An Autonomous Onboard Targeting Algorithm using Finite Thrust Maneuvers

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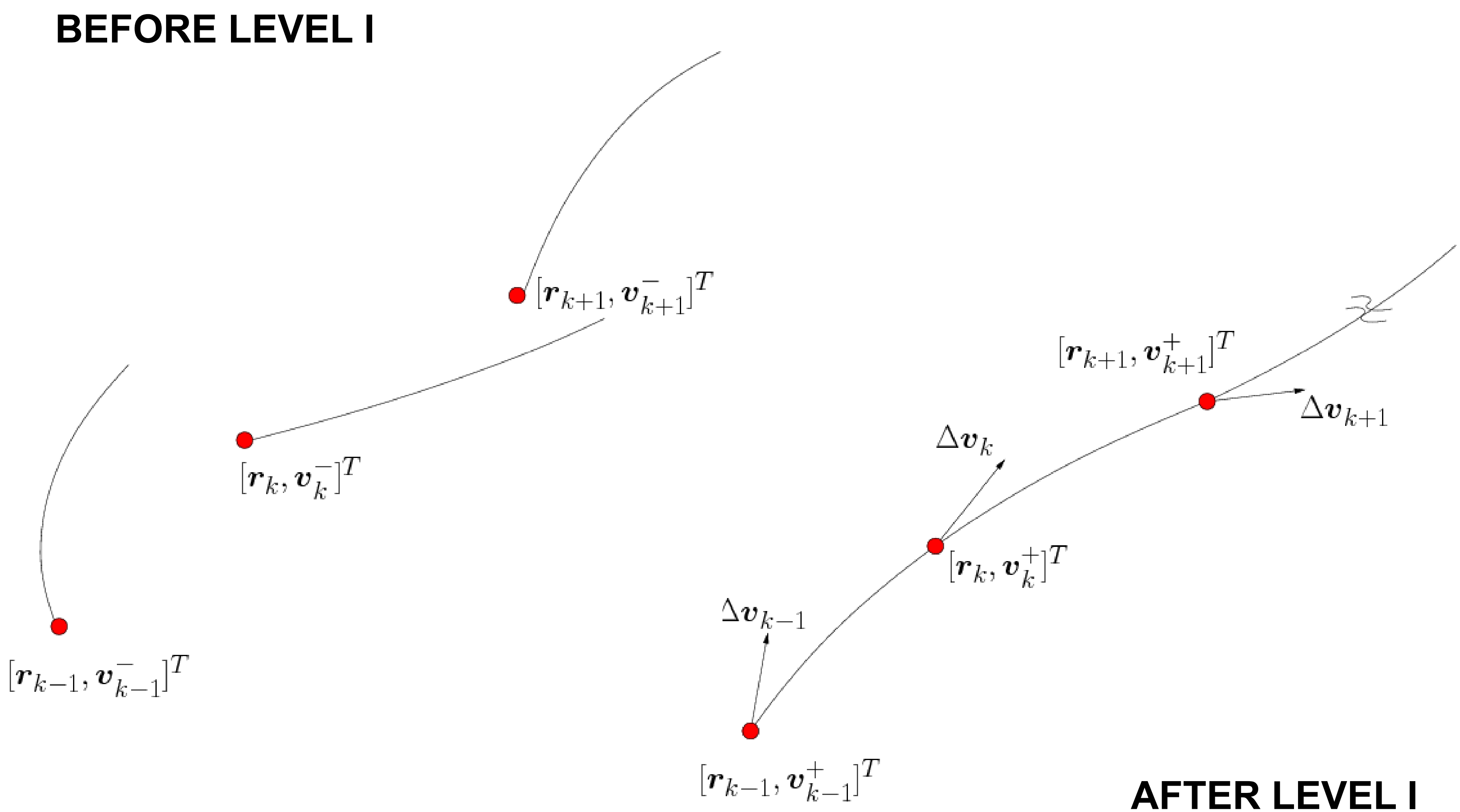
Introduction

- Onboard guidance for Orion lunar return
- Two-level targeting algorithm
  - Based on linear system theory
  - Designed for impulsive maneuvers
- In a main engine failure scenario, impulsive approximation invalid
- Adapt two-level targeter to incorporate finite burns while retaining its simplicity

