ASPHERICAL FORMATIONS NEAR THE LIBRATION POINTS IN THE SUN-EARTH/MOON SYSTEM

B.G. Marchand and K.C. Howell

Purdue University

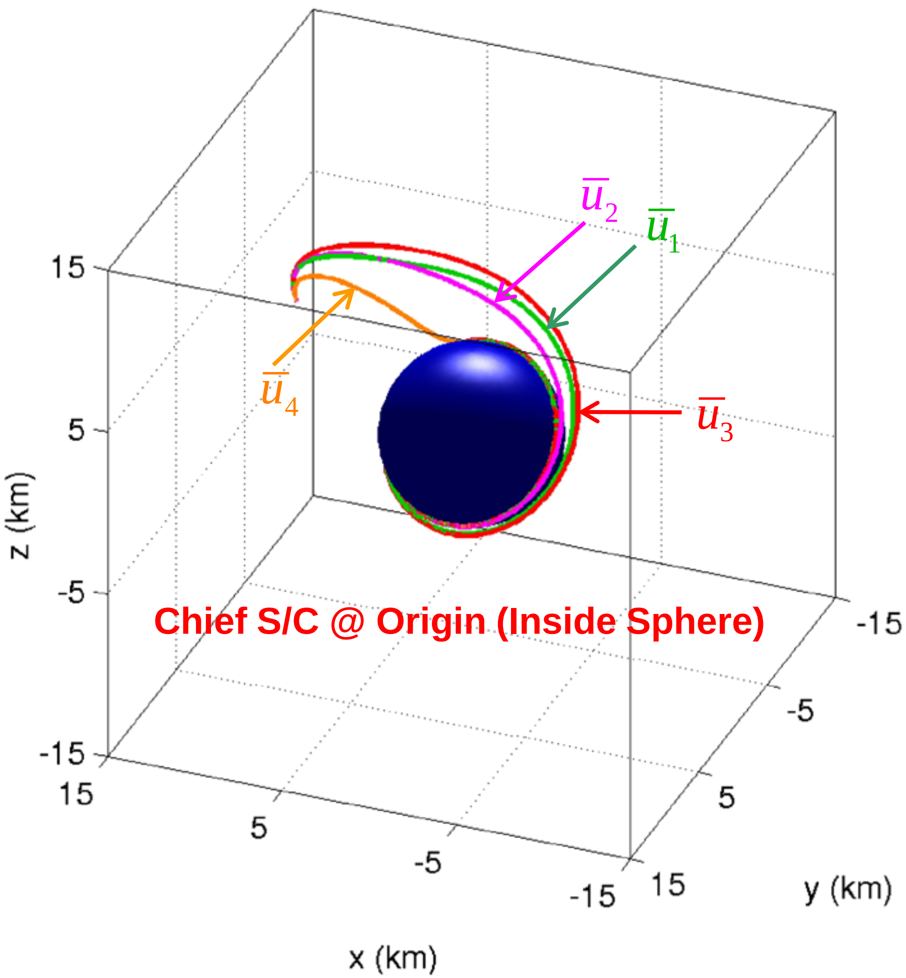
Output Feedback Linearization (OFL) in the Ephemeris Model

- Formation Keeping + Deployment
  - Chief S/C Evolves Along Lissajous Trajectory near L_i
  - Inertial Formation Geometry
    - Spherical Configurations
      - Deputy Constrained to Orbit Chief S/C
      - Fixed Radial Distance
      - Fixed Radial Distance + Rotation Rate
    - Aspherical Configurations → Inertially Fixed Orientation
      - Deputy Constrained to Evolve Along Aspherical Surface
      - Surface may be Offset from Chief S/C

