Design of the Onboard Autonomous Targeting Algorithm for the Trans-Earth Phase of Orion

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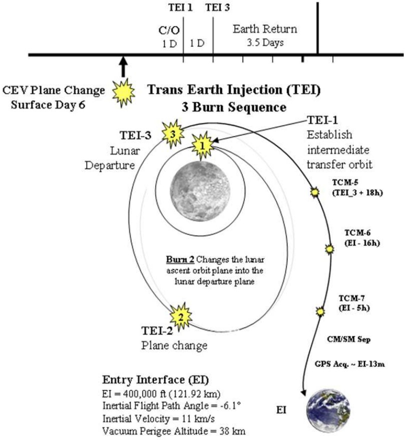
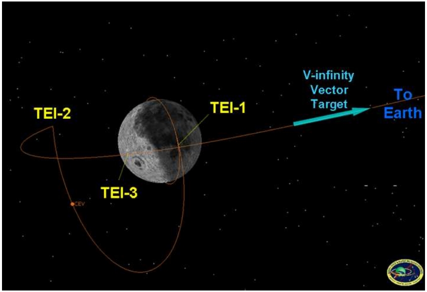
Three dimensional representation of the TEI sequence





Goals of Onboard Targeting Algorithm

- Turn-key autonomous onboard targeting
  - Minimize computational overhead
  - Accurately target entry conditions within available fuel budget
- Address all contingency entry scenarios in case of loss of COMM
- Systematically ID maneuver locations $\Rightarrow$ dynamical sensitivities
- Automated convergence monitoring and adaptive step techniques
- Engine failure contingency $\Rightarrow$ Targeter w/ Finite Burn Model

Targeting Techniques

- Optimization Methods
  - Requires significant computational overhead
  - Many locally optimal arcs exist in 3-Burn TEI scenario
  - Optimization outcome unpredictable

  Optimizers not suitable for autonomous onboard targeting
- Two Level Targeter
  - An enhanced targeting technique for complex dynamical systems
  - Initially applied to trajectory design near the libration points
    - Libration Point Missions: Genesis, Triana, Map, etc.
    - Future Concepts: Formation Flight, L_4/L_5 based telescopes
  - Seeks non-optimal arcs that meet all specified constraints
  - Solution remains in the vicinity of the initial guess
  - Significantly reduced computational overhead relative to NLP

Orion Entry Constraints

- Minimum Entry Constraint Set
  - Altitude (ALT)
  - Flight Path Angle (FPA)
- Additional Constraints for Precision Entry
  - Latitude (LAT)
  - Longitude (LON)
  - Flight Path Azimuth (AZM)
- Fuel budget constraints
  - $\sum_{TEI=1}^{3} ||\Delta V_{TEI}|| < \Delta V_{available}$

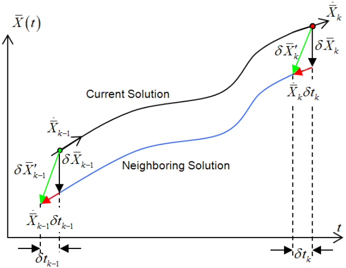
Dynamical Model

- Nonlinear Ephemeris Model (☾, ⊕, ⊙)

$$\dot{\bar{X}}(t) = \bar{f}(\bar{X}) \quad \Rightarrow \quad \text{State Equation} \quad \Rightarrow \quad \delta\bar{X}_k = \Phi(t_k, t_{k-1})\delta\bar{X}_{k-1}$$

$$\bar{y}(t) = \bar{h}\left(\bar{X}\right) \quad \Rightarrow \quad \text{Output Equation} \quad \Rightarrow \quad \delta\bar{y}(t_k) = C(t_k)\delta\bar{X}(t_k)$$

- Linearized Model



$$\underbrace{\begin{bmatrix} \delta\bar{R}'_k - \bar{V}_k^- \delta t_k \\ \delta\bar{V}_k^{-\,\prime} - \bar{a}_k^- \delta t_k \end{bmatrix}}_{\delta\bar{X}_k} = \underbrace{\begin{bmatrix} A_{k,k-1} & B_{k,k-1} \\ C_{k,k-1} & D_{k,k-1} \end{bmatrix}}_{\Phi(t_k, t_{k-1})} \underbrace{\begin{bmatrix} \delta\bar{R}'_{k-1} - \bar{V}_{k-1}^+ \delta t_{k-1} \\ \delta\bar{V}_{k-1}^{+\,\prime} - \bar{a}_{k-1}^+ \delta t_{k-1} \end{bmatrix}}_{\delta\bar{X}_{k-1}}$$