Classical Impulsive Level I Process

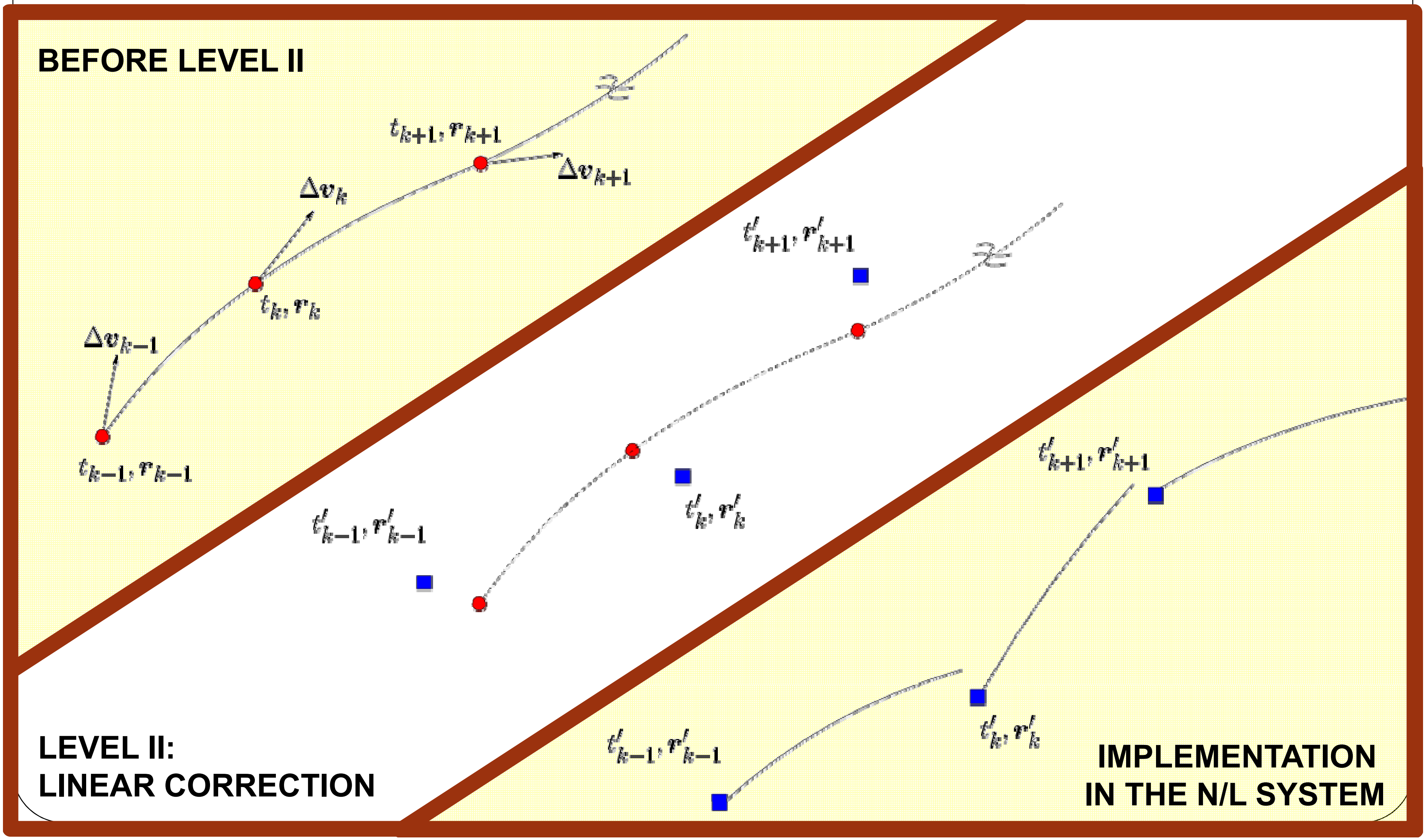
Goal: Position Continuity Only   Control Variables: ΔV's



Classical Level II Process:

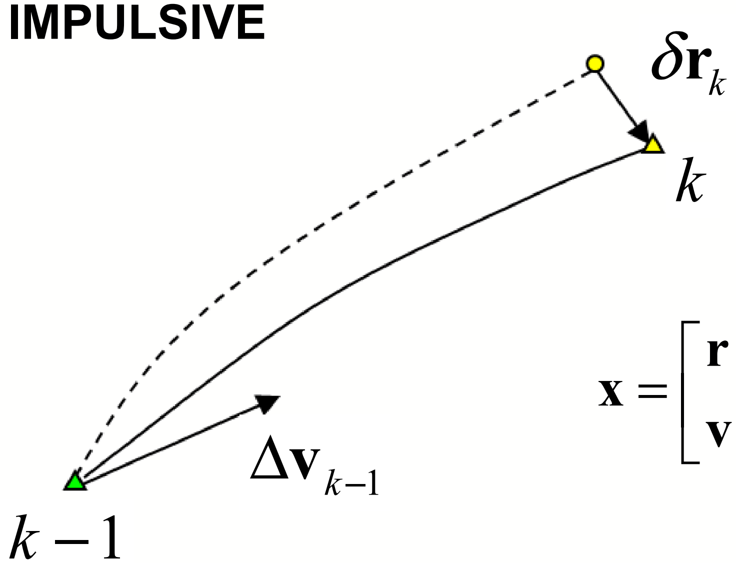
Goal: Meet Specified Constraints (e.g. Velocity Continuity),