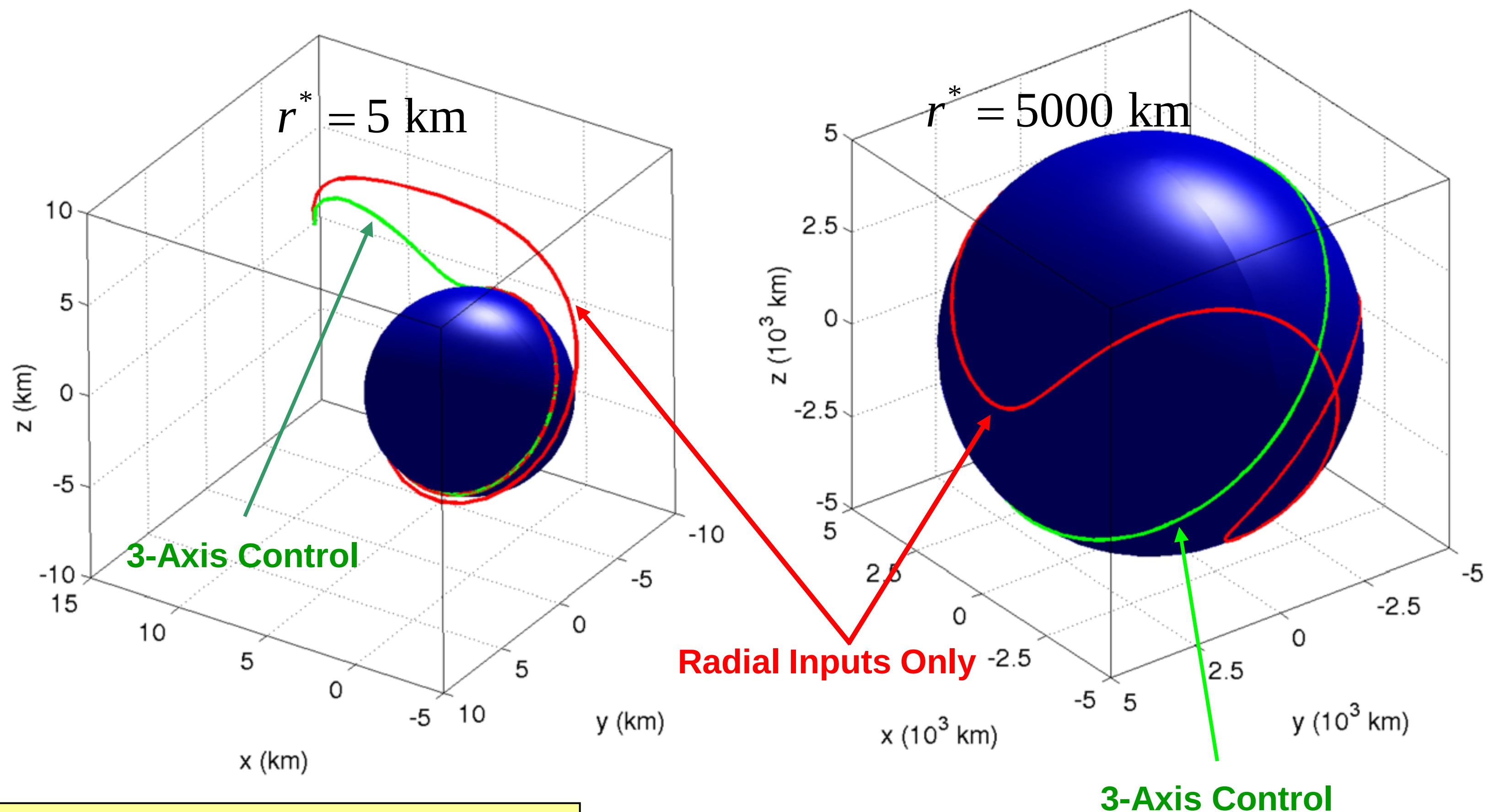
OFL Controlled Response of Deputy S/C

Radial Distance Tracking

	Control Law
1	$\bar{u}(t)=\frac{H(\bar{r},\dot{\bar{r}})}{r}\hat{r}$ Geometric Approach: Radial inputs only
2	$\bar{u}(t)=\left\{\frac{g(\bar{r},\dot{\bar{r}})}{r}-\frac{\dot{\bar{r}}^{T}\dot{\bar{r}}}{r^{2}}\right\}\bar{r}+\left(\frac{\dot{r}}{r}\right)\dot{\bar{r}}-\Delta\bar{f}(\bar{r})$
3	$\bar{u}(t)=\left\{\frac{1}{2}\frac{g(\bar{r},\dot{\bar{r}})}{r^{2}}-\frac{\dot{\bar{r}}^{T}\dot{\bar{r}}}{r^{2}}\right\}\bar{r}-\Delta\bar{f}(\bar{r})$
4	$\bar{u}(t)=\left\{-rg(\bar{r},\dot{\bar{r}})-\frac{\dot{\bar{r}}^{T}\dot{\bar{r}}}{r^{2}}\right\}\bar{r}+3\left(\frac{\dot{r}}{r}\right)\dot{\bar{r}}-\Delta\bar{f}(\bar{r})$



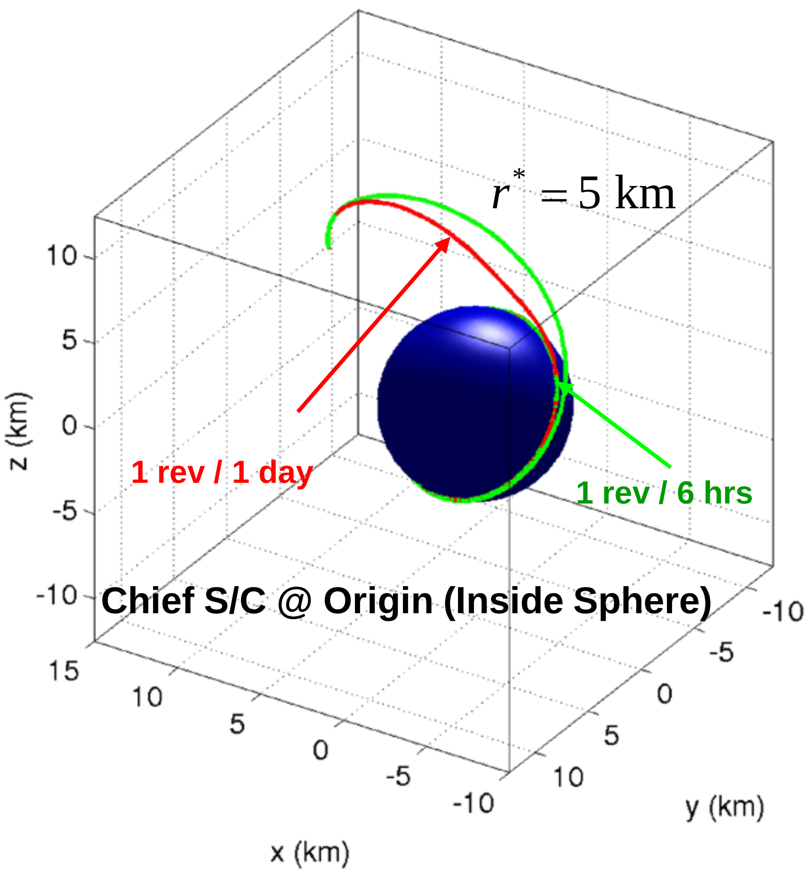
Impact of Nominal Radial Separation on OFL Controlled Response of Deputy S/C

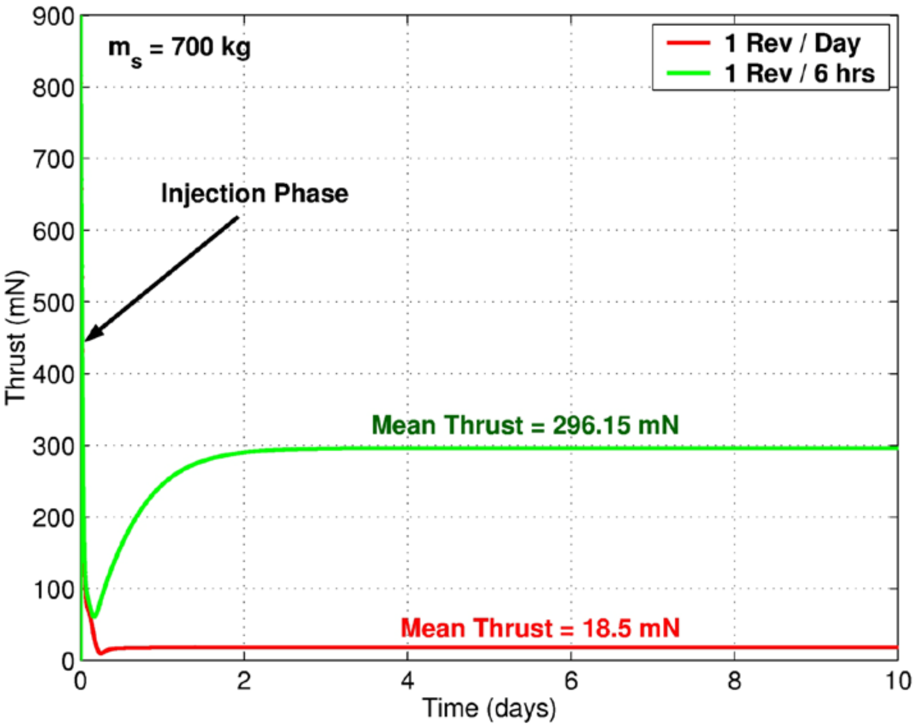


Chief S/C @ Origin (Inside Sphere)