Traditional Level I Targeting

- Terminal State Correction
  - Nullify $\delta \bar{R}_k$ by adjusting initial velocity and TOF

$$\delta \bar{R}_k = \underbrace{\begin{bmatrix} B_{k,k-1} & \bar{V}_k \end{bmatrix}}_{M_p} \underbrace{\begin{bmatrix} \delta \bar{V}_{k-1}^{+} \\ \delta t_k \end{bmatrix}}_{\bar{b}_p}$$

- Terminal Constraint Correction
  - Nullify $\delta \bar{y}_k$ by adjusting initial velocity and TOF

$$\delta \bar{y}_k = C\left(t_k\right) \underbrace{\begin{bmatrix} B_{k,k-1} & \bar{V}_k^{-} \\ D_{k,k-1} & \bar{a}_k^{-} \end{bmatrix}}_{M_c} \underbrace{\begin{bmatrix} \delta \bar{V}_{k-1}^{+} \\ \delta t_k \end{bmatrix}}_{\bar{b}_c}$$

Targeting Entry from TEI-3 via Level I Corrections

Constraint	Initial Error
ALT (km)	5712.2887
FPA (deg)	-34.3866
LAT (deg)	-2.0963
LON (deg)	-75.7495

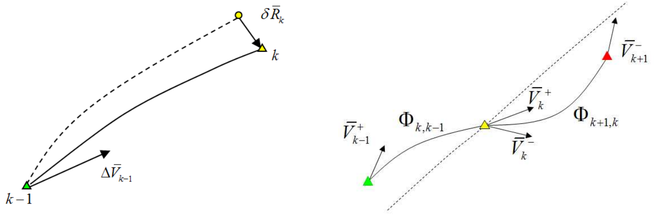
Entry Constraints	TEI-3 ΔV (km/s)	Iterations	Comp. Time (sec)
ALT, FPA	0.8796	14	20.0938
ALT, LAT	0.8704	7	11.4063
ALT, LON	0.8796	7	10.9375
ALT, LAT, LON	0.8796	18	24.5000
ALT, FPA, LAT	0.8796	16	22.7031

Disadvantage of Level I Correction

- Can only efficiently target three constraints at a time
- Initial ΔV (TEI-3) may exceed available budget
- Inconsistent convergence over lunar cycle
- Level I process is extremely sensitive to:
  - The quality of initial state
  - The initial time of flight
  - The Sun-Earth-Moon alignment

Level I correction inadequate for Orion entry targeting from TEI-3

Stylized Representation of Standard Level II Process



(a) (b)

- Segment 1: Forward Propagation

$$\begin{bmatrix} \delta \bar{R}_k' - \bar{V}_k^- \delta t_k \\ \delta \bar{V}_k^{-\,\prime} - \bar{a}_k^- \delta t_k \end{bmatrix} = \begin{bmatrix} A_{k,k-1} & B_{k,k-1} \\ C_{k,k-1} & D_{k,k-1} \end{bmatrix} \begin{bmatrix} \delta \bar{R}_{k-1}' - \bar{V}_{k-1}^+ \delta t_{k-1} \\ \delta \bar{V}_{k-1}^{+\,\prime} - \bar{a}_{k-1}^+ \delta t_{k-1} \end{bmatrix}$$

- Segment 2: Backward Propagation

$$\begin{bmatrix} \delta \bar{R}_k' - \bar{V}_k^+ \delta t_k \\ \delta \bar{V}_k^{+\,\prime} - \bar{a}_k^+ \delta t_k \end{bmatrix} = \begin{bmatrix} A_{k,k+1} & B_{k,k+1} \\ C_{k,k+1} & D_{k,k+1} \end{bmatrix} \begin{bmatrix} \delta \bar{R}_{k+1}' - \bar{V}_{k+1}^- \delta t_{k+1} \\ \delta \bar{V}_{k+1}^{-\,\prime} - \bar{a}_{k+1}^- \delta t_{k+1} \end{bmatrix}$$

Summary of Constrained Level II Process

- Velocity Continuity Constraints $(1 < k < N)$

$$\delta\Delta\bar{V}_k = \underbrace{\begin{bmatrix} \frac{\partial\Delta\bar{V}_k}{\partial\bar{R}_{k-1}} & \frac{\partial\Delta\bar{V}_k}{\partial t_{k-1}} & \frac{\partial\Delta\bar{V}_k}{\partial\bar{R}_k} & \frac{\partial\Delta\bar{V}_k}{\partial t_k} & \frac{\partial\Delta\bar{V}_k}{\partial\bar{R}_{k+1}} & \frac{\partial\Delta\bar{V}_k}{\partial t_{k+1}} \end{bmatrix}}_{M} \underbrace{\begin{bmatrix} \delta\bar{R}_{k-1} \\ \delta t_{k-1} \\ \delta\bar{R}_k \\ \delta t_k \\ \delta\bar{R}_{k+1} \\ \delta t_{k+1} \end{bmatrix}}_{\bar{b}}$$