Control Variables: Time & Position of Patch States



Level 1: Impulsive vs. Finite Burn

IMPULSIVE

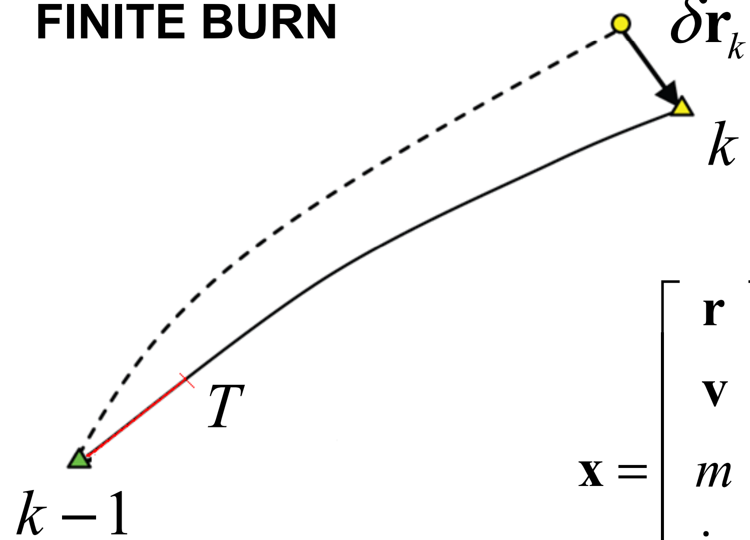


$$\mathbf{x} = \begin{bmatrix} \mathbf{r} \\ \mathbf{v} \end{bmatrix}_{6\times1}$$

Constraint: $\delta\mathbf{r}_k = \mathbf{0}$

Control Variables: $\Delta\mathbf{v}_{k-1}$

FINITE BURN



$$\mathbf{x} = \begin{bmatrix} \mathbf{r} \\ \mathbf{v} \\ m \\ \dot{m}_g \\ \mathbf{u} \end{bmatrix}_{11\times1}$$

Constraint: $\delta\mathbf{r}_k = \mathbf{0}$

Control Variables: $\mathbf{u}_{k-1}^{+}, \delta t_T$

Variational Equations: Impulsive vs. Finite Burn

IMPULSIVE

$$\begin{bmatrix} \delta\mathbf{r}_k - \mathbf{v}_k^- \delta t_k \\ \delta\mathbf{v}_k^- - \mathbf{a}_k^- \delta t_k \end{bmatrix} = \begin{bmatrix} A_{k,k-1} & B_{k,k-1} \\ C_{k,k-1} & D_{k,k-1} \end{bmatrix} \begin{bmatrix} \delta\mathbf{r}_{k-1} - \mathbf{v}_{k-1}^+ \delta t_{k-1} \\ \delta\mathbf{v}_{k-1}^+ - \mathbf{a}_{k-1}^+ \delta t_{k-1} \end{bmatrix}$$

$$\begin{bmatrix} \delta\mathbf{r}_k - \mathbf{v}_k^+ \delta t_k \\ \delta\mathbf{v}_k^+ - \mathbf{a}_k^+ \delta t_k \end{bmatrix} = \begin{bmatrix} A_{k,k+1} & B_{k,k+1} \\ C_{k,k+1} & D_{k,k+1} \end{bmatrix} \begin{bmatrix} \delta\mathbf{r}_{k+1} - \mathbf{v}_{k+1}^- \delta t_{k+1} \\ \delta\mathbf{v}_{k+1}^- - \mathbf{a}_{k+1}^- \delta t_{k+1} \end{bmatrix}$$

FINITE BURN

$$\begin{bmatrix} \delta\mathbf{r}_T - \mathbf{v}_T^- \delta t_T \\ \delta\mathbf{v}_T^- - \mathbf{a}_T^- \delta t_T \\ \delta m_T^- + \dot{m}_{g_T}^- \delta t_T \\ \delta \dot{m}_{g_T}^- - \ddot{m}_{g_T}^- \delta t_T \\ \delta\mathbf{u}_T^- - \dot{\mathbf{u}}_{g_T}^- \delta t_T \end{bmatrix} = \begin{bmatrix} A_{T,k-1} & B_{T,k-1} & E_{T,k-1} & F_{T,k-1} & G_{T,k-1} \\ C_{T,k-1} & D_{T,k-1} & H_{T,k-1} & I_{T,k-1} & J_{T,k-1} \\ K_{T,k-1} & L_{T,k-1} & M_{T,k-1} & N_{T,k-1} & O_{T,k-1} \\ P_{T,k-1} & Q_{T,k-1} & R_{T,k-1} & S_{T,k-1} & T_{T,k-1} \\ U_{T,k-1} & V_{T,k-1} & W_{T,k-1} & X_{T,k-1} & Y_{T,k-1} \end{bmatrix} \begin{bmatrix} \delta\mathbf{r}_{k-1} - \mathbf{v}_{k-1}^+ \delta t_{k-1} \\ \delta\mathbf{v}_{k-1}^+ - \mathbf{a}_{k-1}^+ \delta t_{k-1} \\ \delta m_{k-1}^+ + \dot{m}_{g_{k-1}}^+ \delta t_{k-1} \\ \delta \dot{m}_{g_{k-1}}^+ - \ddot{m}_{g_{k-1}}^+ \delta t_{k-1} \\ \delta\mathbf{u}_{k-1}^+ - \dot{\mathbf{u}}_{k-1}^+ \delta t_{k-1} \end{bmatrix}$$

