FORMATION FLIGHT NEAR L_1 AND L_2 IN THE SUN-EARTH/MOON EPHEMERIS SYSTEM INCLUDING SOLAR RADIATION PRESSURE

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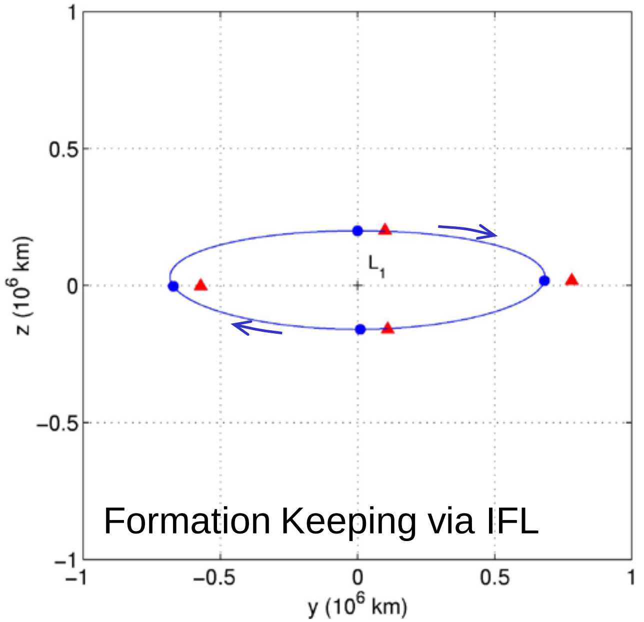
Continuous Control

- LQR and Input Feedback Linearization (IFL)
  - Control Nominal State
    - Fixed in Rotating Frame
    - Fixed in Inertial Frame
  - Critically damped error response
  - LQR & IFL → similar response & control acceleration histories
- Output Feedback Linearization
  - Radial Distance Control
  - Free relative orientation
- Transition from CR3BP
  - Ephemeris Model w/ Solar Radiation Pressure (SRP)



Output Feedback Linearization (Radial Distance Control)