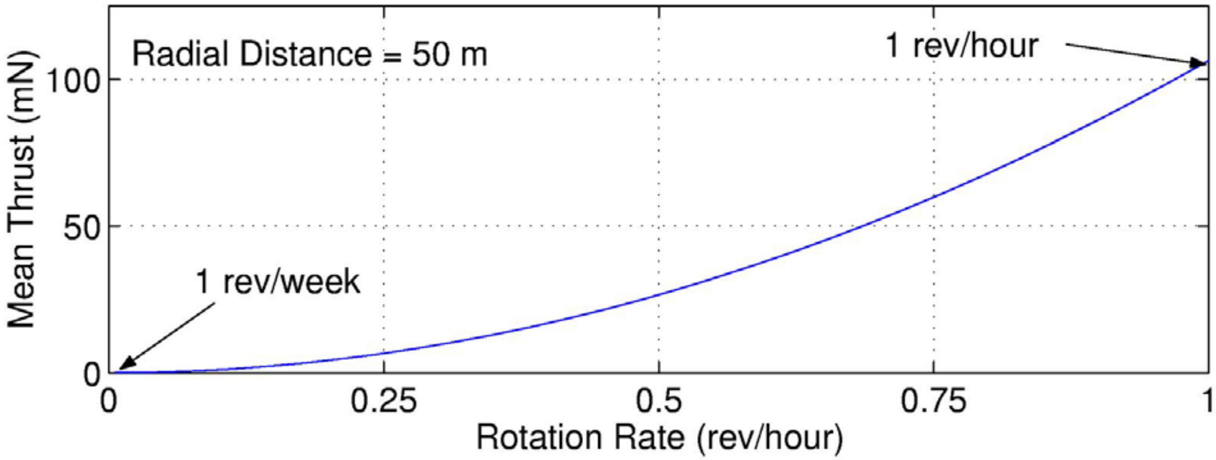
OFL Controlled Response of Deputy S/C

Radial Distance + Rotation Rate Tracking





Impact Commanded Rotation Rate on Cost

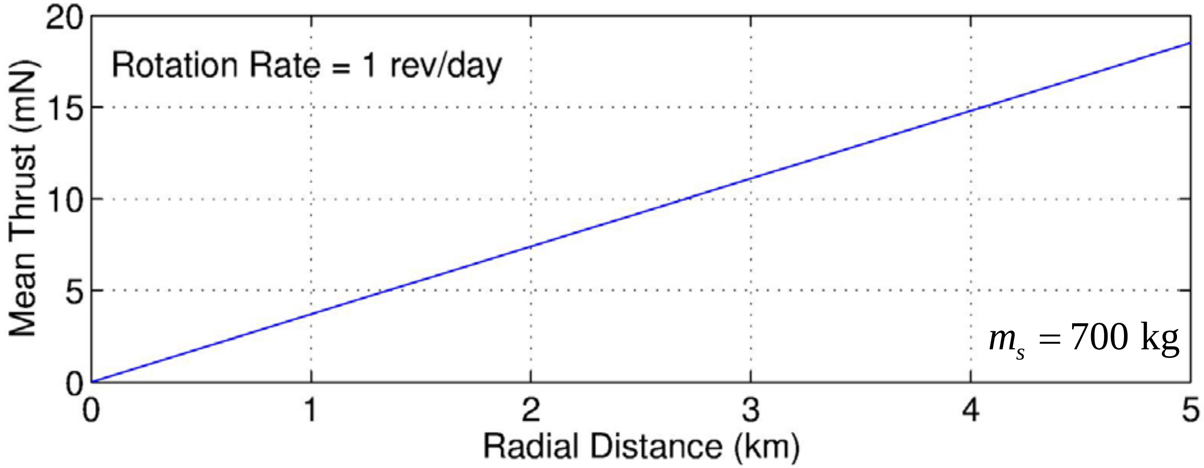


1 rev /24 hrs → 0.19 mN

1 rev /12 hrs → 0.76 mN

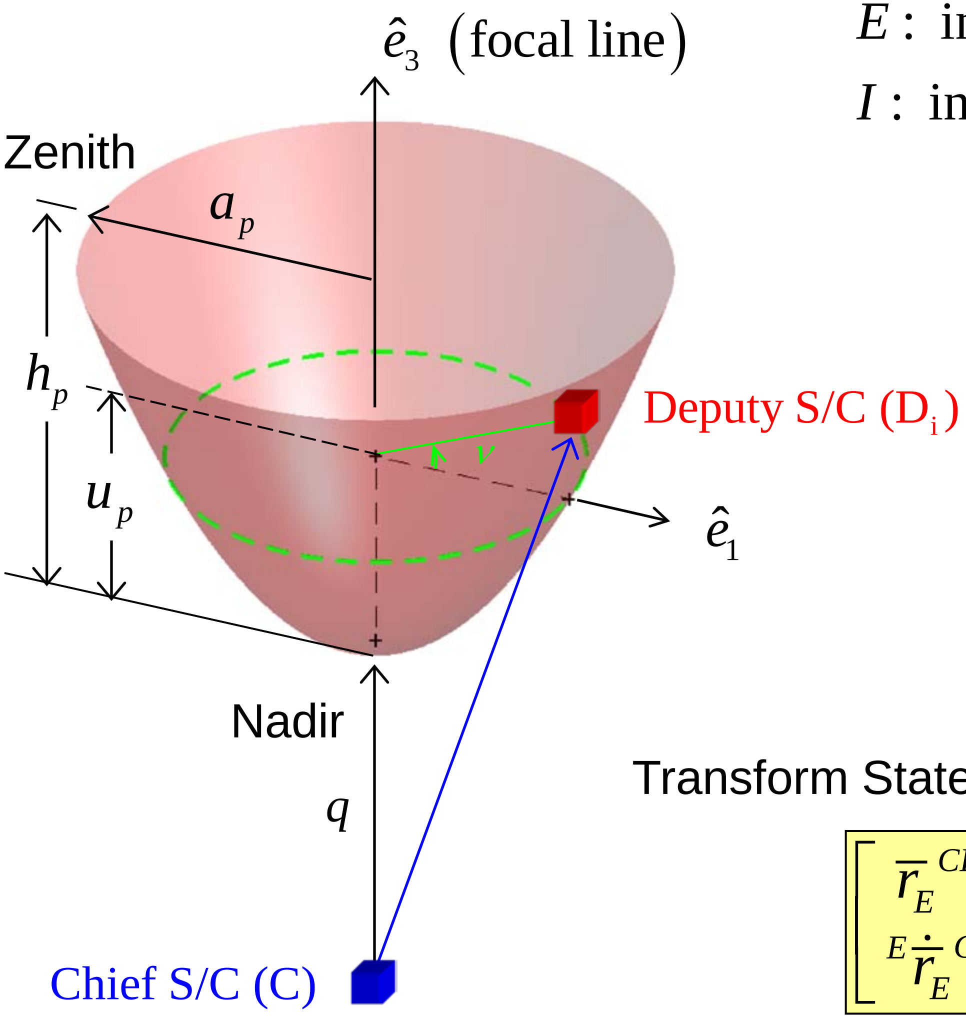
1 rev / 6 hrs → 6.40 mN

1 rev / 1 hrs → 106.50 mN



Application to Aspherical Formations

Parameterization of Parabolic Formation



E : inertially fixed focal frame

I : inertially fixed ephemeris frame

$$\bar{r}_E^{CD_i} = \tilde{x}\hat{e}_1 + \tilde{y}\hat{e}_2 + \tilde{z}\hat{e}_3$$

$$\tilde{x} = a_p \sqrt{u_p / h_p} \cos\nu$$

$$\tilde{y} = a_p \sqrt{u_p / h_p} \sin\nu$$

$$\tilde{z} = u_p + q$$

Transform State from Focal to Ephemeris Frame

$$\begin{bmatrix} \bar{r}_E^{CD_i} \\ {}^E\dot{\bar{r}}_E^{CD_i} \end{bmatrix} = \left\{ {}^I C^E \right\}^T \begin{bmatrix} \bar{r}_I^{CD_i} \\ {}^I\dot{\bar{r}}_I^{CD_i} \end{bmatrix}$$

Controller Development

Desired Response for $u,\ q,$ and $\dot{v}$:

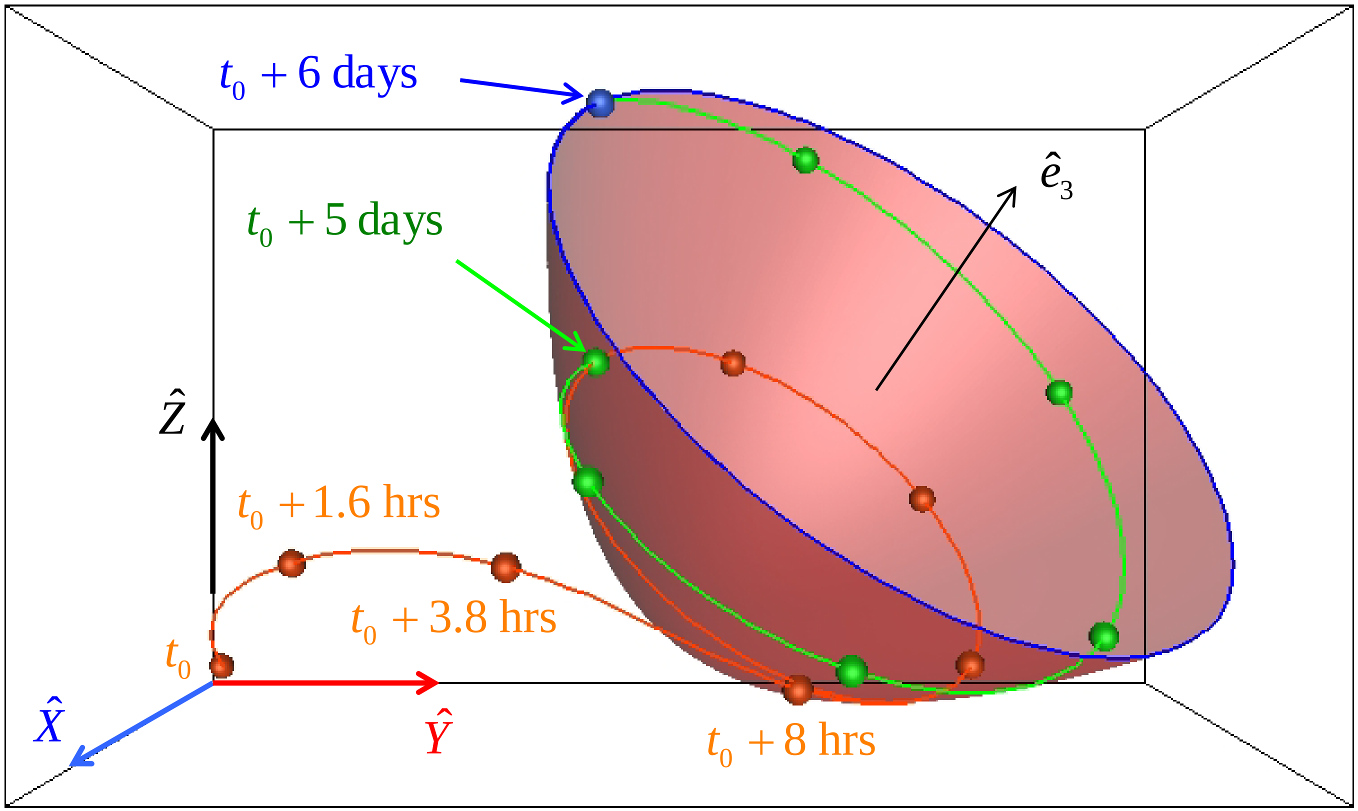
$$\left.\begin{aligned} g_u\left(u_p,\dot{u}_p\right) &= \ddot{u}_p^* - 2\omega_n\left(\dot{u}_p - \dot{u}_p^*\right) - \omega_n{}^2\left(u_p - u_p^*\right) \\ g_q\left(u_p,\dot{u}_p\right) &= \ddot{q}^* - 2\omega_n\left(\dot{q} - \dot{q}^*\right) - \omega_n{}^2\left(q - q^*\right) \end{aligned}\right\} \delta u, \delta q \rightarrow \text{critically damped}$$

$$\left. g_v\left(\dot{v}\right) = \ddot{v}^* - k\omega_n\left(\dot{v} - \dot{v}^*\right) \right\} \ \delta\dot{\theta} \rightarrow \text{exponential decay}$$

Solve for Control Law:

$$\begin{bmatrix} \dfrac{2h}{a^2}\tilde{x} & \dfrac{2h}{a^2}\tilde{y} & 0 \\ -\dfrac{2h}{a^2}\tilde{x} & -\dfrac{2h}{a^2}\tilde{y} & 1 \\ \dfrac{\tilde{x}}{\left(\tilde{x}^2+\tilde{y}^2\right)} & -\dfrac{\tilde{y}}{\left(\tilde{x}^2+\tilde{y}^2\right)} & 0 \end{bmatrix} \begin{bmatrix} \tilde{u}_x \\ \tilde{u}_y \\ \tilde{u}_z \end{bmatrix} = \begin{bmatrix} g_u\left(u,\dot{u}\right) - \dfrac{2h}{a^2}\left(\dot{\tilde{x}}^2 + \dot{\tilde{y}}^2 + \tilde{x}\Delta\tilde{f}_x + \tilde{y}\Delta\tilde{f}_y\right) \\ g_q\left(q,\dot{q}\right) + \dfrac{2h}{a^2}\left(\dot{\tilde{x}}^2 + \dot{\tilde{y}}^2 + \tilde{x}\Delta\tilde{f}_x + \tilde{y}\Delta\tilde{f}_y\right) - \Delta\tilde{f}_z \\ g_{\dot{v}}\left(\dot{v}\right) + 2\dfrac{\left(\tilde{x}\dot{\tilde{x}} + \tilde{y}\dot{\tilde{y}}\right)\left(\tilde{x}\dot{\tilde{y}} - \tilde{y}\dot{\tilde{x}}\right)}{\left(\tilde{x}^2+\tilde{y}^2\right)^2} + \dfrac{\left(\tilde{y}\Delta\tilde{f}_x - \tilde{x}\Delta\tilde{f}_y\right)}{\left(\tilde{x}^2+\tilde{y}^2\right)} \end{bmatrix}$$