$$\begin{bmatrix} \delta\mathbf{r}_k - \mathbf{v}_k^- \delta t_k \\ \delta\mathbf{v}_k^- - \mathbf{a}_k^- \delta t_k \end{bmatrix} = \begin{bmatrix} A_{k,T} & B_{k,T} \\ C_{k,T} & D_{k,T} \end{bmatrix} \begin{bmatrix} \delta\mathbf{r}_T - \mathbf{v}_T^+ \delta t_T \\ \delta\mathbf{v}_T^+ - \mathbf{a}_T^+ \delta t_T \end{bmatrix}$$

$$\begin{bmatrix} \delta\mathbf{r}_k - \mathbf{v}_k^+ \delta t_k \\ \delta\mathbf{v}_k^+ - \mathbf{a}_k^+ \delta t_k \end{bmatrix} = \begin{bmatrix} A_{k,k+1} & B_{k,k+1} \\ C_{k,k+1} & D_{k,k+1} \end{bmatrix} \begin{bmatrix} \delta\mathbf{r}_{k+1} - \mathbf{v}_{k+1}^- \delta t_{k+1} \\ \delta\mathbf{v}_{k+1}^- - \mathbf{a}_{k+1}^- \delta t_{k+1} \end{bmatrix}$$

Level 1 Targeting

- Direct from TEI-3 to Earth entry
- Entry targets:
  - Geodetic Altitude (km) 121.92
  - Longitude (deg) 175.6365
  - Geocentric Azimuth (deg) 49.3291
  - Geocentric Flight Path Angle (deg) -5.86

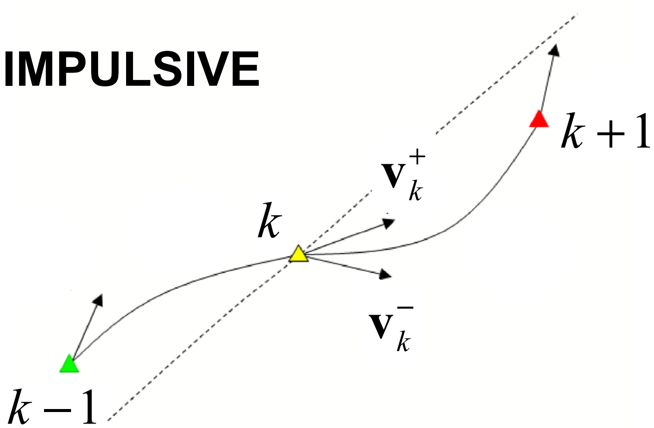
	Impulsive	Finite Burn
Initial Position Error (km)	3.48696e5	3.91018e6
Final Position Error (km)	7.29519e-6	8.83104e-5
FPA Error (deg)	0.0112	0.0123
Azimuth Error (deg)	0.0661	0.0782

Level II Algorithm: Impulsive vs. Finite Burn

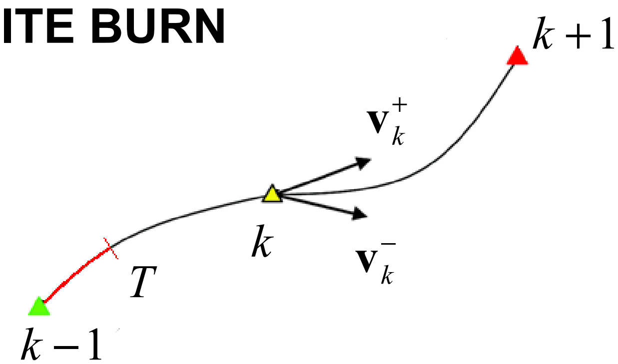
Constraints: $\Delta\mathbf{V} = [\Delta\mathbf{v}_1, \Delta\mathbf{v}_2, \ldots, \Delta\mathbf{v}_{n-1}], \mathbf{A} = \left[h, \lambda, \delta, \gamma, \chi, \ldots, \sum_j \Delta v_{TEI_j} \right]$