Nonlinear Scalar Constraint on $\bar{u}(t)$

$$h\left(\bar{r}(t),\dot{\bar{r}}(t)\right)-\bar{u}(t)^{T}\,\bar{r}(t)=0$$

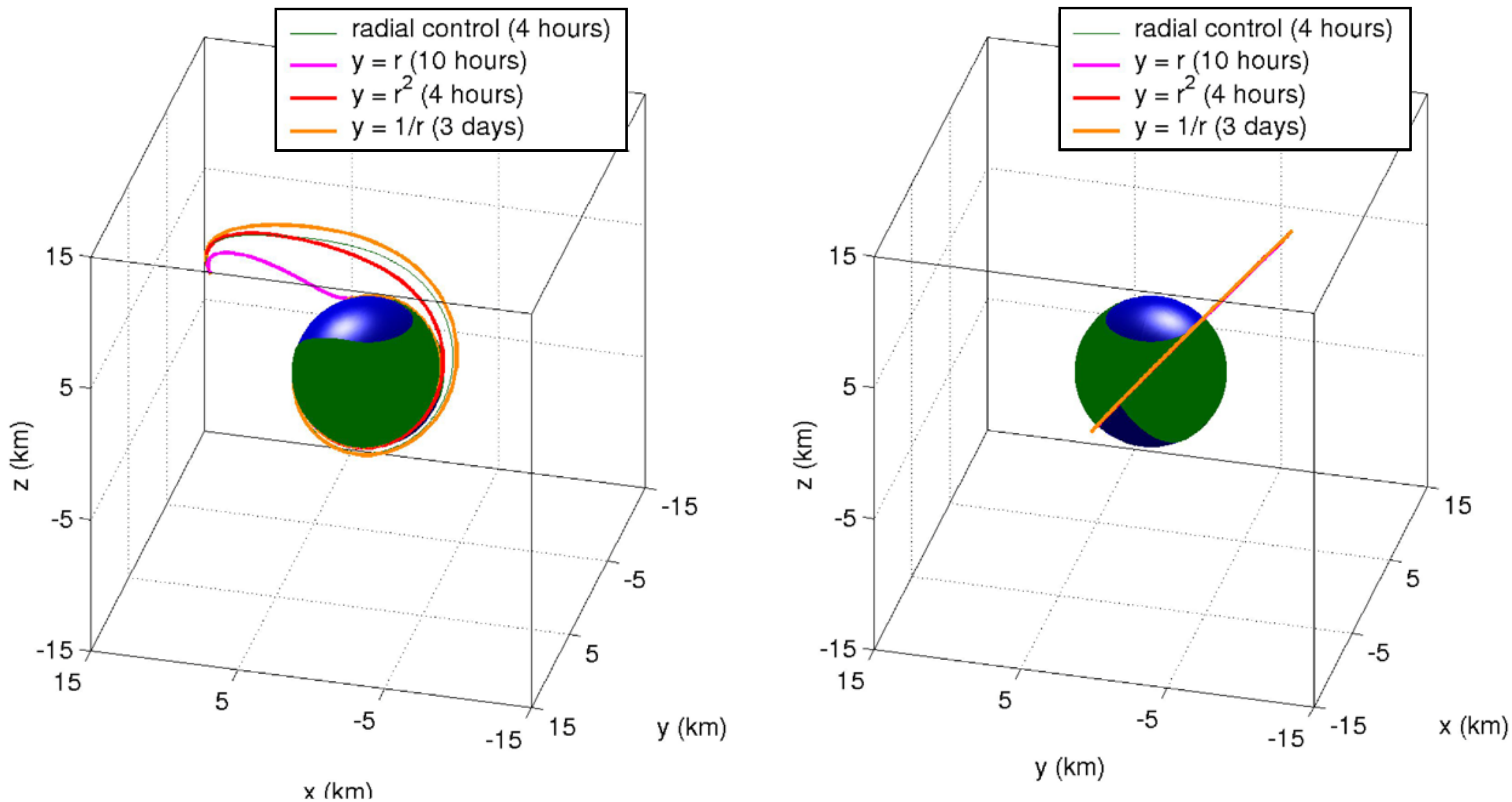
Target Nominal: $r=5$ km

Mission Time: 180 days

$y=l\left(\bar{r},\dot{\bar{r}}\right)$	Control Law	Cumulative ΔV (180 days)
r	$\bar{u}(t)=\dfrac{h\left(\bar{r},\dot{\bar{r}}\right)}{r}\hat{r}$ Geometric Approach: Radial inputs only	16,392.4 m/sec
r	$\bar{u}(t)=\left\{\dfrac{g\left(\bar{r},\dot{\bar{r}}\right)}{r}-\dfrac{\dot{\bar{r}}^{T}\dot{\bar{r}}}{r^{2}}\right\}\bar{r}+\left(\dfrac{\dot{r}}{r}\right)\dot{\bar{r}}-\Delta\bar{f}\left(\bar{r}\right)$	2,310.0 m/sec
r^{2}	$\bar{u}(t)=\left\{\dfrac{1}{2}\dfrac{g\left(\bar{r},\dot{\bar{r}}\right)}{r^{2}}-\dfrac{\dot{\bar{r}}^{T}\dot{\bar{r}}}{r^{2}}\right\}\bar{r}-\Delta\bar{f}\left(\bar{r}\right)$	16,442.0 m/sec
$1/r$	$\bar{u}(t)=\left\{-rg\left(\bar{r},\dot{\bar{r}}\right)-\dfrac{\dot{\bar{r}}^{T}\dot{\bar{r}}}{r^{2}}\right\}\bar{r}+3\left(\dfrac{\dot{r}}{r}\right)\dot{\bar{r}}-\Delta\bar{f}\left(\bar{r}\right)$	49.8 m/sec