OFL Controlled Parabolic Formation



$q = 10$ km

$\dot{\nu} = 1$ rev/day

$h_p = 500$ m

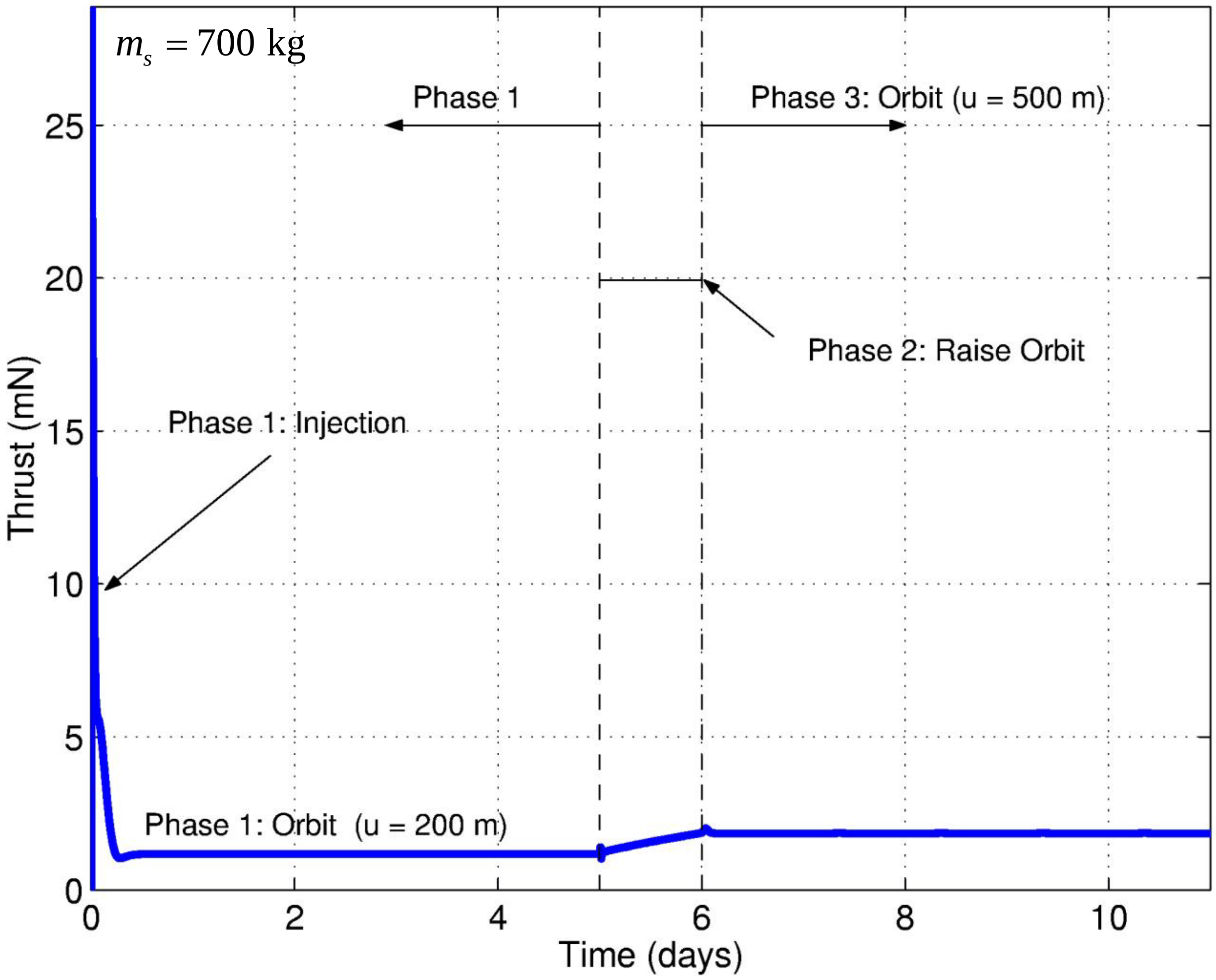
$a_p = 500$ m

Phase I: $u_p = 200$ m

Phase II: $\dot{u}_p = 300$ m / 1 day

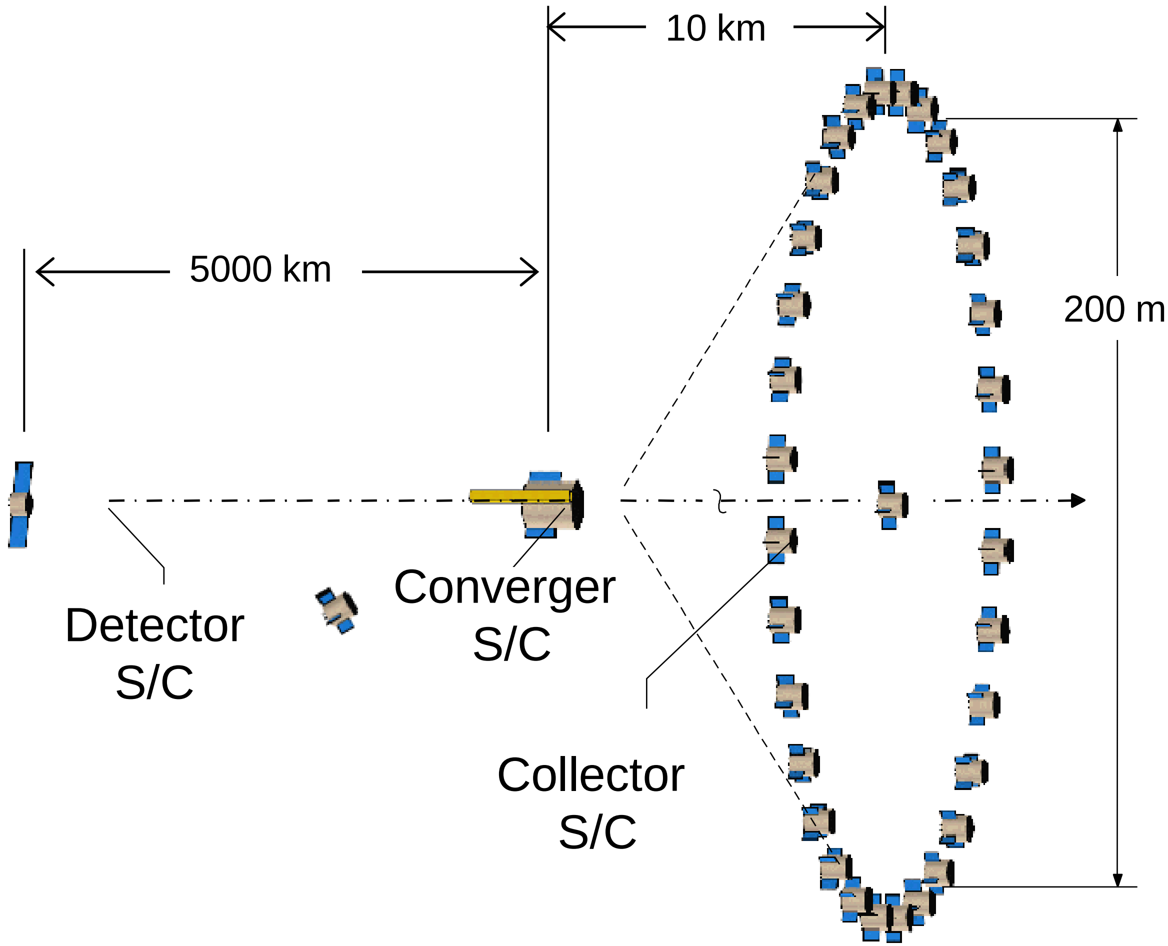
Phase III: $u_p = 500$ m

OFL Thrust Profile



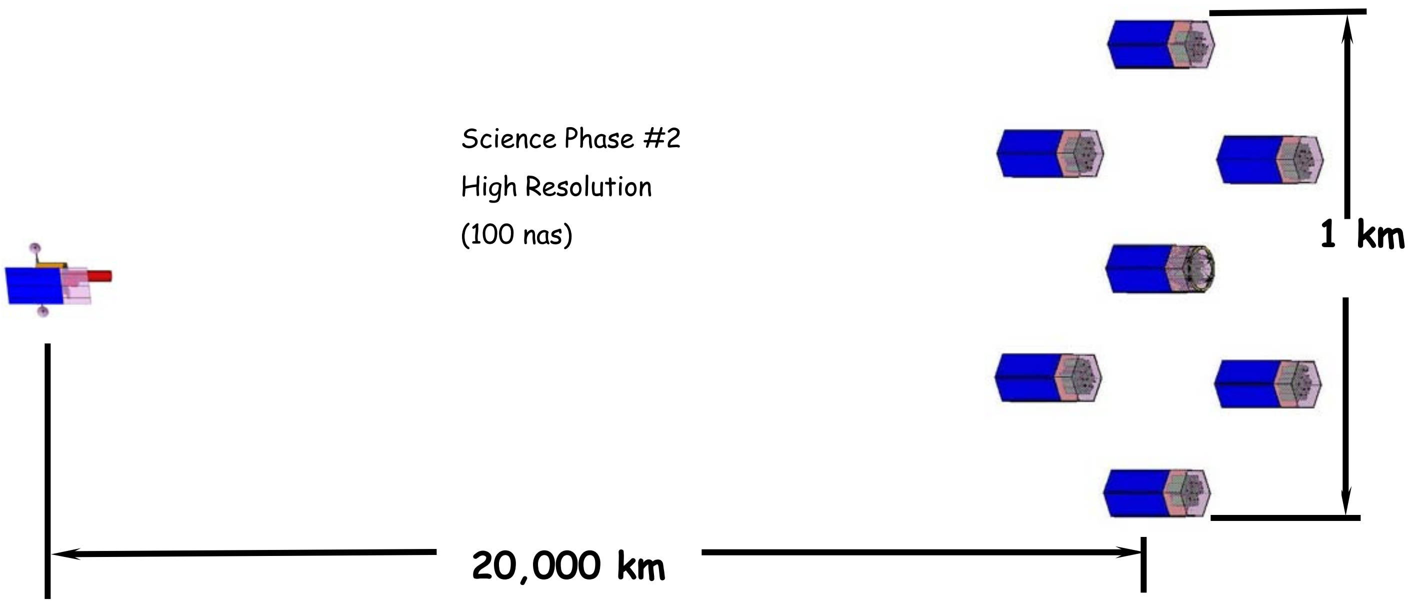
PURDUE UNIVERSITY

Original Maxim Design



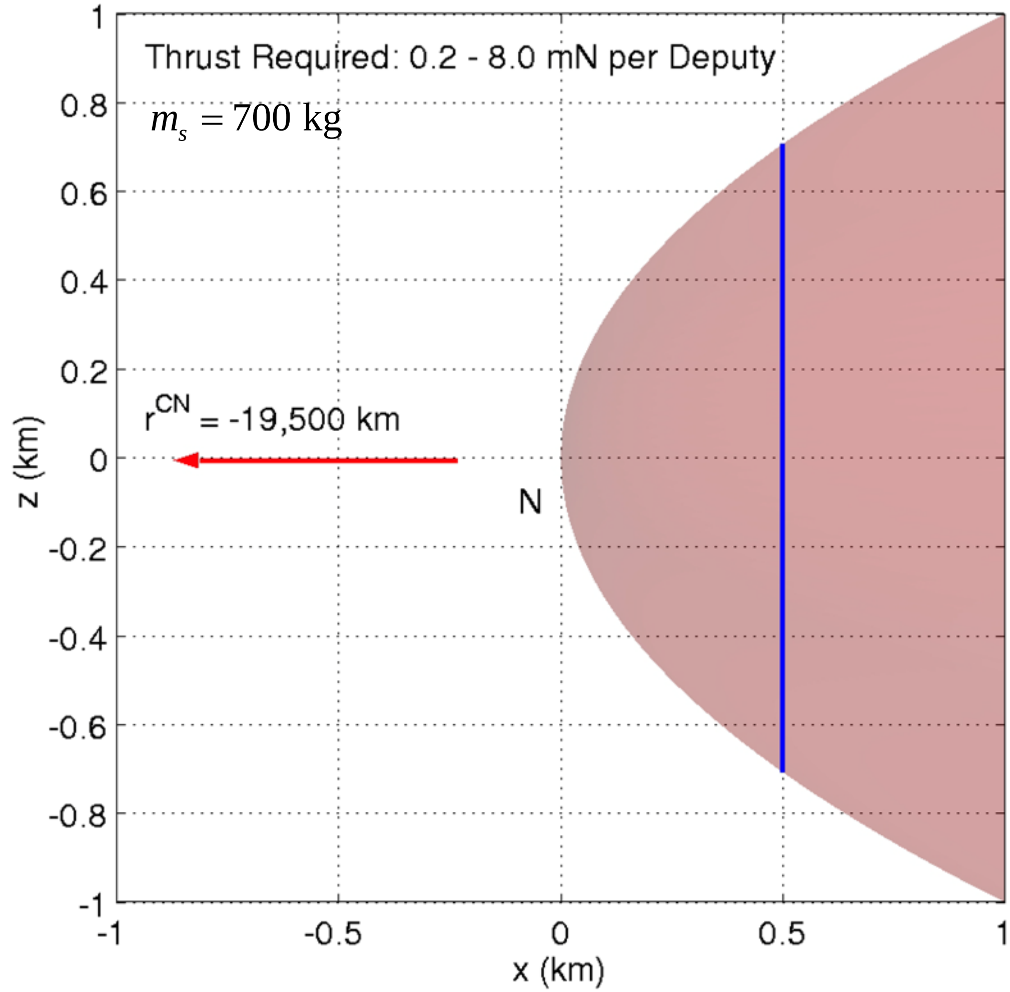