Control Variables: $\mathbf{b} = [\delta\mathbf{r}_0, \delta t_0, \delta\mathbf{r}_1, \delta t_1, \ldots, \delta\mathbf{r}_n, \delta t_n]$

IMPULSIVE



FINITE BURN



$$\begin{Bmatrix} \delta\Delta\mathbf{V} \\ \delta\mathbf{A} \end{Bmatrix} = \underbrace{\begin{bmatrix} \dfrac{\partial\Delta\mathbf{V}}{\partial\mathbf{b}} \\ \dfrac{\partial\mathbf{A}}{\partial\mathbf{b}} \end{bmatrix}}_{M} \mathbf{b} \quad \rightarrow \quad \mathbf{b} = M^T \left(MM^T\right)^{-1} \begin{Bmatrix} \delta\Delta\mathbf{V} \\ \delta\mathbf{A} \end{Bmatrix}$$

Variational Equations: Impulsive vs. Finite Burn

IMPULSIVE

$$\begin{bmatrix} \delta\mathbf{r}_k - \mathbf{v}_k^-\delta t_k \\ \delta\mathbf{v}_k^- - \mathbf{a}_k^-\delta t_k \end{bmatrix} = \begin{bmatrix} A_{k,k-1} & B_{k,k-1} \\ C_{k,k-1} & D_{k,k-1} \end{bmatrix} \begin{bmatrix} \delta\mathbf{r}_{k-1} - \mathbf{v}_{k-1}^+\delta t_{k-1} \\ \delta\mathbf{v}_{k-1}^+ - \mathbf{a}_{k-1}^+\delta t_{k-1} \end{bmatrix}$$

$$\begin{bmatrix} \delta\mathbf{r}_k - \mathbf{v}_k^+\delta t_k \\ \delta\mathbf{v}_k^+ - \mathbf{a}_k^+\delta t_k \end{bmatrix} = \begin{bmatrix} A_{k,k+1} & B_{k,k+1} \\ C_{k,k+1} & D_{k,k+1} \end{bmatrix} \begin{bmatrix} \delta\mathbf{r}_{k+1} - \mathbf{v}_{k+1}^-\delta t_{k+1} \\ \delta\mathbf{v}_{k+1}^- - \mathbf{a}_{k+1}^-\delta t_{k+1} \end{bmatrix}$$

FINITE BURN

$$\begin{bmatrix} \delta\mathbf{r}_T - \mathbf{v}_T^-\delta t_T \\ \delta\mathbf{v}_T^- - \mathbf{a}_T^-\delta t_T \\ \delta m_T^- + \dot{m}_{g_T}^-\delta t_T \\ \delta\dot{m}_{g_T}^- - \ddot{m}_{g_T}^-\delta t_T \\ \delta\mathbf{u}_T^- - \dot{\mathbf{u}}_{g_T}^-\delta t_T \end{bmatrix} = \begin{bmatrix} A_{T,k-1} & B_{T,k-1} & E_{T,k-1} & F_{T,k-1} & G_{T,k-1} \\ C_{T,k-1} & D_{T,k-1} & H_{T,k-1} & I_{T,k-1} & J_{T,k-1} \\ K_{T,k-1} & L_{T,k-1} & M_{T,k-1} & N_{T,k-1} & O_{T,k-1} \\ P_{T,k-1} & Q_{T,k-1} & R_{T,k-1} & S_{T,k-1} & T_{T,k-1} \\ U_{T,k-1} & V_{T,k-1} & W_{T,k-1} & X_{T,k-1} & Y_{T,k-1} \end{bmatrix} \begin{bmatrix} \delta\mathbf{r}_{k-1} - \mathbf{v}_{k-1}^+\delta t_{k-1} \\ \delta\mathbf{v}_{k-1}^+ - \mathbf{a}_{k-1}^+\delta t_{k-1} \\ \delta m_{k-1}^+ + \dot{m}_{g_{k-1}}^+\delta t_{k-1} \\ \delta\dot{m}_{g_{k-1}}^+ - \ddot{m}_{g_{k-1}}^+\delta t_{k-1} \\ \delta\mathbf{u}_{k-1}^+ - \dot{\mathbf{u}}_{k-1}^+\delta t_{k-1} \end{bmatrix}$$

$$\begin{bmatrix} \delta\mathbf{r}_k - \mathbf{v}_k^-\delta t_k \\ \delta\mathbf{v}_k^- - \mathbf{a}_k^-\delta t_k \end{bmatrix} = \begin{bmatrix} A_{k,T} & B_{k,T} \\ C_{k,T} & D_{k,T} \end{bmatrix} \begin{bmatrix} \delta\mathbf{r}_T - \mathbf{v}_T^+\delta t_T \\ \delta\mathbf{v}_T^+ - \mathbf{a}_T^+\delta t_T \end{bmatrix}$$

$$\begin{bmatrix} \delta\mathbf{r}_k - \mathbf{v}_k^+\delta t_k \\ \delta\mathbf{v}_k^+ - \mathbf{a}_k^+\delta t_k \end{bmatrix} = \begin{bmatrix} A_{k,k+1} & B_{k,k+1} \\ C_{k,k+1} & D_{k,k+1} \end{bmatrix} \begin{bmatrix} \delta\mathbf{r}_{k+1} - \mathbf{v}_{k+1}^-\delta t_{k+1} \\ \delta\mathbf{v}_{k+1}^- - \mathbf{a}_{k+1}^-\delta t_{k+1} \end{bmatrix}$$