- Additional Constraints $(\forall\, k \in [1, N])$

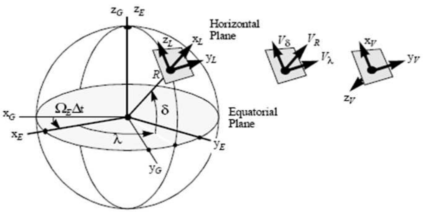
$$\begin{aligned} \delta\alpha_{kj} = {} & \left(\frac{\partial\alpha_{kj}}{\partial\bar{V}_k^-}\frac{\partial\bar{V}_k^-}{\partial\bar{R}_{k-1}}\right)\delta\bar{R}_{k-1} + \left(\frac{\partial\alpha_{kj}}{\partial\bar{V}_k^-}\frac{\partial\bar{V}_k^-}{\partial t_{k-1}}\right)\delta t_{k-1} \\ & + \left(\frac{\partial\alpha_{kj}}{\partial\bar{R}_k} + \frac{\partial\alpha_{kj}}{\partial\bar{V}_k^+}\frac{\partial\bar{V}_k^+}{\delta\bar{R}_k} + \frac{\partial\alpha_{kj}}{\partial\bar{V}_k^-}\frac{\partial\bar{V}_k^-}{\delta\bar{R}_k}\right)\delta\bar{R}_k + \left(\frac{\partial\alpha_{kj}}{\partial t_k} + \frac{\partial\alpha_{kj}}{\partial\bar{V}_k^+}\frac{\partial\bar{V}_k^+}{\partial t_k} + \frac{\partial\alpha_{kj}}{\partial\bar{V}_k^-}\frac{\partial\bar{V}_k^-}{\partial t_k}\right)\delta t_k \\ & + \left(\frac{\partial\alpha_{kj}}{\partial\bar{V}_k^+}\frac{\partial\bar{V}_k^+}{\partial\bar{R}_{k+1}}\right)\delta\bar{R}_{k+1} + \left(\frac{\partial\alpha_{kj}}{\partial\bar{V}_k^+}\frac{\partial\bar{V}_k^+}{\partial t_{k+1}}\right)\delta t_{k+1} \end{aligned}$$

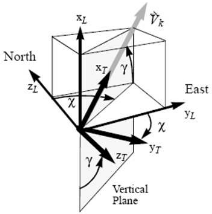
Level II Correction via Minimum Norm Solution

$$\underbrace{\begin{bmatrix} \vdots \\ \delta\Delta\bar{V}_k \\ \vdots \\ \delta\alpha_{kj} \\ \vdots \end{bmatrix}}_{\delta\tilde{\alpha}} = \underbrace{\tilde{M}}_{\frac{\partial\tilde{\alpha}}{\partial\tilde{b}}} \underbrace{\begin{bmatrix} \delta\bar{R}_1 \\ \delta t_1 \\ \vdots \\ \delta\bar{R}_{k-1} \\ \delta t_{k-1} \\ \delta\bar{R}_k \\ \delta t_k \\ \delta\bar{R}_{k+1} \\ \delta t_{k+1} \\ \vdots \\ \delta\bar{R}_N \\ \delta t_N \end{bmatrix}}_{\tilde{b}}$$

$$\tilde{b} = \tilde{M}^T \left(\tilde{M}\tilde{M}^T\right)^{-1} \delta\tilde{\alpha}$$

Longitude, Flight Path Angle and Azimuth



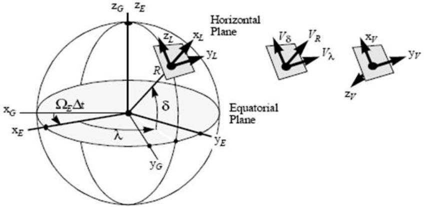


$$\alpha_{k\lambda} = \lambda_k - \lambda_{des} = \tan^{-1}\left(\frac{\bar{R}_k^T \hat{y}_G}{\bar{R}_k^T \hat{x}_G}\right) - \theta_{g0} - \omega_e\left(t_k - t_0\right) - \lambda_{des}$$

$$\alpha_{k\gamma} = \sin\gamma_k - \sin\gamma_{des} = \frac{\bar{R}_k^T \bar{V}_k^-}{\left|\bar{R}_k\right|\left|\bar{V}_k^-\right|} - \sin\gamma_{des}$$

$$\alpha_{k\chi} = \sin\chi_k - \sin\chi_{des} = \frac{\left(V_k^-\right)^T \hat{e}_k}{\sqrt{\left(\left(V_k^-\right)^T \hat{n}_k\right)^2 + \left(\left(V_k^-\right)^T \hat{e}_k\right)^2}} - \sin\chi_{des}$$