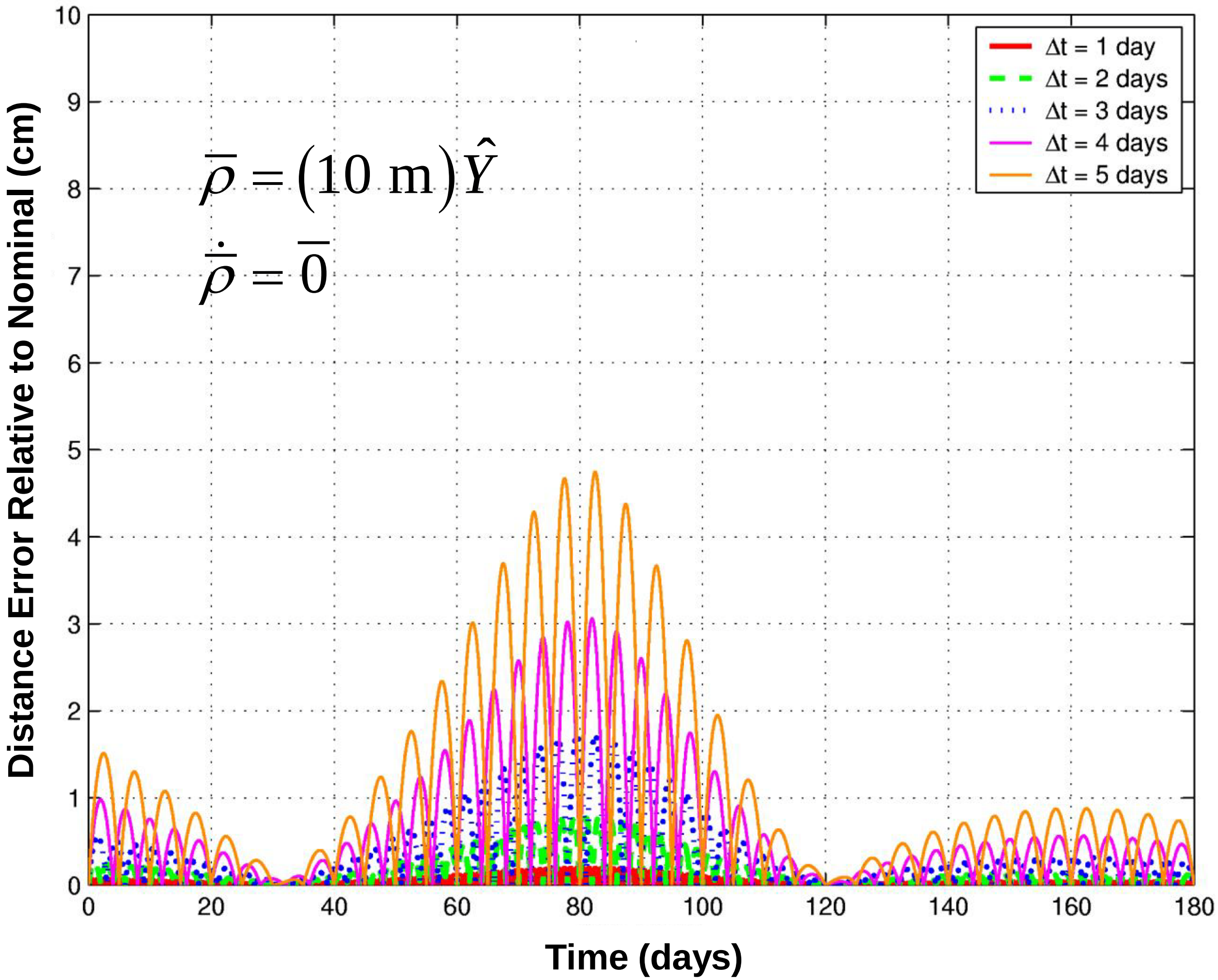
Partial Feedback Control of Spherical Formations

$$\bar{r}(0) = [12 \quad -5 \quad 3]\ \text{km} \qquad \dot{\bar{r}}(0) = [1 \quad -1 \quad 1]\ \text{m/sec}$$