Total Cost Constraint: Impulsive vs. Finite Burn

IMPULSIVE

$$\Delta \mathrm{v}_k = \left| \mathbf{v}_k^+ - \mathbf{v}_k^- \right|$$

$$\begin{cases} \mathbf{v}_k^- = \mathbf{v}_k^- \left(\mathbf{r}_{k-1}, t_{k-1}, \mathbf{r}_k, t_k \right) \\ \mathbf{v}_k^+ = \mathbf{v}_k^+ \left(\mathbf{r}_k, t_k, \mathbf{r}_{k+1}, t_{k+1} \right) \end{cases}$$

FINITE BURN

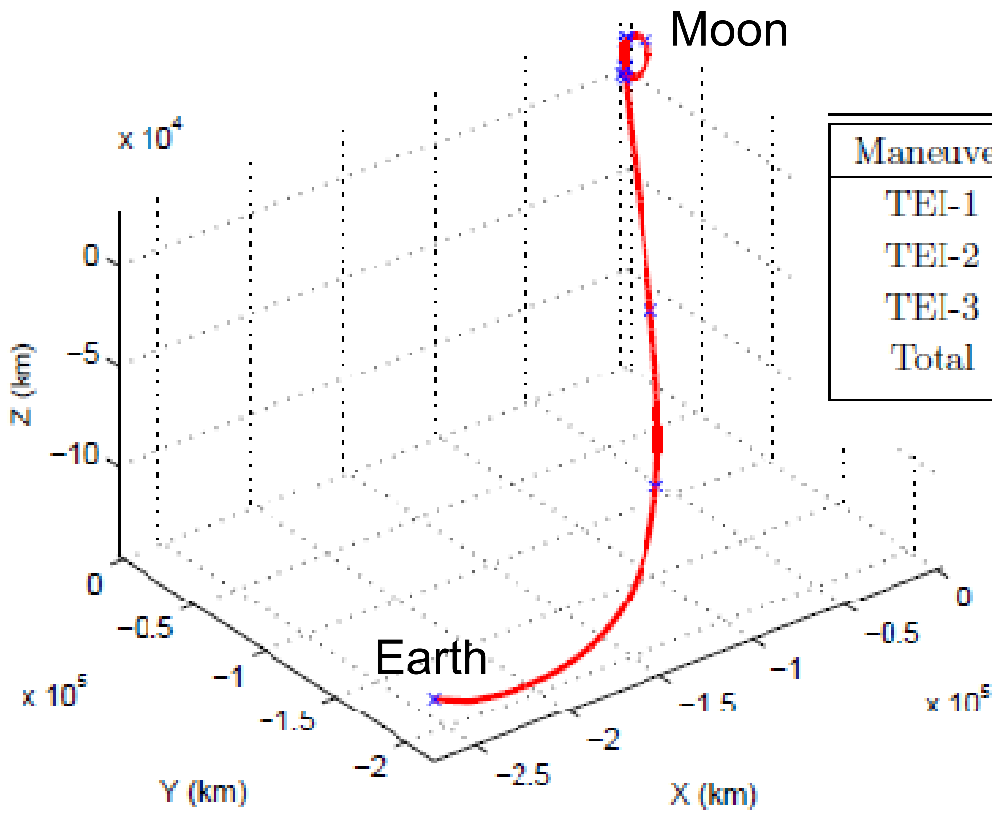
$$\Delta \mathrm{v}_k = -I_{sp} g_0 \ln \left(1 - \frac{\dot{m}_{g_k} \left(t_T - t_k \right)}{m_k} \right)$$

$$\begin{cases} \Delta \mathrm{v}_k = f\left(t_k, t_T, m_k \right) \\ m_k = m_0 - \sum_{j=1}^{n-1} \left[\dot{m}_g \Delta t_{burn} \right] \Big|_j \end{cases}$$