Entry Constraints: Coordinate Systems



- $\hat{x}_L = \hat{r}_k \Rightarrow$ radial unit vector
- $\hat{y}_L = \hat{e}_k \Rightarrow$ unit vector due east
- $\hat{z}_L = \hat{n}_k \Rightarrow$ unit vector due north

$$\hat{r}_k = \hat{r}_k\left(\bar{R}_k\right) = \frac{\bar{R}_k}{R_k}$$

$$\hat{e}_k = \hat{e}_k\left(\bar{R}_k\right) = \left(\frac{\hat{z}_G \times \hat{r}_k}{\left|\hat{z}_G \times \hat{r}_k\right|}\right) = \left(\frac{\hat{z}_G \times \bar{R}_k}{\left|\hat{z}_G \times \bar{R}_k\right|}\right)$$

$$\hat{n}_k = \hat{n}_k\left(\bar{R}_k\right) = \hat{r}_k \times \left(\frac{\hat{z}_G \times \bar{R}_k}{\left|\hat{z}_G \times \bar{R}_k\right|}\right) = \frac{1}{\left|\bar{R}_k\right|\left|\hat{z}_G \times \bar{R}_k\right|}\left(\bar{R}_k \times \left(\hat{z}_G \times \bar{R}_k\right)\right)$$

Individual Maneuver Constraint

- ΔV_k Constraint

$$\alpha_{k\Delta V} = \sqrt{\Delta \bar{V}_k^T \Delta \bar{V}_k} - \Delta V_{k_{desired}} = \sqrt{\left(\bar{V}_k^+ - \bar{V}_k^-\right)^T \left(\bar{V}_k^+ - \bar{V}_k^-\right)} - \Delta V_{k_{desired}}$$

- Constraint Partials

$$\frac{\partial \alpha_{k\Delta V}}{\partial \bar{V}_k^+} = \frac{\bar{V}_k^{+T}}{|\bar{V}_k^+|}$$

$$\frac{\partial \alpha_{k\Delta V}}{\partial \bar{V}_k^-} = -\frac{\bar{V}_k^{-T}}{|\bar{V}_k^-|}$$

Total ΔV Constraint

$$\delta\Delta V = \delta\sum_{k=1}^{n_{\Delta V}}\Delta V_k = \sum_{k=1}^{n_{\Delta V}}\delta\Delta V_k$$

- Individual Maneuver Constraint

$$\begin{aligned}\delta\Delta V_k = {} & \frac{\partial\Delta V_k}{\partial\bar{R}_{k-1}}\delta\bar{R}_{k-1} + \frac{\partial\Delta V_k}{\partial t_{k-1}}\delta t_{k-1} \\ & + \frac{\partial\Delta V_k}{\partial\bar{R}_k}\delta\bar{R}_k + \frac{\partial\Delta V_k}{\partial t_k}\delta t_k \\ & + \frac{\partial\Delta V_k}{\partial\bar{R}_{k+1}}\delta\bar{R}_{k+1} + \frac{\partial\Delta V_k}{\partial t_{k+1}}\delta t_{k+1}\end{aligned}$$

- Total ΔV constraint

$$\begin{aligned}\delta\Delta V = \sum_{k=1}^{n_{\Delta V}} & \frac{\partial\Delta V_k}{\partial\bar{R}_{k-1}}\delta\bar{R}_{k-1} + \frac{\partial\Delta V_k}{\partial t_{k-1}}\delta t_{k-1} \\ & + \frac{\partial\Delta V_k}{\partial\bar{R}_k}\delta\bar{R}_k + \frac{\partial\Delta V_k}{\partial t_k}\delta t_k \\ & + \frac{\partial\Delta V_k}{\partial\bar{R}_{k+1}}\delta\bar{R}_{k+1} + \frac{\partial\Delta V_k}{\partial t_{k+1}}\delta t_{k+1}\end{aligned}$$