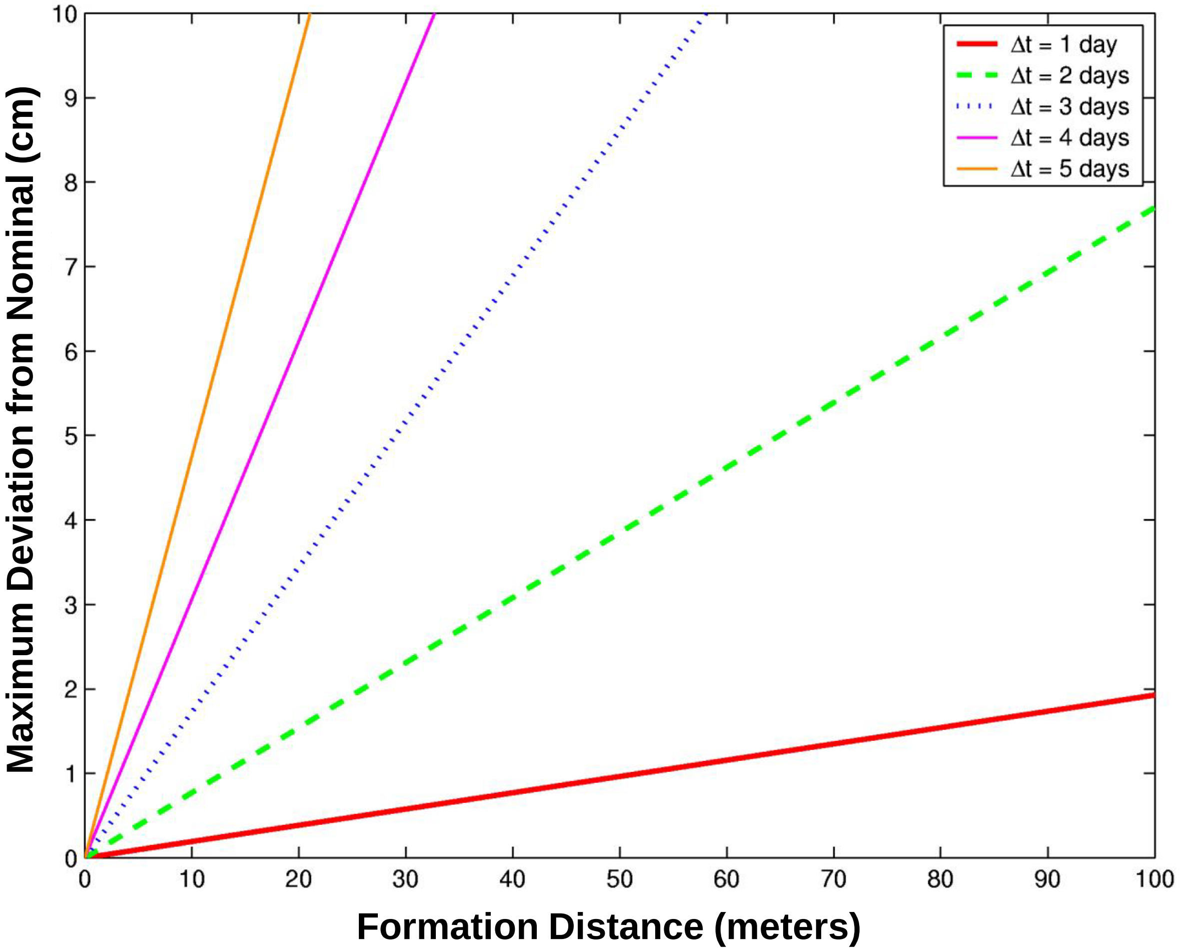


Formation Keeping in the Ephemeris Model via Discrete Control

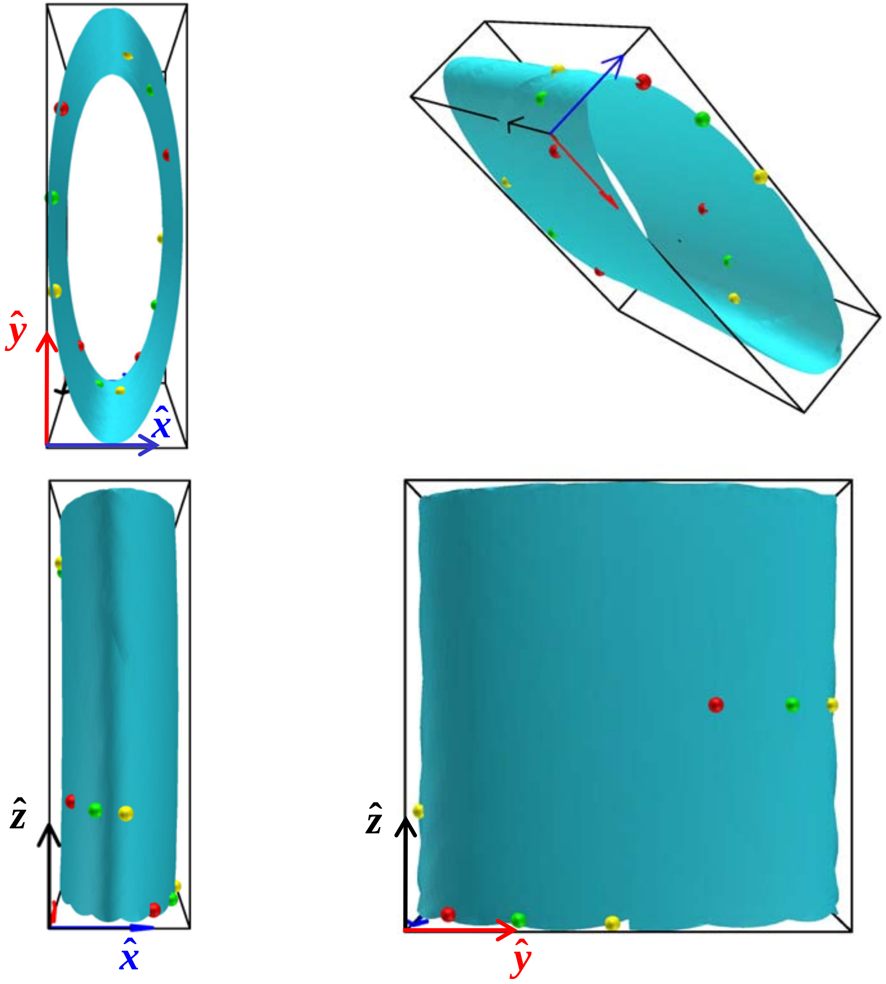
Linear Targeter Approach



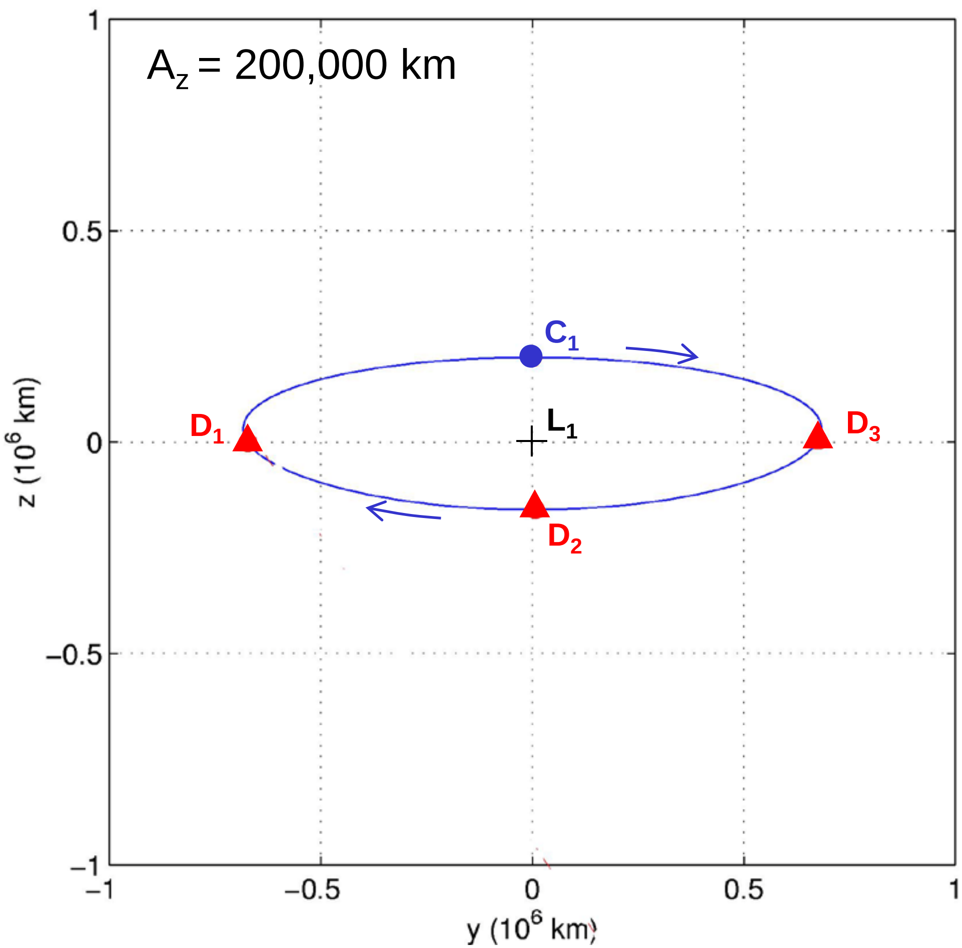
Achievable Accuracy via Targeter Scheme



