}^2 - y_{A_1}{}^2} \end{aligned}$$

Intersection of the TL with the LTAS

The equation for the TL that connects the satellite to the THS is given by,

$$y = m\left(x - x_s\right) + y_s$$

Where m denotes the slope of the line,

$$m = \frac{y_t - y_s}{x_t - x_s}$$

and

$$x_t = r_t \sin\theta_t, \quad y_t = r_t \cos\theta_t, \quad \text{and} \quad \theta_t = \cos^{-1}\left(\frac{r_t}{r_s}\right).$$

The intersection of the TL with the LTAS is identified from the solution to the following system of equations:

$$\left.\begin{aligned} x_{L_{1A/B}}{}^2 + y_{L_{1A/B}}{}^2 &= r_l{}^2 \\ y_{L_{1A/B}} &= m x_{L_{1A/B}} + r_s \end{aligned}\right\} \quad \begin{aligned} x_{L_{1A/B}} &= -x_{L_{2A/B}} = \frac{-2mr_s \pm \sqrt{4m^2 r_s{}^2 - 4\left(1+m^2\right)\left(r_s{}^2 - r_l{}^2\right)}}{2\left(1+m^2\right)} \\ y_{L_{1A/B}} &= y_{L_{2A/B}} = m x_{L_{1A/B}} + r_s \end{aligned}$$

Intersection of the TL with the UTAS

The intersections of the TL with the UTAS are similarly identified through the solution to the following system of equations,

$$x_{U_{1A/B}}{}^2 + y_{U_{1A/B}}{}^2 = r_u{}^2$$

$$y_{U_{1A/B}} = m x_{U_{1A/B}} + r_s$$

The solution is subsequently identified as,

$$x_{U_{1A/B}} = -x_{U_{2A/B}} = \frac{-2mr_s \pm \sqrt{4m^2 r_s{}^2 - 4\left(1+m^2\right)\left(r_s{}^2 - r_u{}^2\right)}}{2\left(1+m^2\right)}$$

$$y_{U_{1A/B}} = y_{U_{2A/B}} = m x_{U_{1A/B}} + r_s$$

Intersection of the TL with the RS

The intersection of the TL with the RS is identified from the solution to the following system of equations,