Initial Guess

- Identifying a suitable end-to-end initial guess not trivial
- n-Body sensitivities impact success of TEI-3 for precision entry
- TEI ΔV's relative location and timing impact targeting success
- A 3-burn TEI not always the best option
- Convergence of any iterative process is affected by
  - An initial ΔV that significantly exceeds the budget
  - Large position discontinuities in the startup arc
  - Significant constraint violations at entry $\Rightarrow$ constraint coupling

Initial Guess Generation

- Employ 2-Body ☾-centered Approx. for first two TEI maneuvers
- Propagate results in ⊙-⊕-☾ model up to the TEI-3 point
  - Method 1 (M1): Use level I from TEI-3 to entry
    - Limited success provided no. of constraints is less than 3
    - Resulting initial guess is feasible
    - Resulting ΔV can significantly exceed available budget
    - Convergence is not consistent enough for onboard determination
  - Method 2 (M2): Use level II targeter from TEI-3 to entry
    - Resulting initial guess is feasible and meets all entry constraints
    - Significantly reduces the overall computational time required

Summary of the 2-level Targeter Results

- Initial Guess Information (ALT/LAT/LON)

Case	ΔV (km/s)	Iter.	Time(s)
1	1.16	9	134.5
2	1.16	9	134.5
3	1.21	4	32.6
4	1.28	0	14.3
5	2.3	25	364

- Sample Level II Corrector Results

Active Constraints	Iter.	Time(s)	ΔV_1	ΔV_2	ΔV_3	ΔV_4	ΔV
M3: h, ϕ, θ, γ, $\Delta V \leq 0.9$ (km/s)	10	4 m 4.5 s	.609	.044	.248	0	.900
M3: h, ϕ, θ, γ	4	2 m 9.9 s	.608	.057	.460	.105	.124
M3: h, γ	3	1 m 31.4 s	.608	.057	.545	0	.122
M2: h, γ	13	6 m 11.2 s	.603	.106	.541	0	.125
M3: h, ϕ, θ, γ, Az, $\Delta V \leq 1.35$ (km/s)	15	6 m 11.0 s	.629	.071	.389	.261	.135

Level I, Level II, and Optimal Solutions Over the Lunar Cycle

- Target: Altitude, Latitude, Longitude, Total ΔV
- M2 (Level I) Initial Guess Only

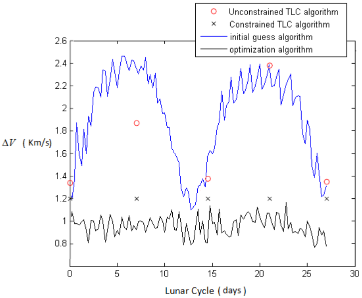


Table: ΔV Values for Unconstrained L-II (km/s)

Cycle Day	TEI-1	TEI-2	TEI-3	Total
0	0.6078	0.0580	0.6716	1.3374
7	0.6098	0.1190	1.1396	1.8684
14.5	0.6081	0.0599	0.7091	1.3771
21	0.6080	0.0244	1.7496	2.3820
27	0.6078	0.0579	0.6887	1.3544

Table: ΔV Values for Constrained L-II (km/s)

Cycle Day	TEI-1	TEI-2	TEI-3	Total
0	0.6075	0.0594	0.5330	1.20
7	0.6055	0.1082	0.4863	1.20
14.5	0.6076	0.0730	0.5195	1.20
21	0.6076	0.1075	0.4849	1.20
27	0.6076	0.0525	0.5398	1.20

Performance Comparison: 2-level Targeter vs. NLP Algorithm

TLC Case No.	2-Level	NLP
M3: h, ϕ, θ, γ, $\Delta V \leq 0.9$ (m/s)	6 min 19 sec	14 min 48 sec
M3: h, ϕ, θ, γ, Az, $\Delta V \leq 1.35$ (m/s)	12 min 15 sec	22 min 11 sec

Incorporating Finite Burns in a Level II Process

Process requires augmented state and variational equations

- Level I Targeter
  - Fixed parameters: Initial State, TOF, Propellant Flow Rate
  - Control variables: Thrust direction and Burn duration
- Level II Targeter
  - Control variables: Position and time of all patch states
  - Partial derivatives of $\bar{V}^+$ and $\bar{V}^-$ change
  - Constraint Partials are unchanged relative to Impulsive Targeter
  - Total Cost Constraint
    - Different ΔV equation and control variables

$$\Delta V_k = -I_{sp} g_0 \ln(1 - \frac{\dot{m}_{g_k} \Delta t_{burn}}{m_k})$$

    - Associated explicit constraint partials must be reformulated
    - $\sum_{TEI=1}^{3} \Delta V_{TEI}$ variational equation still applicable