http://maxim.gsfc.nasa.gov/documents/SPIE-2002/spie2002.ppt

New Maxim Pathfinder



http://maxim.gsfc.nasa.gov/documents/SPIE-2002/spie2002.ppt

Maxim Configuration Example



$q = 19{,}500\text{km}$

$\dot{\nu} = 1 \text{ rev/day}$

$u_p = 500 \text{ m}$

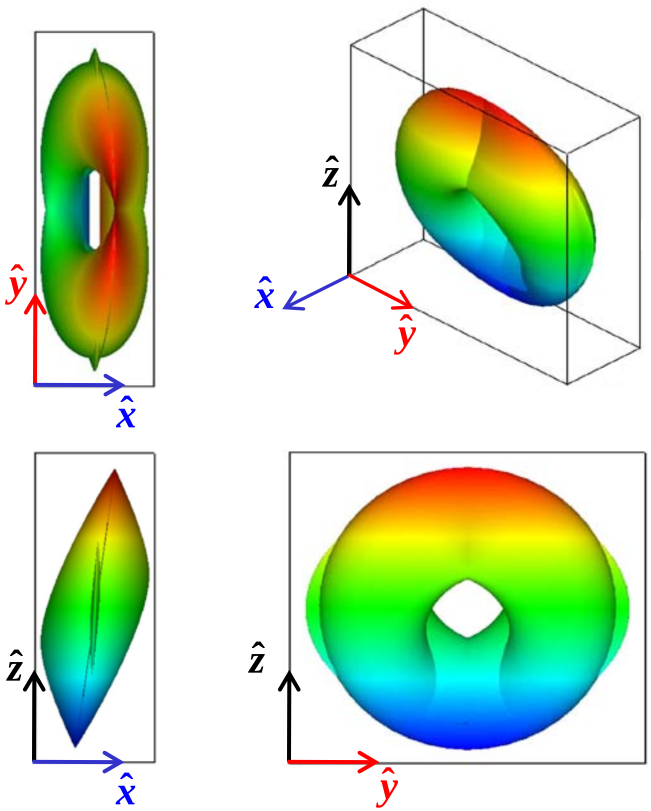
$h_p = 1 \text{ km}$

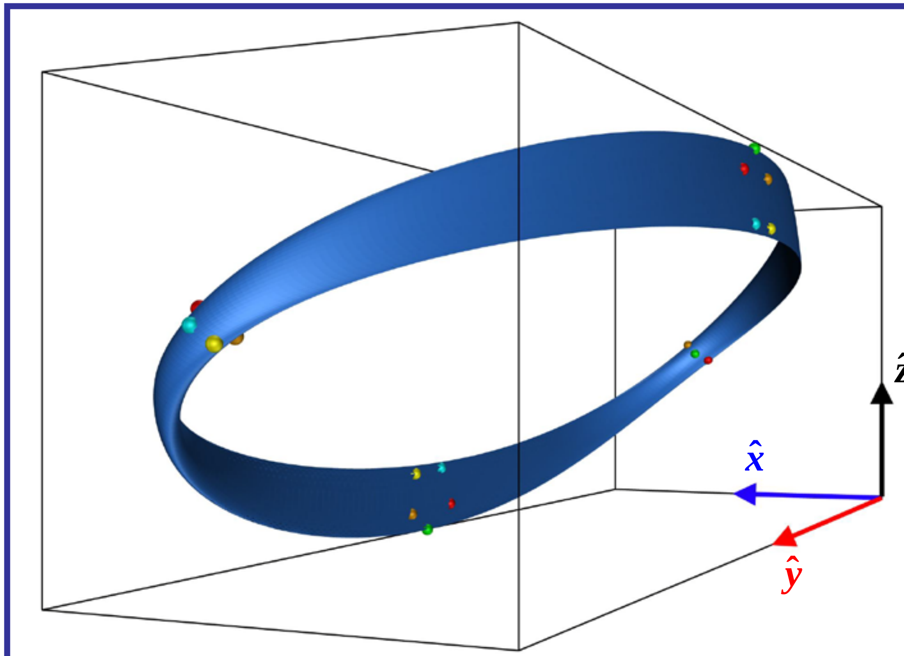
$a_p = 1 \text{ km}$

Building Non-Natural Formations Using Naturally Existing Solutions & Impulsive Control