$$x_{T_1}{}^2 + \left(y_{T_1} - y_s\right)^2 = R^2,$$

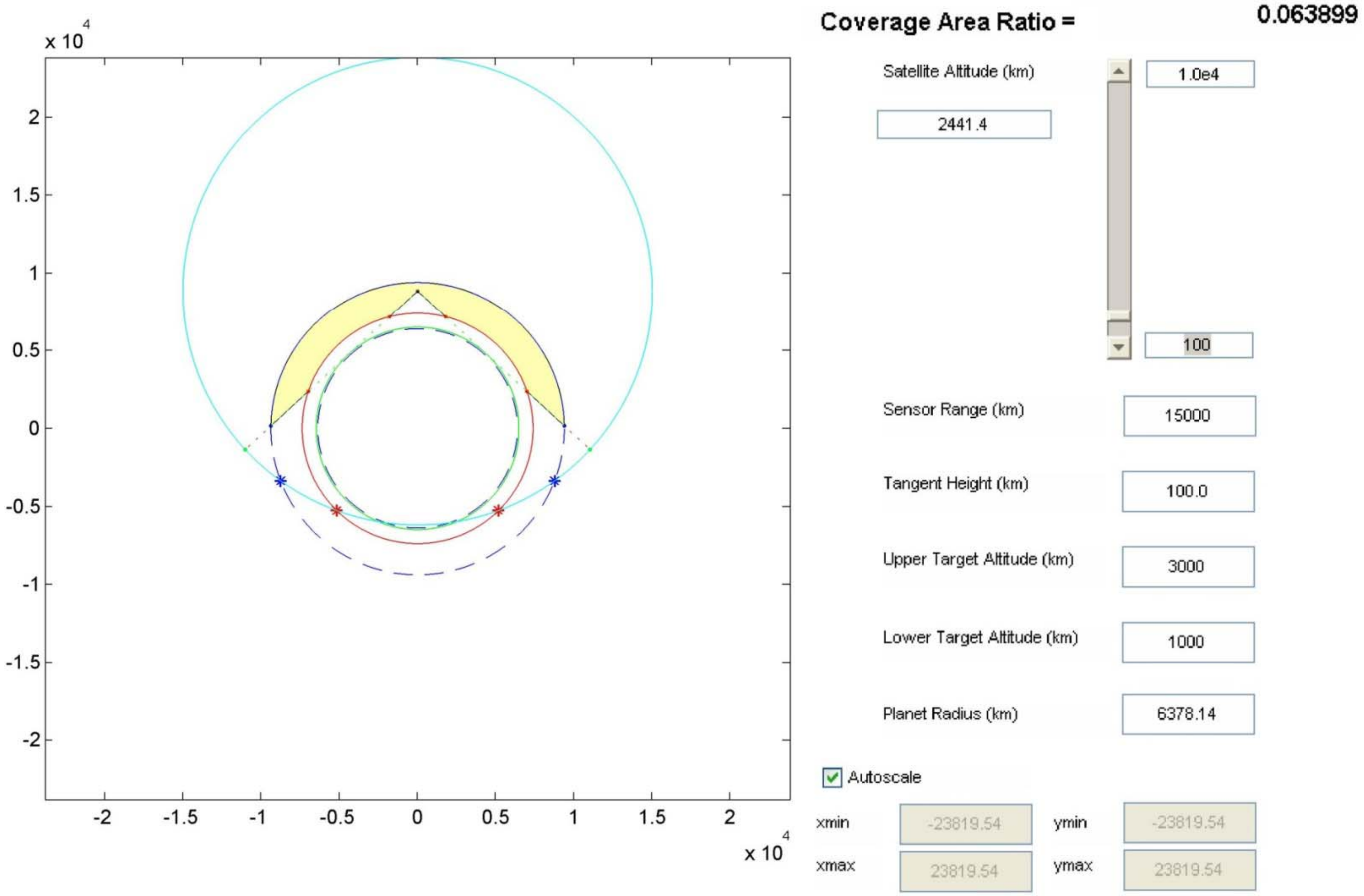
$$y_{T_1} = m x_{T_1} + y_s.$$

The solution to the above system is given by,

$$x_{T_1} = -x_{T_2} = \frac{R}{\sqrt{1+m^2}},$$

$$y_{T_1} = y_{T_2} = m x_{T_1} + r_s.$$

Sample Area Calculation



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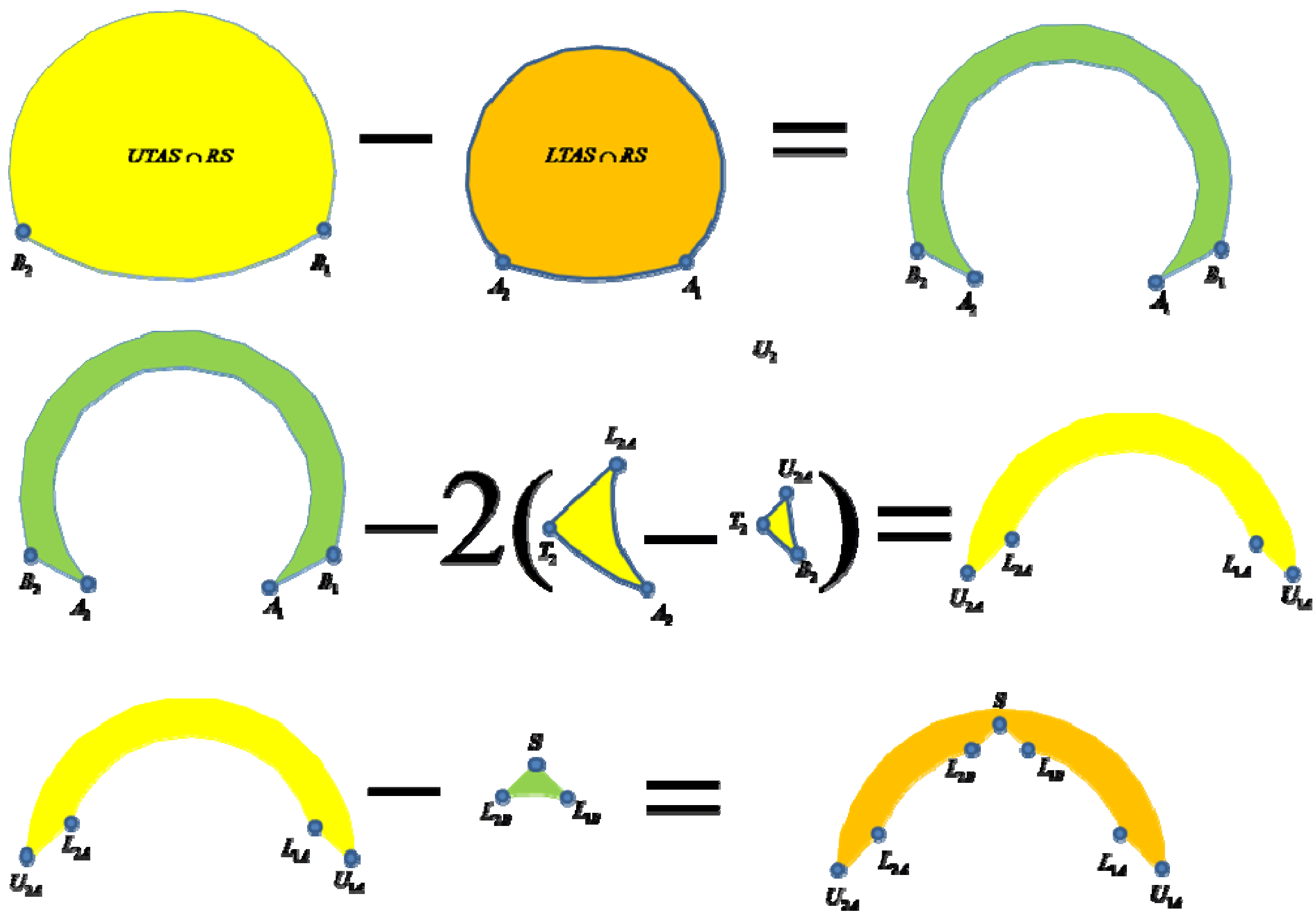
Geometrical Components



Triangle Area and Semiperimeter

- Δ's are a large component of the coverage area geometry.
- Define the area of a Δ as a function of the semiperimeter, "s", and the sides of the Δ; "a", "b", and "c":

$$s = \frac{(a+b+c)}{2},$$

- "s" easily computed from available shell intersections
- Subsequently, the area of a triangular section is given by:

$$\mathrm{A}_{\triangle}(a,b,c) = \sqrt{s(s-a)(s-b)(s-c)}$$

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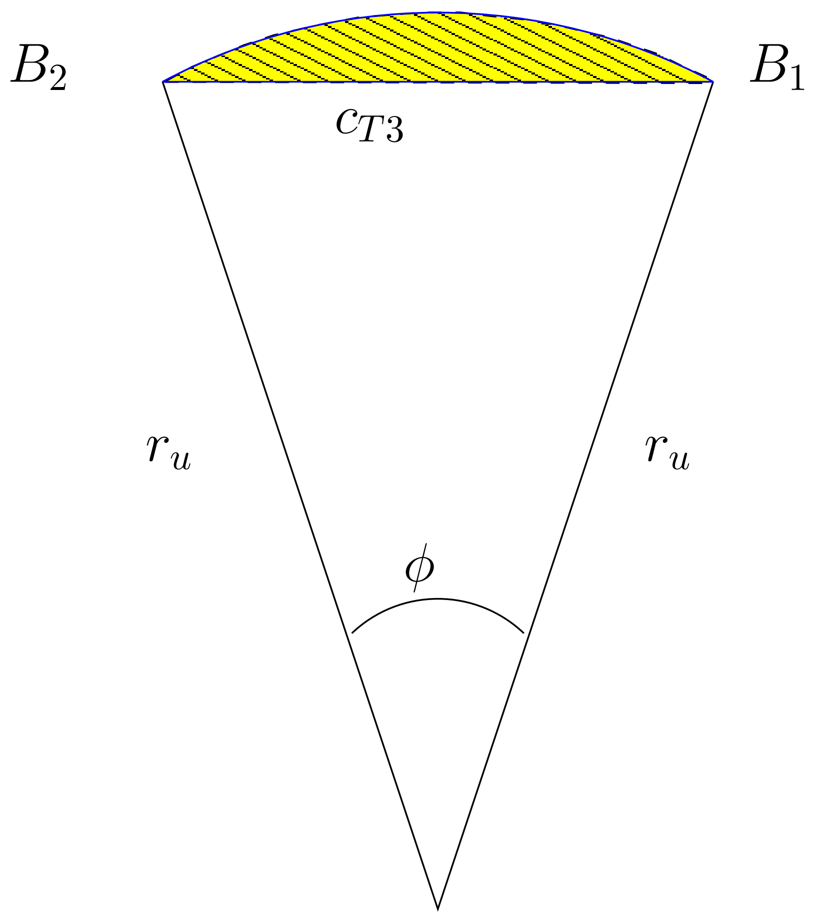
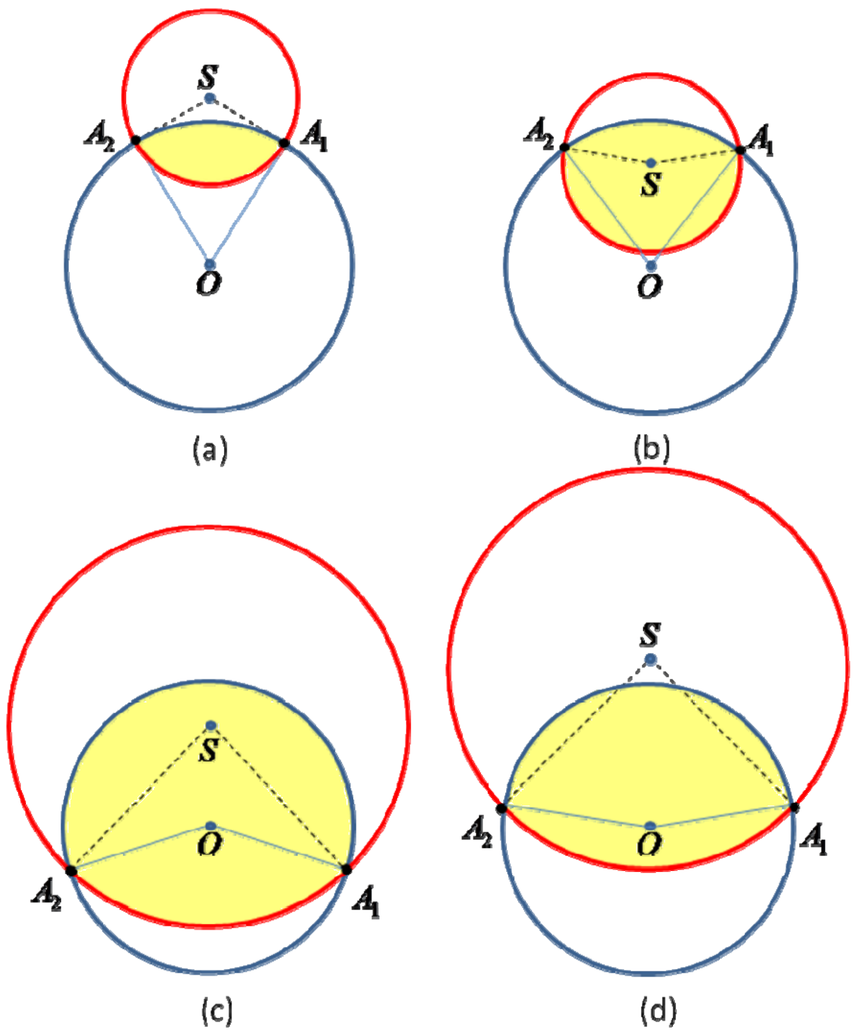
Arc Segments