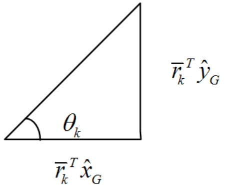
Conclusions

- Constraints for LON, AZ, and ΔV developed and tested
- Traditional L1 targeting alone not suitable for Orion TEI
- 2-Level corrector $\Rightarrow$ efficient and viable option for TEI targeting
- Superior computational performance relative to NLP process
- Computational improvements with enhanced IG algorithm
- Startup arc algorithm should account for constraint coupling
- Finite burn targeter developed and successfully tested

Backup Slides

Longitude Constraint Geometry

$$\cos\lambda_k = \cos(\theta_k - \theta_{g0} - \omega_e \Delta t_k) = \cos\theta_k \cos(\theta_{g0} + \omega_e \Delta t) + \sin\theta_k \sin(\theta_{g0} + \omega_e \Delta t)$$



$$\cos\theta_k = \frac{\bar{R}_k^T \hat{x}_G}{\sqrt{\left(\bar{R}_k^T \hat{x}_G\right)^2 + \left(\bar{R}_k^T \hat{y}_G\right)^2}}$$

$$\sin\theta_k = \frac{\bar{R}_k^T \hat{y}_G}{\sqrt{\left(\bar{R}_k^T \hat{x}_G\right)^2 + \left(\bar{R}_k^T \hat{y}_G\right)^2}}$$

Longitude Constraint Partials

- Longitude Partials

$$\frac{\partial \cos \lambda_k}{\partial \bar{R}_k} = \frac{\partial \cos \theta_k}{\partial \bar{R}_k} \cos\left(\theta_{g0} + \omega_e \Delta t\right) + \frac{\partial \sin \theta_k}{\partial \bar{R}_k} \sin\left(\theta_{g0} + \omega_e \Delta t\right)$$

$$\frac{\partial \cos \lambda_k}{\partial t_k} = \omega_e \sin\left(\theta_k - \theta_{g0} - \omega_e \Delta t\right)$$

$$\frac{\partial \cos \lambda_k}{\partial t_k} = -\omega_e \cos \theta_k \sin\left(\theta_{g0} + \omega_e \Delta t\right) + \omega_e \sin \theta_k \cos\left(\theta_{g0} + \omega_e \Delta t\right)$$

- Right Ascension Partials

$$\frac{\partial \cos \theta_k}{\partial \bar{R}_k} = \frac{\hat{x}_G^T \sqrt{\left(\bar{R}_k^T \hat{x}_G\right)^2 + \left(\bar{R}_k^T \hat{y}_G\right)^2} - \left(\bar{R}_k^T \hat{x}_G\right) \frac{\left(\left(\bar{R}_k^T \hat{x}_G\right)\hat{x}_G^T + \left(\bar{R}_k^T \hat{y}_G\right)\hat{y}_G^T\right)}{\sqrt{\left(\bar{R}_k^T \hat{x}_G\right)^2 + \left(\bar{R}_k^T \hat{y}_G\right)^2}}}{\left(\bar{R}_k^T \hat{x}_G\right)^2 + \left(\bar{R}_k^T \hat{y}_G\right)^2}$$

$$\frac{\partial \sin \theta_k}{\partial \bar{R}_k} = \frac{\hat{y}_G^T \sqrt{\left(\bar{R}_k^T \hat{x}_G\right)^2 + \left(\bar{R}_k^T \hat{y}_G\right)^2} - \left(\bar{R}_k^T \hat{y}_G\right) \frac{\left(\left(\bar{R}_k^T \hat{x}_G\right)\hat{x}_G^T + \left(\bar{R}_k^T \hat{y}_G\right)\hat{y}_G^T\right)}{\sqrt{\left(\bar{R}_k^T \hat{x}_G\right)^2 + \left(\bar{R}_k^T \hat{y}_G\right)^2}}}{\left(\bar{R}_k^T \hat{x}_G\right)^2 + \left(\bar{R}_k^T \hat{y}_G\right)^2}$$