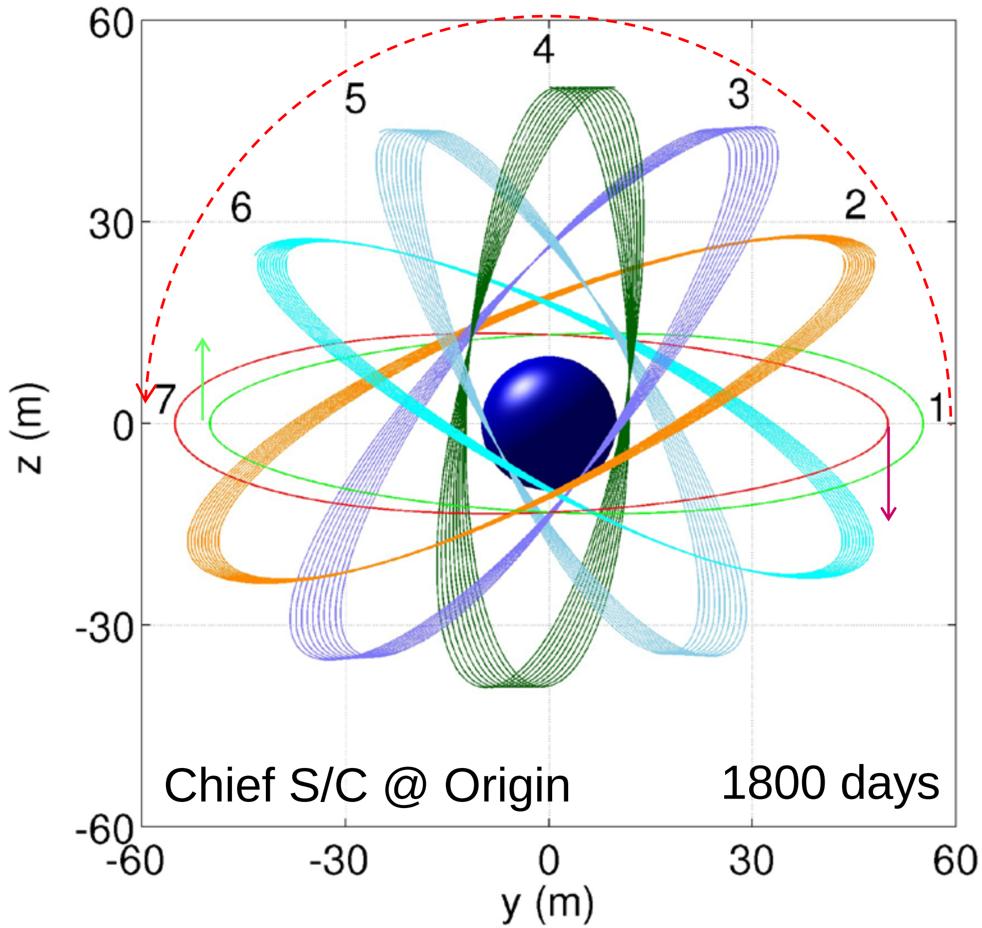
Natural Formations: Quasi-Periodic Relative Orbits → 2-D Torus

Chief S/C Centered View (RLP Frame)