$$\mathbf{A}_{\Sigma}\left(r_u, c_{T_3}\right) = \underbrace{\frac{1}{2}\phi r_u^{\;2}}_{SECTOR} - \underbrace{\frac{c_{T_3} r_u}{2}\cos\frac{\phi}{2}}_{TRIANGLE}$$

$$\phi = 2\sin^{-1}\left(\frac{c_{T_3}}{2r_u}\right)$$



Area of Intersection Between Two Circles

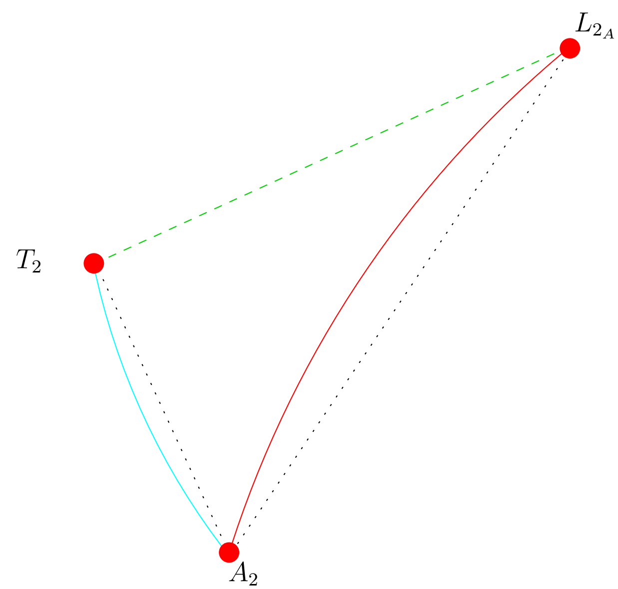


The example to the left focuses on the Intersection of the RS with the LTAS. Note, in each case, the area of intersection is given by the sum of the area of two arc segments. However, that equation can vary by a constant factor depending on the geometry of the intersection (4 types)

Area of Intersection of RS with L/UTAS

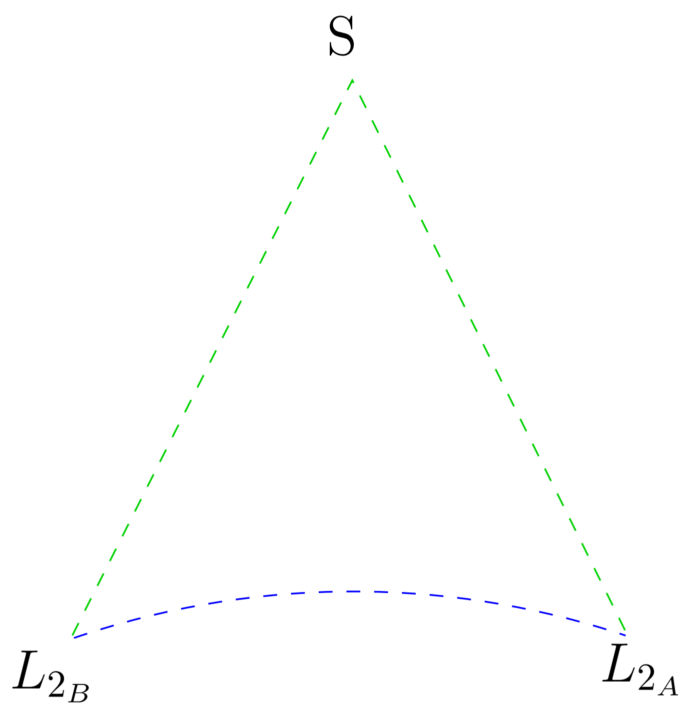
Condition	Area of Intersection
$\left(r_l > R \text{ and } r_s > \sqrt{r_l^2 - R^2}\right)$ or $\left(r_l \le R \text{ and } r_s > \sqrt{R^2 - r_l^2}\right)$	$\mathbf{A}_{RS \cap LTAS} = \mathbf{A}_\Sigma\left(R, \left\lvert\overline{A_1 A_2}\right\rvert\right) + \mathbf{A}_\Sigma\left(r_l, \left\lvert\overline{A_1 A_2}\right\rvert\right)$
$\left(r_l > R \text{ and } r_s \le \sqrt{r_l^2 - R^2}\right)$	$\mathbf{A}_{RS \cap LTAS} = \pi R^2 - \mathbf{A}_\Sigma\left(R, \left\lvert\overline{A_1 A_2}\right\rvert\right) + \mathbf{A}_\Sigma\left(r_l, \left\lvert\overline{A_1 A_2}\right\rvert\right)$
$\left(r_l \le R \text{ and } r_s \le \sqrt{R^2 - r_l^2}\right)$	$\mathbf{A}_{RS \cap LTAS} = \pi r_l^2 - \mathbf{A}_\Sigma\left(r_l, \left\lvert\overline{A_1 A_2}\right\rvert\right) + \mathbf{A}_\Sigma\left(R, \left\lvert\overline{A_1 A_2}\right\rvert\right)$