Flight Path Azimuth Constraint Partials

$$\frac{\partial \sin \chi_k}{\partial \bar{R}_k} = \frac{\left\{ \begin{array}{l} \left(\left(\bar{V}_k^-\right)^T \frac{\partial \hat{e}_k}{\partial \bar{R}_k}\right) \sqrt{\left(\left(\bar{V}_k^-\right)^T \hat{e}_k\right)^2 + \left(\left(\bar{V}_k^-\right)^T \hat{n}_k\right)^2} \\ - \frac{\left(\left(\left(\bar{V}_k^-\right)^T \hat{e}_k\right)\left(\left(\bar{V}_k^-\right)^T \frac{\partial \hat{e}_k}{\partial \bar{R}_k}\right) + \left(\left(\bar{V}_k^-\right)^T \hat{n}_k\right)\left(\left(\bar{V}_k^-\right)^T \frac{\partial \hat{n}_k}{\partial \bar{R}_k}\right)\right)\left(\left(\bar{V}_k^-\right)^T \hat{e}_k\right)}{\sqrt{\left(\left(\bar{V}_k^-\right)^T \hat{e}_k\right)^2 + \left(\left(\bar{V}_k^-\right)^T \hat{n}_k\right)^2}} \end{array} \right\}}{\left(\left(\bar{V}_k^-\right)^T \hat{e}_k\right)^2 + \left(\left(\bar{V}_k^-\right)^T \hat{n}_k\right)^2}$$

$$\frac{\partial \left(\sin \chi_k\right)}{\partial \bar{V}_k^-} = \frac{\left(\hat{e}_k^T\right) \sqrt{\left(\left(V_k^-\right)^T \hat{n}_k\right)^2 + \left(\left(V_k^-\right)^T \hat{e}_k\right)^2} - \left[\left(V_k^-\right)^T \hat{e}_k\right] \left[\frac{\left(\left(V_k^-\right)^T \hat{n}_k\right) \hat{n}_k^T + \left(\left(V_k^-\right)^T \hat{e}_k\right) \hat{e}_k^T}{\sqrt{\left(\left(V_k^-\right)^T \hat{n}_k\right)^2 + \left(\left(V_k^-\right)^T \hat{e}_k\right)^2}} \right]}{\left(\left(V_k^-\right)^T \hat{n}_k\right)^2 + \left(\left(V_k^-\right)^T \hat{e}_k\right)^2}$$

Coordinate Frame Partials

$$\frac{\partial \hat{e}_k}{\partial \bar{R}_k} = \left(I - \frac{1}{\left| Z_\times \bar{R}_k \right|^2} \left(Z_\times \bar{R}_k \right) \left(Z_\times \bar{R}_k \right)^T \right) \frac{Z_\times}{\left| Z_\times \bar{R}_k \right|}$$

$$\frac{\partial \hat{n}}{\partial \bar{R}_k} = \frac{1}{\left|\bar{R}_k\right|^2 \left| Z_\times \bar{R}_k \right|^2} \left\{ \begin{array}{l} \left|\bar{R}_k\right| \left| Z_\times \bar{R}_k \right| \left[2 \hat{z}_G \bar{R}_k^T - \bar{R}_k \hat{z}_G^T - \left(\bar{R}_k^T \hat{z}_G \right) I \right] \\ - \left(\hat{z}_G \left(\bar{R}_k^T \bar{R}_k \right) - \bar{R}_k \left(\bar{R}_k^T \hat{z}_G \right) \right) \left[\left(\frac{\left| Z_\times \bar{R}_k \right|}{\left|\bar{R}_k\right|} \right) \bar{R}_k^T + \frac{\left|\bar{R}_k\right|}{\left| Z_\times \bar{R}_k \right|} \left(Z_\times \bar{R}_k \right)^T Z_\times \right] \end{array} \right\}$$

$$Z_\times = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Finite Burn Targeter: Variational Equations

- Burn subsegment
  - Augment state equations: $\dot{m} = -\dot{m}_g$, $\ddot{m}_g = 0$, $\dot{\bar{u}} = \bar{0}$
  - Augment variational equations

$$\begin{bmatrix} \delta\bar{R}_T - \bar{V}_T^- \delta t_T \\ \delta\bar{V}_T^- - \bar{a}_T^- \delta t_T \\ \delta m_T^- + \dot{m}_{g_T}^- \delta t_T \\ \delta\dot{m}_{g_T}^- - \ddot{m}_{g_T}^- \delta t_T \\ \delta\bar{u}_T^- - \dot{\bar{u}}_T^- \delta t_T \end{bmatrix} =$$