Main Engine Simulation

- Initial guess data
  - Epoch: 4-Apr-2024 15:30:00 TDT
  - Initial mass: 20339.9 kg (total fuel = 8063.65 kg)
  - Main Engine Thrust: 33,361.6621 N
  - Main Engine Isp: 326 sec
  - State (J2000 Moon-centered inertial frame):
    - X: -1236.7970783385588 km
    - Y: 1268.1142350088496 km
    - Z: 468.38317094160635 km
    - Vx: 0.0329108058365355 km/sec
    - Vy: 0.589269803607714 km/sec
    - Vz -1.528058717568413 km/sec
- Entry constraints:
  - Geodetic Altitude (km): 121.92
  - Longitude (deg): 175.6365
  - Geocentric Azimuth (deg): 49.3291
  - Geocentric Flight Path Angle (deg): -5.86

Results (1/2)



Maneuver	Duration (s)	Prop. Mass Consumed (kg)
TEI-1	400.2308	4175.128
TEI-2	119.2402	1243.890
TEI-3	171.6074	1790.174
Total	691.0784	7209.192

MCI Frame Perspective

Results (2/2)

- Comparison of finite burn and impulsive algorithms:

Maneuver	Impulsive ΔV (km/s)	Finite Burn ΔV (km/s)
TEI-1	0.6619	0.7348
TEI-2	0.3257	0.2561
TEI-3	0.4115	0.4087
Total	1.3991	1.3996

	Impulsive Algorithm	Finite Burn Algorithm
Iterations	20	6
Altitude (km)	-1.0e-8	-5.2e-8
Flight Path Angle (deg)	2.8e-10	-3.3e-10
Longitude (deg)	4.0e-8	-5.0e-7
Flight Path Azimuth (deg)	-2.0e-7	1.6e-7

Auxiliary Engine Simulation

- Same initial guess data and constraints
- Assume main engine failure after TEI-1
- TEI-2 and TEI-3 performed using auxiliary engines:
  - Auxiliary Engine Thrust: 4,448.0 N
  - Auxiliary Engine Isp: 309 sec

Results

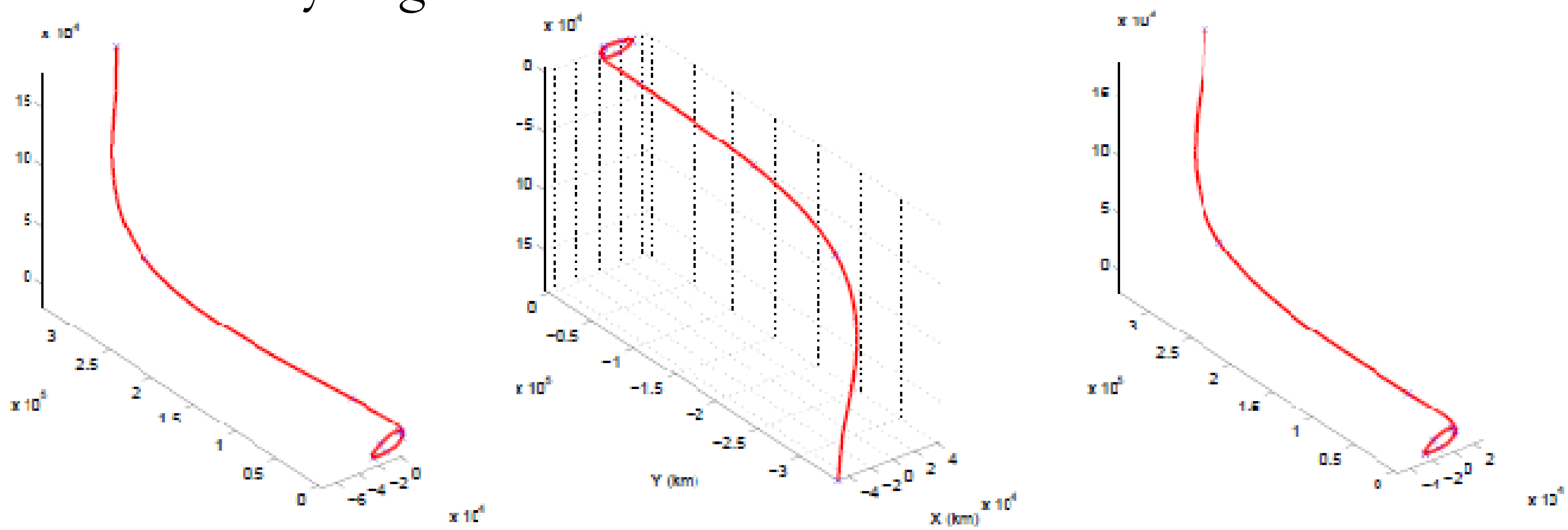
- Maneuver and final constraint data:

Maneuver	Duration (s)	Prop. Mass Consumed (kg)	ΔV (km/s)
TEI-1	363.5548	3792.531	0.6255
TEI-2	949.2614	1392.946	0.2666
TEI-3	1400.5756	2055.205	0.4418
Total	2713.3918	7240.682	1.3339

Iterations	23
Altitude (km)	4.1e-7
Flight Path Angle (deg)	4.5e-10
Longitude (deg)	2.1e-6
Flight Path Azimuth (deg)	-2.6e-7