Composite Triangles: Type 1



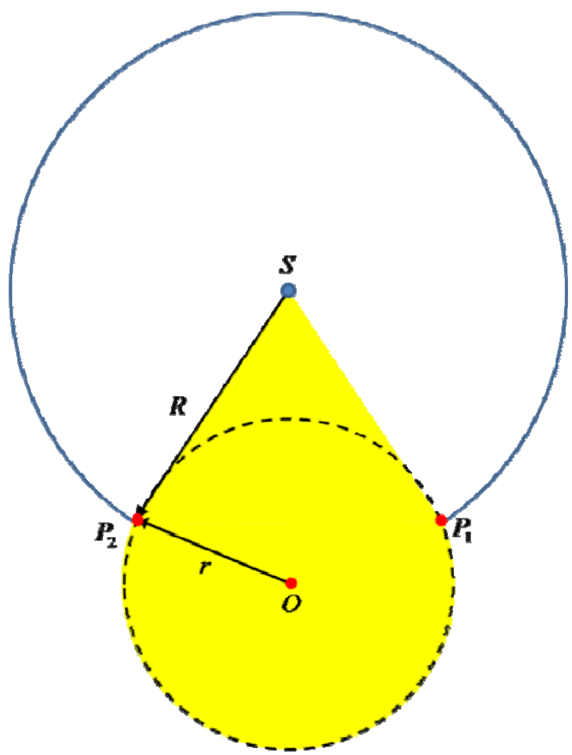
$$\mathbf{A}_{\Lambda_1}\left(r_l, R, \left|\overline{B_2T_2}\right|, \left|\overline{B_2L_2}\right|, \left|\overline{T_2L_2}\right|\right) = \mathbf{A}_{\triangle}\left(\left|\overline{B_2T_2}\right|, \left|\overline{B_2L_2}\right|, \left|\overline{T_2L_2}\right|\right) - \mathbf{A}_{\Sigma}\left(r_l, \left|\overline{T_2L_2}\right|\right) + \mathbf{A}_{\Sigma}\left(R, \left|\overline{B_2T_2}\right|\right)$$

Composite Triangle: Type 2



$$\mathbf{A}_{\Lambda_2}\left(r_l, \left|\overline{L_2 S}\right|, \left|\overline{L_1 L_2}\right|, \left|\overline{L_1 S}\right|\right) = \mathbf{A}_{\triangle}\left(\left|\overline{L_2 S}\right|, \left|\overline{L_1 L_2}\right|, \left|\overline{L_1 S}\right|\right) - \mathbf{A}_{\Sigma}\left(r_l, \left|\overline{L_1 L_2}\right|\right)$$

"Teardrop" Sections



$$\mathbf{A}_{\pi_2}\left(r,R,\overline{P_1P_2}\right)=\begin{cases}\mathbf{A}_{\triangle}\left(R,\left|\overline{P_1P_2}\right|,R\right)+\mathbf{A}_{\Sigma}\left(r,\left|\overline{P_1P_2}\right|\right); & R<\sqrt{r_s^{\,2}+r^2}\\ \mathbf{A}_{\triangle}\left(R,\left|\overline{P_1P_2}\right|,R\right)+\tilde{\mathbf{A}}_{\Sigma}\left(r,\left|\overline{P_1P_2}\right|\right); & R\geq\sqrt{r_s^{\,2}+r^2}\end{cases}$$

Summary of Special Cases

- Primary cases:
  - $R_t \le R_s < R_l$
  - $R_l \le R_s < R_u$
  - $R_u \le R_s < R_{s3}$
- Subcases due to existence of intersections

$$\left|\overline{A_1A_2}\right| = 0 \qquad (\text{entry/exit})$$

$$\left|\overline{B_1B_2}\right| = 0 \qquad (\text{entry/exit})$$

- Subcases due to RS Size

$$\left|\overline{T_2S}\right| < \left|\overline{U_{2b}S}\right|$$

$$\left|\overline{U_{2b}S}\right| \le \left|\overline{T_2S}\right| < \left|\overline{L_{2b}S}\right|$$

$$\left|\overline{L_{2b}S}\right| \le \left|\overline{T_2S}\right| < \left|\overline{L_{2a}S}\right|$$

$$\left|\overline{L_{2a}S}\right| \le \left|\overline{T_2S}\right| < \left|\overline{U_{2a}S}\right|$$

$$\left\|\overline{L_{2a}S}\right| \le \overline{T_2S}\,\Big|$$

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Area Geometry: Satellite Below UTAS

Coverage Area Subcases for $r_t \le r_s < r_l$

$\lvert\overline{T_2S}\rvert < \lvert\overline{L_{2A}S}\rvert$	$\lvert\overline{L_{2A}S}\rvert \le \lvert\overline{T_2S}\rvert < \lvert\overline{U_{2A}S}\rvert$	$\lvert\overline{U_{2A}S}\rvert \le \lvert\overline{T_2S}\rvert$
1(a)	1 (b.i): $\lvert\overline{A_1A_2}\rvert \ne \varnothing$	1 (c.i): $\lvert\overline{A_1A_2}\rvert \ne \varnothing$
	1 (b.ii): $\lvert\overline{A_1A_2}\rvert = \varnothing$	1 (c.ii): $\lvert\overline{A_1A_2}\rvert = \varnothing$

Coverage Area Subcases for $r_l \le r_s < r_u$

$\lvert\overline{T_2S}\rvert < \lvert\overline{L_{2B}S}\rvert$	$\lvert\overline{L_{2B}S}\rvert \le \lvert\overline{T_2S}\rvert < \lvert\overline{L_{2A}S}\rvert$	$\lvert\overline{L_{2A}S}\rvert \le \lvert\overline{T_2S}\rvert < \lvert\overline{U_{2A}S}\rvert$	$\lvert\overline{U_{2A}S}\rvert \le \lvert\overline{T_2S}\rvert$
2(a)	2(b)	2(c.i): $\lvert\overline{A_1A_2}\rvert \ne \varnothing$	2(d.i): $\lvert\overline{A_1A_2}\rvert \ne \varnothing$
		2(c.ii) $\lvert\overline{A_1A_2}\rvert = \varnothing$	2(d.ii): $\lvert\overline{A_1A_2}\rvert = \varnothing$

Area Geometry: Satellite Above UTAS

Coverage Area Subcases for $r_u \le r_s < r_{s_3}$

$\lvert\overline{T_2S}\rvert < \lvert\overline{U_{2B}S}\rvert$	$\lvert\overline{U_{2B}S}\rvert \le \lvert\overline{T_2S}\rvert < \lvert\overline{L_{2B}S}\rvert$	$\lvert\overline{L_{2B}S}\rvert \le \lvert\overline{T_2S}\rvert < \lvert\overline{L_{2A}S}\rvert$	$\lvert\overline{L_{2A}S}\rvert \le \lvert\overline{T_2S}\rvert < \lvert\overline{U_{2A}S}\rvert$	$\lvert\overline{U_{2A}S}\rvert \le \lvert\overline{T_2S}\rvert$
3(a)	3(b)	3(c)	3(d.i): $\lvert\overline{A_1A_2}\rvert \neq \varnothing$	3(e.i): $\lvert\overline{A_1A_2}\rvert \neq \varnothing$
			3(d.ii): $\lvert\overline{A_1A_2}\rvert = \varnothing$	3(e.ii): $\lvert\overline{A_1A_2}\rvert = \varnothing$