$$\begin{bmatrix} A_{T,k-1} & B_{T,k-1} & E_{T,k-1} & F_{T,k-1} & G_{T,k-1} \\ C_{T,k-1} & D_{T,k-1} & H_{T,k-1} & I_{T,k-1} & J_{T,k-1} \\ K_{T,k-1} & L_{T,k-1} & M_{T,k-1} & N_{T,k-1} & O_{T,k-1} \\ P_{T,k-1} & Q_{T,k-1} & R_{T,k-1} & S_{T,k-1} & T_{T,k-1} \\ U_{T,k-1} & V_{T,k-1} & W_{T,k-1} & X_{T,k-1} & Y_{T,k-1} \end{bmatrix} \begin{bmatrix} \delta\bar{R}_{k-1} - \bar{V}_{k-1}^+ \delta t_{k-1} \\ \delta\bar{V}_{k-1}^+ - \bar{a}_{k-1}^+ \delta t_{k-1} \\ \delta m_{k-1}^+ + \dot{m}_{g_{k-1}}^+ \delta t_{k-1} \\ \delta\dot{m}_{g_{k-1}}^+ - \ddot{m}_{g_{k-1}}^+ \delta t_{k-1} \\ \delta\bar{u}_{k-1}^+ - \dot{\bar{u}}_{k-1}^+ \delta t_{k-1} \end{bmatrix}$$

- Coast subsegment

$$\begin{bmatrix} \delta\bar{R}_k - \bar{V}_k^- \delta t_k \\ \delta\bar{V}_k^- - \bar{a}_k^- \delta t_k \end{bmatrix} = \begin{bmatrix} A_{k,T} & B_{k,T} \\ C_{k,T} & D_{k,T} \end{bmatrix} \begin{bmatrix} \delta\bar{R}_T - \bar{V}_T^+ \delta t_T \\ \delta\bar{V}_T^+ - \bar{a}_T^+ \delta t_T \end{bmatrix}$$

Level I Targeting: Finite Burn

- 1 segment = 1 thrust sub-arc + 1 coast sub-arc
- Fixed Initial parameters:
  - $\bar{X}_{k-1}$, initial state vector
  - $\dot{m}_{g_k}$, propellant mass flow rate over burn arc
  - t_k, total time-of-flight
- Control variables
  - $\bar{u}_{k-1}$, inertial thrust direction (assumed constant over burn arc)
  - t_T, duration of burn sub-segment
- Target quantities
  - $\bar{R}_k$, inertial position vector

Level I Finite Burn Correction

- Position Variational Equation

$$\delta\bar{R}_k = \underbrace{\begin{bmatrix} (A_{k,T}G_{T,k-1} + B_{k,T}J_{T,k-1}) & (A_{k,T}\bar{V}_T^+ + B_{k,T}\bar{a}_T^+) \end{bmatrix}}_{M_{f_b}} \begin{bmatrix} \delta\bar{u}_{k-1}^+ \\ \delta t_T \end{bmatrix}$$

- Suggested Correction (Minimum Norm Solution)

$$\begin{bmatrix} \delta\bar{u}_{k-1}^+ \\ \delta t_T \end{bmatrix} = \tilde{M_{f_b}}^T (\tilde{M_{f_b}} \tilde{M_{f_b}}^T)^{-1} \delta\bar{R}_k$$

Level II Finite Burn Correction

- Finite burn and impulsive Level II correction process is similar
  - Control variables are the position and time of each patch state
  - Explicit constraint partials are unchanged
  - Only velocity partials, $\bar{V}^+$ and $\bar{V}^-$, are affected due to modified variational equations. Otherwise, the process of identifying these partials is exactly the same as that employed in the impulsive targeter
- Total Cost Constraint
  - Different ΔV equation and control variables

$$\Delta V_k = -I_{sp} g_0 ln(1 - \frac{\dot{m}_{g_k} \Delta t_{burn}}{m_k}) \quad (1)$$

    for $\Delta t_{burn} = t_T - t_k$.
  - $\sum_{TEI=1}^{3} \Delta V_{TEI}$ variational equation still applicable
  - Associated explicit constraint partials must be reformulated

Example: Finite Burn Level II Targeter

- Targets: ALT, LAT, LON, FPA, $\Delta V < 1.2$ km/sec

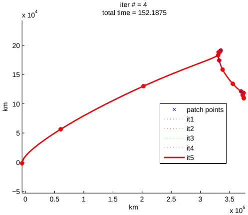


Figure: (a) - Initial Guess 1

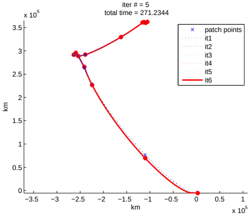


Figure: (b) - Initial Guess 2

Burn	Figure (a)	Figure (b)
TEI-1	302.3305 s	324.5929 s
TEI-2	74.7421 s	27.2148 s
TEI-3	159.6147 s	118.6056 s
TEI-4	N/A	50.9205 s

- Fig (a): ΔV of 1.1108 km/s, 4 iterations, 152.1875 sec CPU Time
- Fig (b): ΔV of 1.0731 km/s, 5 iterations, 271.2344 sec CPU Time