Lunar Cycle Simulations

- Simulations run for 10 different days spanning February 2024
  - Patch points from converged impulsive runs
- Initial lunar orbit of 100 km, targeting altitude (121.9 km) and flight path angle (-5.86°)
- Auxiliary engines used for TEI-2 and TEI-3



(a) Day 1 (b) Day 13 (c) Day 28

Results

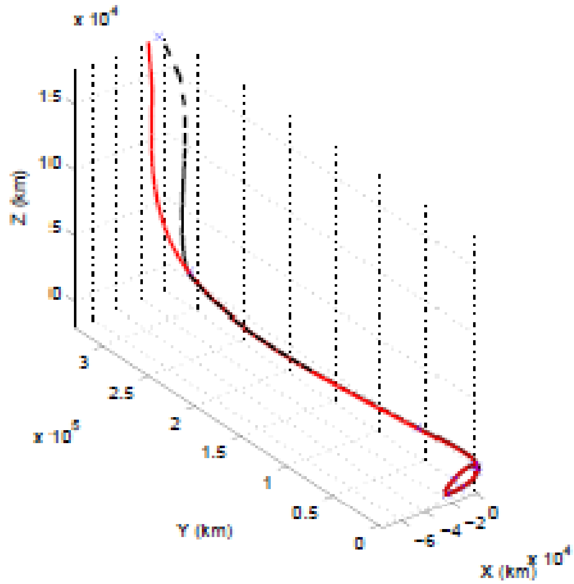
Day	TEI-1		TEI-2		TEI-3		Total Cost	
	ΔV (km/s)	Duration (s)	ΔV	Duration	ΔV	Duration	Total ΔV	Iterations
1	0.5963	348.1673	0.4741	1648.5129	0.3995	1202.2754	1.4698	4
3	0.6837	393.6876	0.4393	1492.3184	0.3585	1067.4428	1.4814	4
6	0.5901	344.8973	0.5271	1820.9042	0.3929	1165.7458	1.5101	15
10	0.5849	342.1671	0.5808	1992.6327	0.3259	961.9933	1.4916	4
13	0.5869	343.1966	0.3661	1299.5151	0.5079	1561.4187	1.4609	4
16	0.6806	392.0921	0.3205	1111.0622	0.6251	1856.5447	1.6262	39
19	0.5778	338.3921	0.5925	2033.8077	0.3189	940.8576	1.4892	4
22	0.7004	402.2770	0.3915	1332.7907	0.4080	1217.7097	1.4999	11
25	0.5862	342.8320	0.6318	2149.2973	0.2740	801.4648	1.4919	4
28	0.5861	342.7945	0.4912	1709.0768	0.3882	1167.9040	1.4656	4

Delayed Patch Points

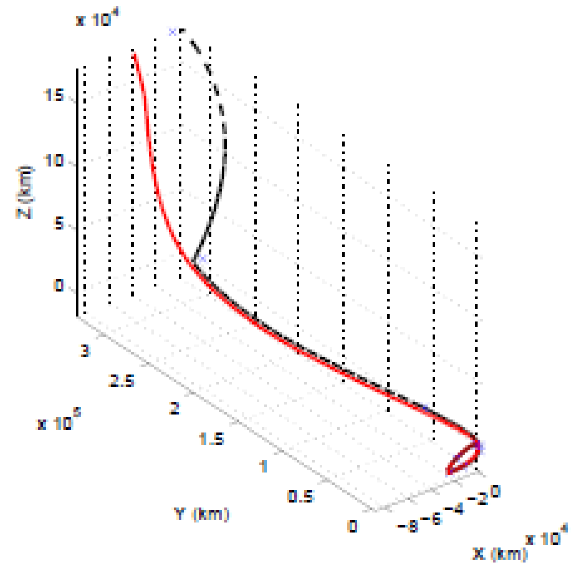
- Patch points associated with specific epoch
- Targeter must converge even if the patch points are not current
- Using February 1 input file from previous example, initial epoch delayed for (a) 3 hours and (b) 12 hours

Delay (hr)	Altitude Error (km)	FPA Error (deg)
3	1.1040e4	1.6464
12	4.5332e4	10.4268

Results



(a) 3 Hours



(b) 12 Hours

Delay (hr)	Total ΔV (km/s)	Altitude Error (km)	FPA Error (deg)	Iterations
3	1.4655	-2.8218e-5	-9.4476e-8	10
12	1.2961	6.0979e-6	1.2184e-8	12

Conclusions and Future Work

- Two-level targeting algorithm developed for finite burn maneuvers
- Algorithm successfully targets lunar return trajectory
  - Using main engines
  - Using auxiliary engines following simulated failure of main engines after TEI-1
- Future work
  - Implementing thruster steering law
  - Automated patch point selection