Coverage Area for $r_t \le r_s < r_l$

Case	Area
1(a)	$\mathbf{A} = \mathbf{A}_{UTAS \cap RS} - \mathbf{A}_{LTAS \cap RS}$
1(b.i)	$\mathbf{A} = \mathbf{A}_{UTAS \cap RS} - \mathbf{A}_{LTAS \cap RS}$ $\mathbf{A} = \mathbf{A} - 2\mathbf{A}_{\Lambda_1}\left(R, r_l, \left\lvert\overline{T_2L_{2A}}\right\rvert, \left\lvert\overline{T_2A_2}\right\rvert, \left\lvert\overline{A_2L_{2A}}\right\rvert\right)$
1(b.ii)	$\mathbf{A} = \mathbf{A}_{UTAS \cap RS} - \pi r_l^2$ $\mathbf{A} = \mathbf{A} - \mathbf{A}_{\pi_1}\left(R, \left\lvert\overline{T_1T_2}\right\rvert\right)$ $\mathbf{A} = \mathbf{A} + \mathbf{A}_{\pi_2}\left(r_l, R, \left\lvert\overline{L_{1A}L_{2A}}\right\rvert\right)$
1 (c.i)	$\mathbf{A} = \mathbf{A}_{UTAS \cap RS} - \mathbf{A}_{LTAS \cap RS}$ $\mathbf{A} = \mathbf{A} - 2\mathbf{A}_{\Lambda_1}\left(r_l, R, \left\lvert\overline{T_2L_{2A}}\right\rvert, \left\lvert\overline{T_2A_2}\right\rvert, \left\lvert\overline{A_2L_{2A}}\right\rvert\right)$ $\mathbf{A} = \mathbf{A} + 2\mathbf{A}_{\Lambda_1}\left(r_u, R, \left\lvert\overline{T_2U_{2A}}\right\rvert, \left\lvert\overline{T_2B_2}\right\rvert, \left\lvert\overline{B_2U_{2A}}\right\rvert\right)$
1 (c.ii)	$\mathbf{A} = \pi r_u^2 - \pi r_l^2$ $\mathbf{A} = \mathbf{A} - \mathbf{A}_{\pi_2}\left(r_u, R, \left\lvert\overline{U_{1A}U_{2A}}\right\rvert\right)$ $\mathbf{A} = \mathbf{A} + \mathbf{A}_{\pi_2}\left(r_l, R, \left\lvert\overline{L_{1A}L_{2A}}\right\rvert\right)$