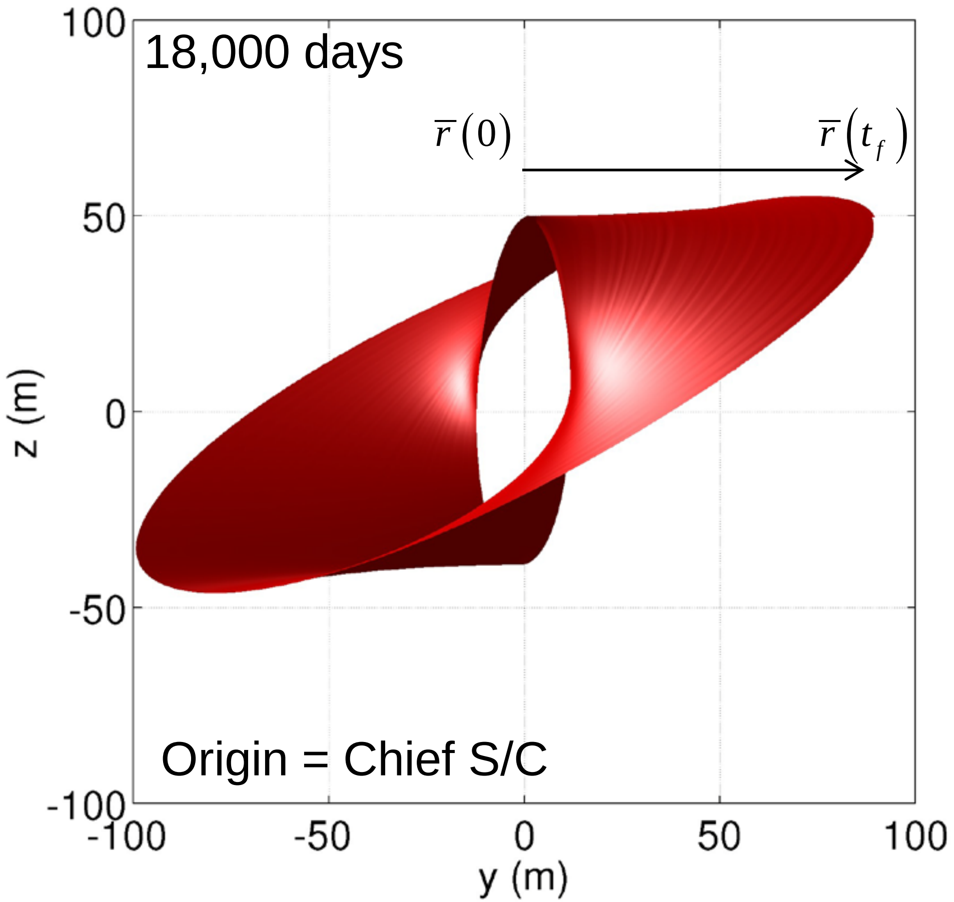




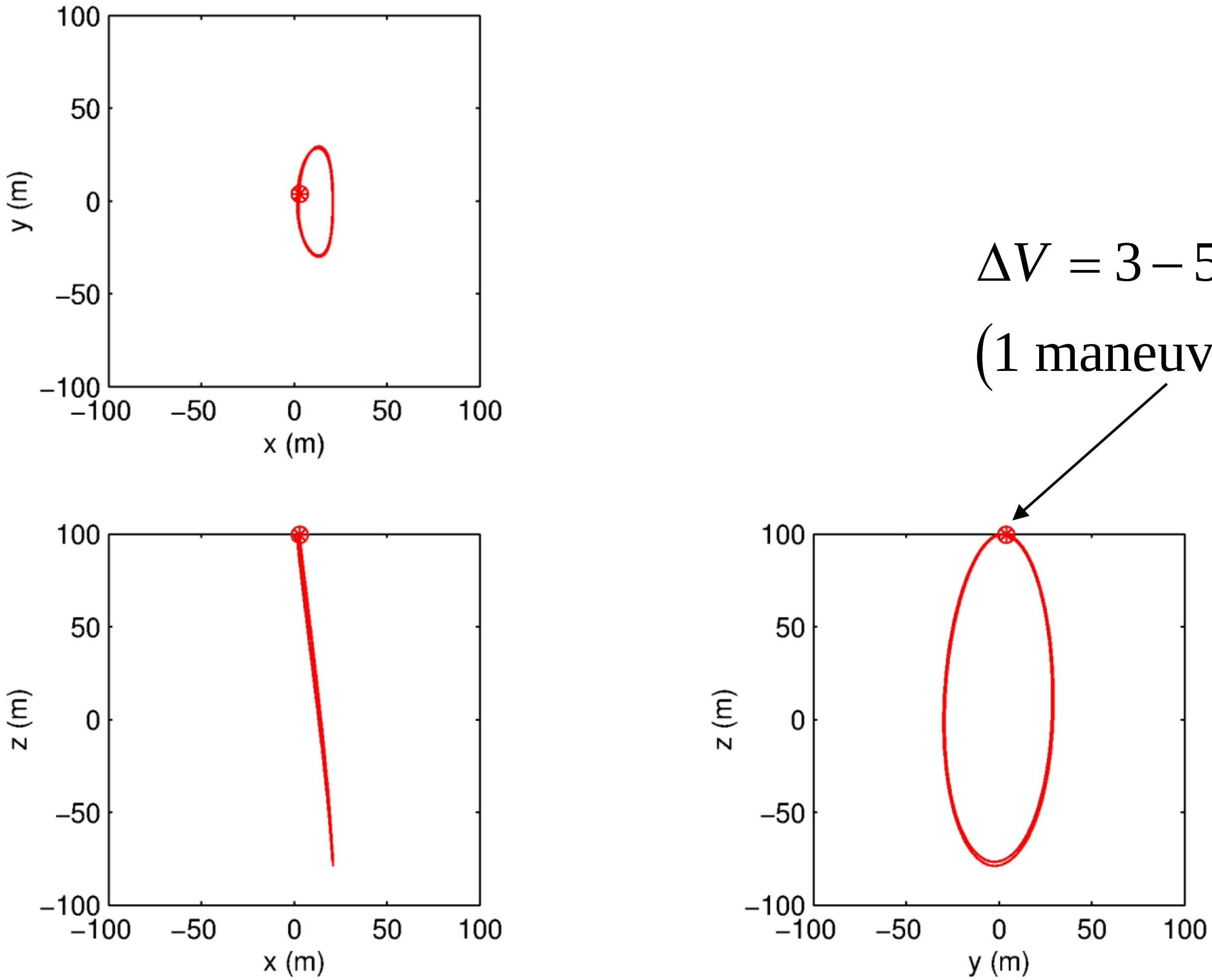
Natural Formations:
Nearly Periodic + Slowly Expanding Orbits



Evolution of Nearly Vertical Orbit Over 100 Orbital Periods



Enforcing Periodicity in the Ephemeris Model

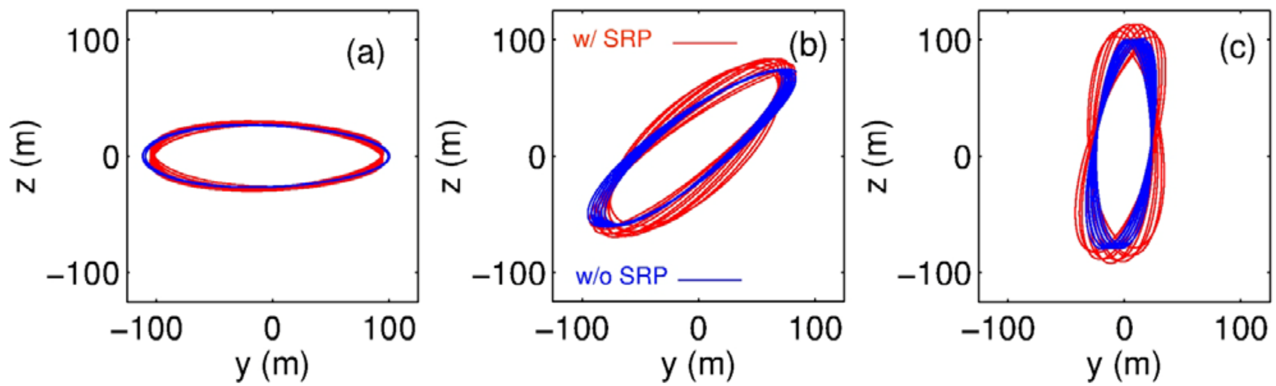


$\Delta V = 3 - 5$ m/sec

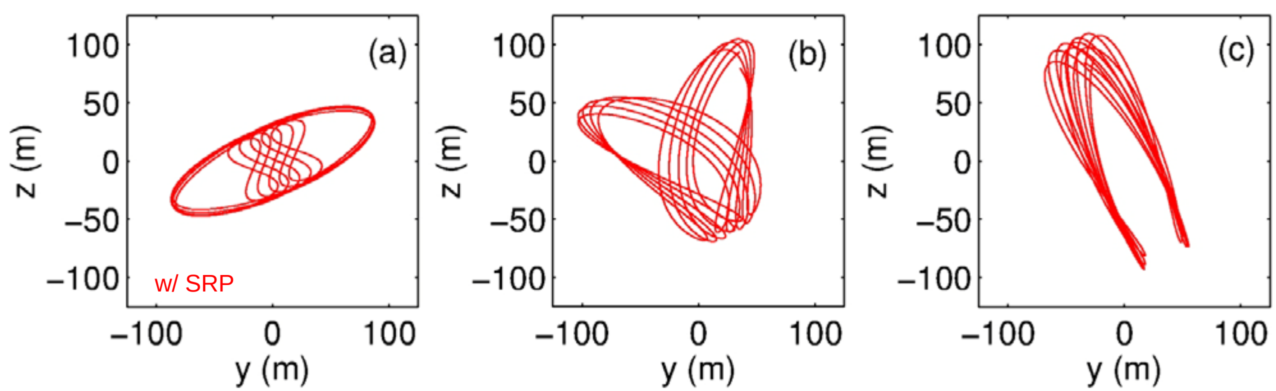
(1 maneuver/year)

Geometry of Natural Solutions in the Ephemeris Model

Rotating Frame Perspective:



Inertial Frame Perspective:



Concluding Remarks

- OFL Control
  - Successful for Formation Keeping & Reconfiguration
    - Spherical + Parabolic Formations
    - ↑ Rotation Rate = ↑ Thrust Level
      - w/o Rotation Rate → Thrust ~ O(nN)
      - w/ Rotation Rate → Thrust ~ O(mN)
- Natural to Non-Natural Formations
  - Differential Corrector → 1 small maneuver/year
    - Can work well in the rotating frame
      - Depends on Impact of SRP
    - Difficult in the inertial frame due to Geometry of Initial Guess