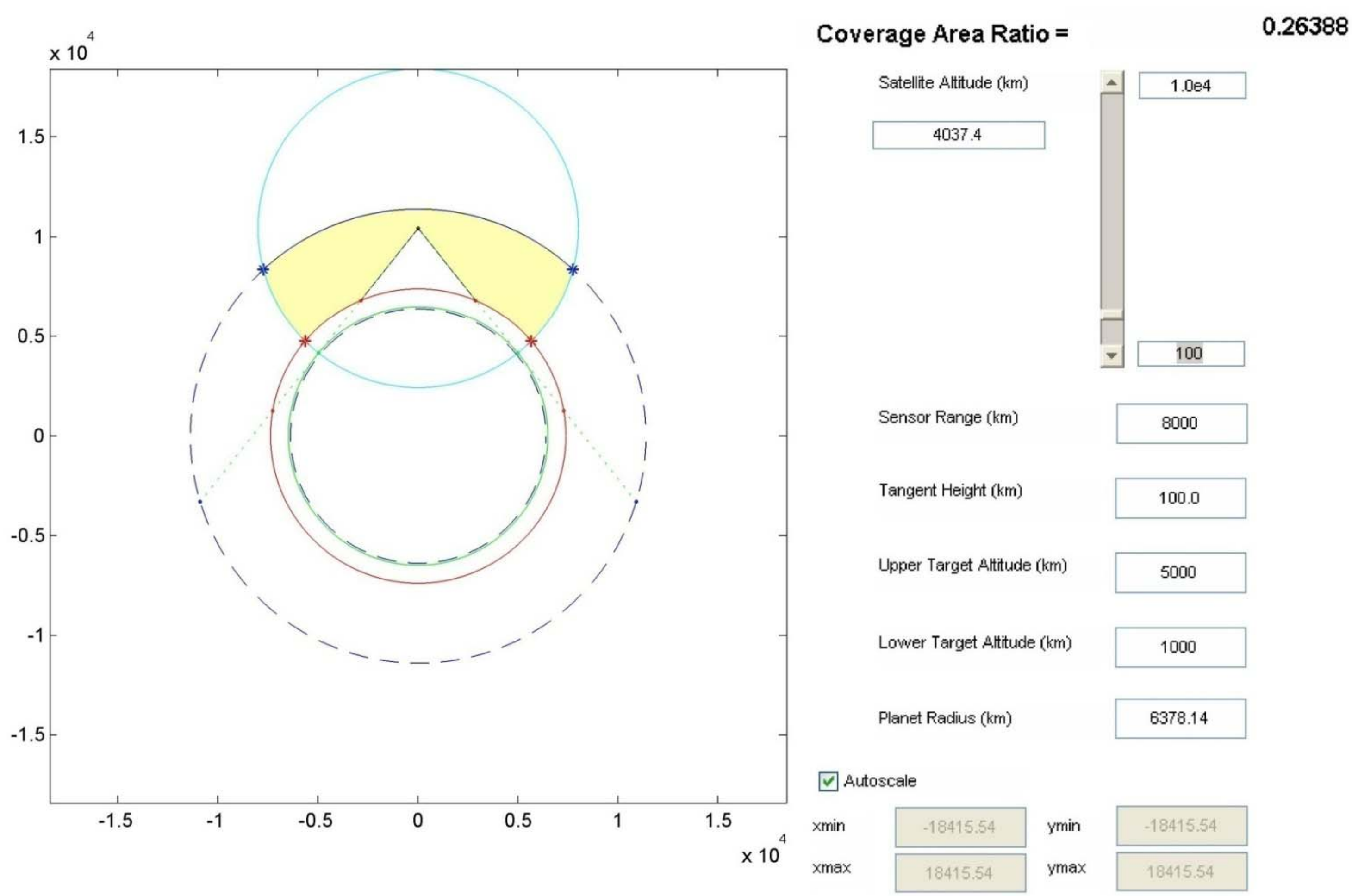
Coverage Area for $r_l \le r_s < r_u$

Case	Area
2(a)	$\mathbf{A} = \mathbf{A}_{UTAS \cap RS} - \mathbf{A}_{\pi_1}\left(R, \left\lvert\overline{T_1T_2}\right\rvert\right)$
2(b)	$\mathbf{A} = \mathbf{A}_{UTAS \cap RS} - \mathbf{A}_{LTAS \cap RS}$ $\mathbf{A} = \mathbf{A} - \mathbf{A}_{\Lambda_2}\left(r_l, \left\lvert\overline{L_{1B}S}\right\rvert, \left\lvert\overline{L_{1B}L_{2B}}\right\rvert, \left\lvert\overline{L_{2B}S}\right\rvert\right)$
2(c.i)	$\mathbf{A} = \mathbf{A}_{UTAS \cap RS} - \mathbf{A}_{LTAS \cap RS}$ $\mathbf{A} = \mathbf{A} - \mathbf{A}_{\Lambda_2}\left(r_l, \left\lvert\overline{L_{1B}S}\right\rvert, \left\lvert\overline{L_{1B}L_{2B}}\right\rvert, \left\lvert\overline{L_{2B}S}\right\rvert\right)$ $\mathbf{A} = \mathbf{A} - 2\mathbf{A}_{\Lambda_1}\left(r_l, R, \left\lvert\overline{T_2L_{2A}}\right\rvert, \left\lvert\overline{T_2A_2}\right\rvert, \left\lvert\overline{A_2L_{2A}}\right\rvert\right)$
2(c.ii)	$\mathbf{A} = \mathbf{A}_{UTAS \cap RS} - \mathbf{A}_{LTAS \cap RS}$ $\mathbf{A} = \mathbf{A} - \mathbf{A}_{\Lambda_2}\left(r_l, \left\lvert\overline{L_{1B}S}\right\rvert, \left\lvert\overline{L_{1B}L_{2B}}\right\rvert, \left\lvert\overline{L_{2B}S}\right\rvert\right)$ $\mathbf{A} = \mathbf{A} - \mathbf{A}_{\pi_1}\left(R, \left\lvert\overline{T_1T_2}\right\rvert\right)$ $\mathbf{A} = \mathbf{A} + \mathbf{A}_{\pi_2}\left(r_l, R, \left\lvert\overline{L_{1A}L_{2A}}\right\rvert\right)$
2(d.i)	$\mathbf{A} = \mathbf{A}_{UTAS \cap RS} - \mathbf{A}_{LTAS \cap RS}$ $\mathbf{A} = \mathbf{A} - \mathbf{A}_{\Lambda_2}\left(r_l, \left\lvert\overline{L_{1B}S}\right\rvert, \left\lvert\overline{L_{1B}L_{2B}}\right\rvert, \left\lvert\overline{L_{2B}S}\right\rvert\right)$ $\mathbf{A} = \mathbf{A} - 2\mathbf{A}_{\Lambda_1}\left(r_l, R, \overline{T_2L_{2A}}, \overline{T_2A_2}, \overline{A_2L_{2A}}\right)$ $\mathbf{A} = \mathbf{A} + 2\mathbf{A}_{\Lambda_1}\left(r_u, R, \overline{T_2U_{2A}}, \overline{T_2B_2}, \overline{B_2U_{2A}}\right)$
2(d.ii)	$\mathbf{A} = \pi r_u^2 - \pi r_l^2$ $\mathbf{A} = \mathbf{A} - \mathbf{A}_{\pi_2}\left(r_u, R, \left\lvert\overline{U_{1A}U_{2A}}\right\rvert\right)$ $\mathbf{A} = \mathbf{A} + \mathbf{A}_{\pi_2}\left(r_l, R, \left\lvert\overline{L_{1A}L_{2A}}\right\rvert\right)$ $\mathbf{A} = \mathbf{A} - \mathbf{A}_{\mathbf{\Lambda}_2}\left(r_l, \left\lvert\overline{L_{1B}S}\right\rvert, \left\lvert\overline{L_{1B}L_{2B}}\right\rvert\left\lvert\overline{L_{2B}S}\right\rvert\right)$

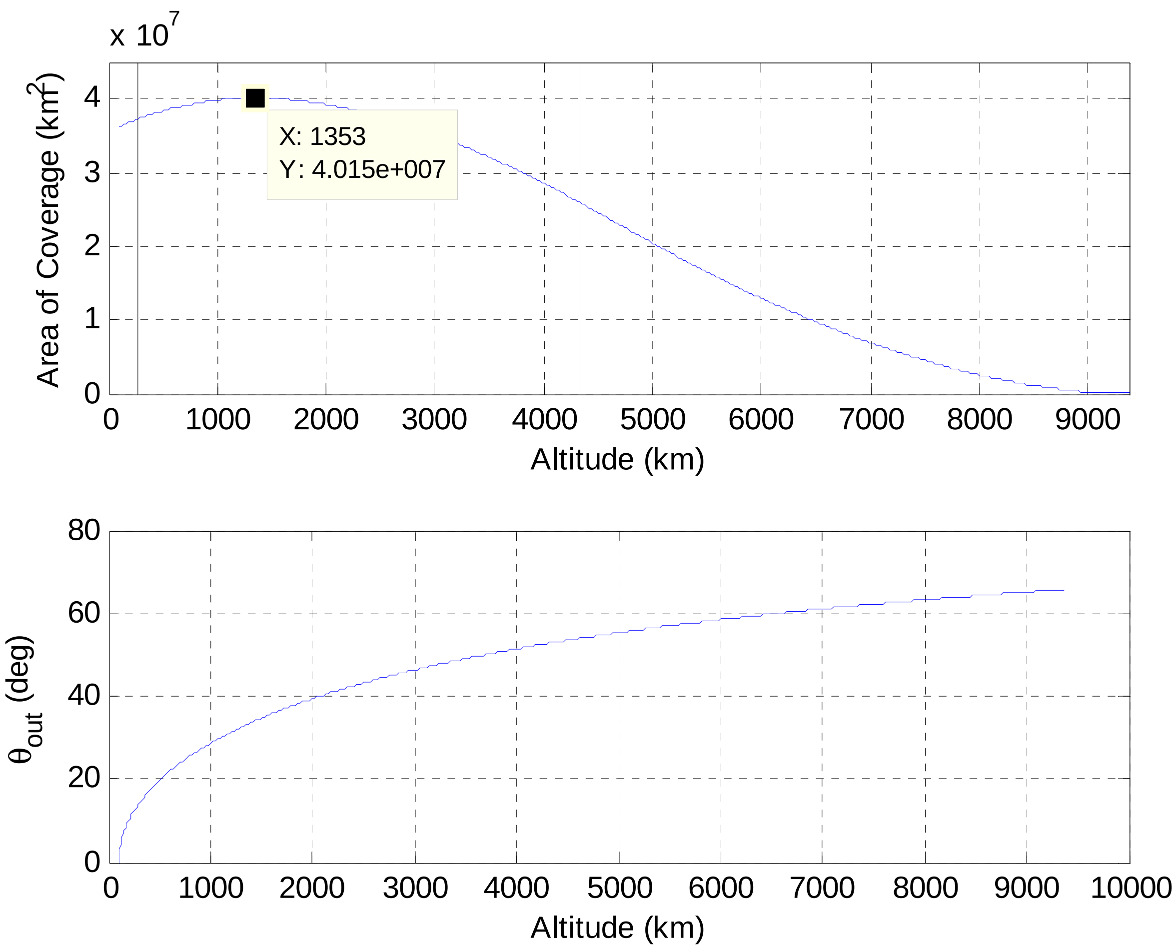
Table 8 - Coverage Area for $r_l \le r_s < r_u$

Case	Area
3(a)	$\mathbf{A} = \mathbf{0}$
3(b)	$\mathbf{A} = \mathbf{A}_{UTAS \frown RS} - \mathbf{A}_{\pi_2}\left(R, \left\|\overline{T_1T_2}\right\|\right)$ $\mathbf{A} = \mathbf{A} + \mathbf{A}_{\Lambda_2}\left(r_u, \overline{U_{2B}S}, \overline{U_{1B}U_{2B}}, \overline{U_{1B}S}\right)$
3(c)	$\mathbf{A} = \mathbf{A}_{UTAS \frown RS}$ $\mathbf{A} = \mathbf{A} - \mathbf{A}_{\Lambda_2}\left(r_l, \left\|\overline{L_{1B}S}\right\|, \left\|\overline{L_{1B}L_{2B}}\right\|, \left\|\overline{L_{2B}S}\right\|\right)$ $\mathbf{A} = \mathbf{A} + \mathbf{A}_{\Lambda_2}\left(r_u, \left\|\overline{U_{1B}S}\right\|, \left\|\overline{U_{1B}U_{2B}}\right\|, \left\|\overline{U_{2B}S}\right\|\right)$
3(d.i)	$\mathbf{A} = \mathbf{A}_{UTAS \frown RS}$ $\mathbf{A} = \mathbf{A} - \mathbf{A}_{\Lambda_2}\left(r_l, \left\|\overline{L_{1B}S}\right\|, \left\|\overline{L_{1B}L_{2B}}\right\|, \left\|\overline{L_{2B}S}\right\|\right)$ $\mathbf{A} = \mathbf{A} + \mathbf{A}_{\Lambda_2}\left(r_u, \left\|\overline{U_{1B}S}\right\|, \left\|\overline{U_{1B}U_{2B}}\right\|, \left\|\overline{U_{2B}S}\right\|\right)$ $\mathbf{A} = \mathbf{A} - 2\mathbf{A}_{\Lambda_2}\left(r_l, R, \left\|\overline{T_2L_{2A}}\right\|, \left\|\overline{T_2A_2}\right\|, \left\|\overline{A_2L_{2A}}\right\|\right)$
3(d.ii)	$\mathbf{A} = \mathbf{A}_{UTAS \frown RS} - \pi r_l^2$ $\mathbf{A} = \mathbf{A} - \mathbf{A}_{\Lambda_2}\left(r_l, \left\|\overline{L_{1B}S}\right\|, \left\|\overline{L_{1B}L_{2B}}\right\|, \left\|\overline{L_{2B}S}\right\|\right)$ $\mathbf{A} = \mathbf{A} + \mathbf{A}_{\Lambda_2}\left(r_u, \left\|\overline{U_{1B}S}\right\|, \left\|\overline{U_{1B}U_{2B}}\right\|, \left\|\overline{U_{2B}S}\right\|\right)$ $\mathbf{A} = \mathbf{A} - \mathbf{A}_{\pi_2}\left(R, \left\|\overline{T_1T_2}\right\|\right)$ $\mathbf{A} = \mathbf{A} + \mathbf{A}_{\pi_2}\left(r_l, R, \overline{L_{1A}L_{2A}}\right)$
3(e.i)	$\mathbf{A} = \mathbf{A}_{UTAS \frown RS} - \mathbf{A}_{LTAS \frown RS}$ $\mathbf{A} = \mathbf{A} - \mathbf{A}_{\Lambda_2}\left(r_l, \left\|\overline{L_{1B}S}\right\|, \left\|\overline{L_{1B}L_{2B}}\right\|, \left\|\overline{L_{2B}S}\right\|\right)$ $\mathbf{A} = \mathbf{A} + \mathbf{A}_{\Lambda_2}\left(r_u, \left\|\overline{U_{1B}S}\right\|, \left\|\overline{U_{1B}U_{2B}}\right\|, \left\|\overline{U_{2B}S}\right\|\right)$ $\mathbf{A} = \mathbf{A} - 2\mathbf{A}_{\Lambda_2}\left(r_l, R, \left\|\overline{T_2L_{2A}}\right\|, \left\|\overline{T_2A_2}\right\|, \left\|\overline{A_2L_{2A}}\right\|\right)$ $\mathbf{A} = \mathbf{A} + 2\mathbf{A}_{\Lambda_2}\left(r_u, R, \left\|\overline{T_2U_{2A}}\right\|, \left\|\overline{T_2B_2}\right\|, \left\|\overline{B_2U_{2A}}\right\|\right)$
3(e.ii)	$\mathbf{A} = \pi r_u^2 - \pi r_l^2 - \mathbf{A}_{\pi_2}\left(r_u, R, \overline{U_{1A}U_{2A}}\right) + \mathbf{A}_{\pi_2}\left(r_l, R, \overline{L_{1A}L_{2A}}\right)$ $\mathbf{A} = \mathbf{A} - \mathbf{A}_{\Lambda_2}\left(r_l, \left\|\overline{L_{1B}S}\right\|, \left\|\overline{L_{1B}L_{2B}}\right\|, \left\|\overline{L_{2B}S}\right\|\right)$ $\mathbf{A} = \mathbf{A} + \mathbf{A}_{\Lambda_2}\left(r_u, \left\|\overline{U_{1B}S}\right\|, \left\|\overline{U_{1B}U_{2B}}\right\|, \left\|\overline{U_{2B}S}\right\|\right)$

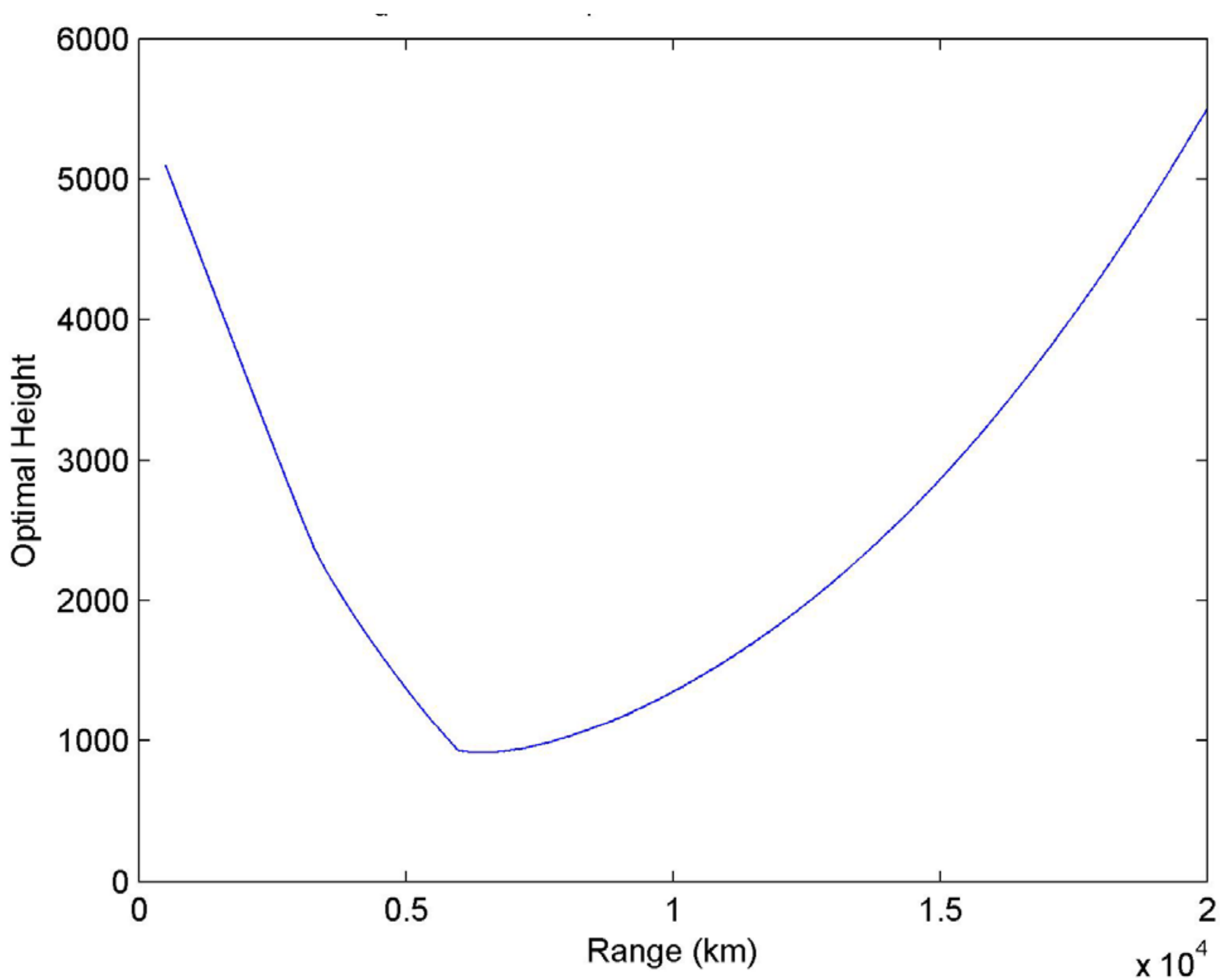
Coverage Area Analysis Tool



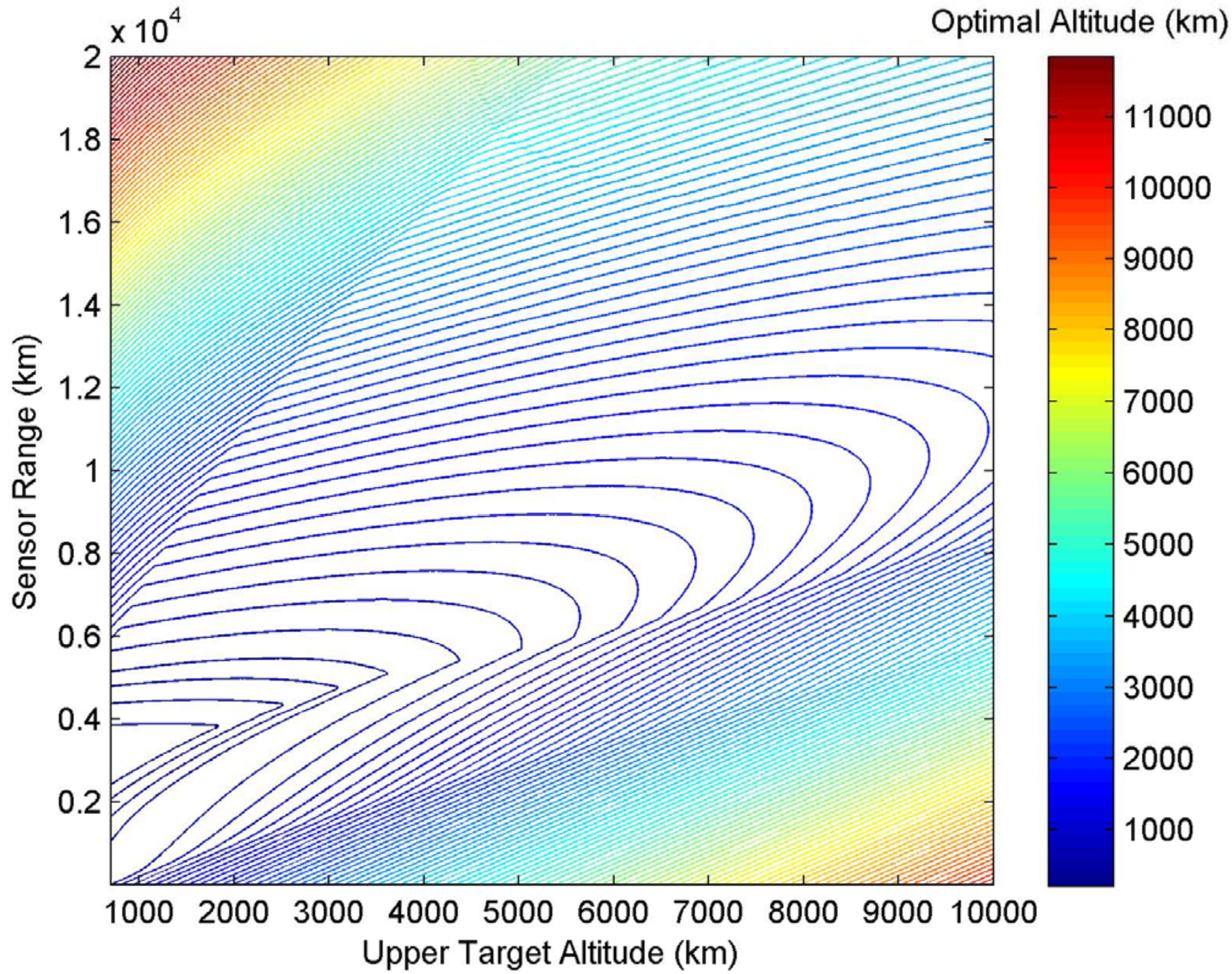
R = 5000 km, h_l = 1000 km, h_u = 5000 km, and h_t = 100 km.



h_u = 5600 km, h_l = 600 km (5000x5000 grid)



Optimal Altitude Space



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Conclusions

- Optimal satellite altitude non-intuitive
- Graphical tools helpful in design process
- Ongoing work
  - Multi-objective optimization for constellation design with applications to constrained ATH coverage problem.
- The results of the current investigation represent useful startup solutions for numerical optimization process.
- Results also provide physical insight into the expected trends.