Natural Formations: String of Pearls



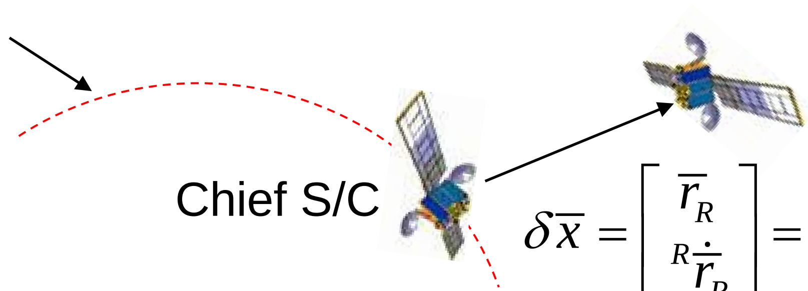
Natural Formations: Phased Vehicles Along Halo Orbit



Unstable Halo Orbit

1 Unstable Eigenvalue (γ_1)
1 Stable Eigenvalue (γ_2)
4 Center Eigenvalues (γ_3-γ_6)

Eigenstructure Near Halo Orbit



$$\delta\bar{x} = \begin{bmatrix} \bar{r}_R \\ {}^{R}\dot{\bar{r}}_R \end{bmatrix} = \Phi(t,0)\,\delta\bar{x}(0)$$

Floquet Decomposition of $\Phi(t,0)$:

$$\Phi(t,0) = \{P(t)S\}e^{Jt}\{P(0)S\}^{-1}$$

Floquet Modal Matrix:

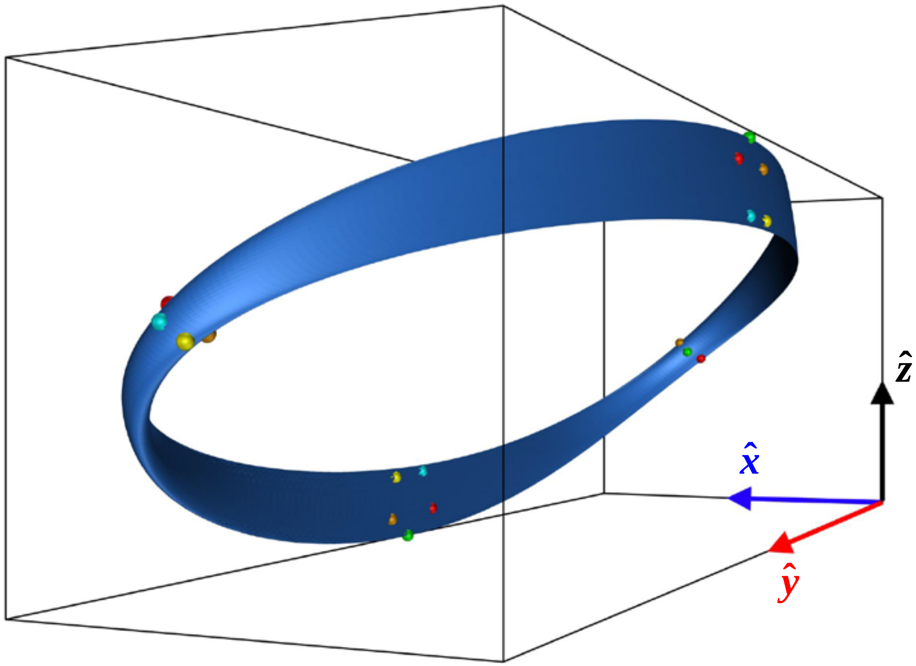
$$E(t) = P(t)S = \Phi(t,0)E(0)e^{-Jt}$$

Solution to Variational Eqn. in terms of Floquet Modes:

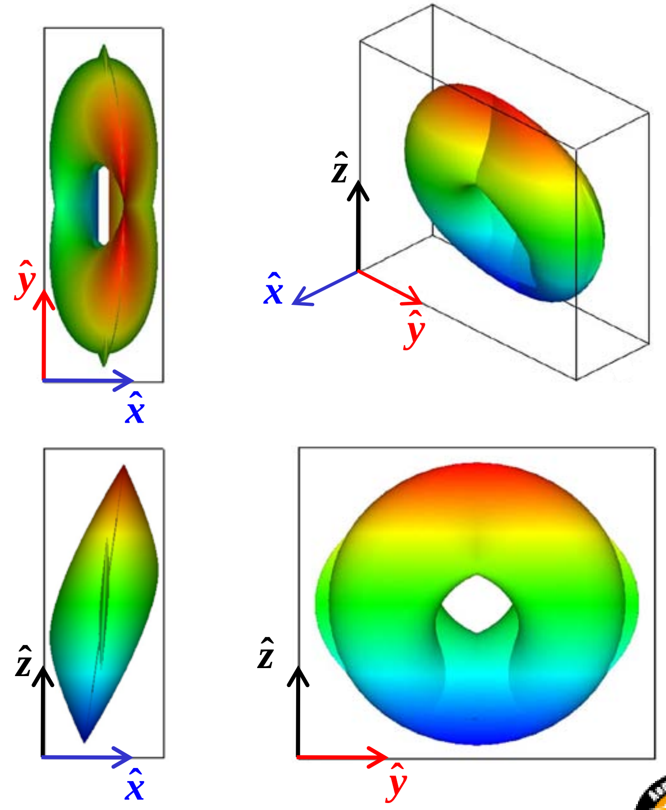
$$\delta\bar{x}(t) = \sum_{j=1}^{6} \delta\bar{x}_j(t) = \sum_{j=1}^{6} c_j(t)\,\bar{e}_j(t) = E(t)\,\bar{c}$$

Natural Solutions:
Torus (Associated with Modes 3 and 4)

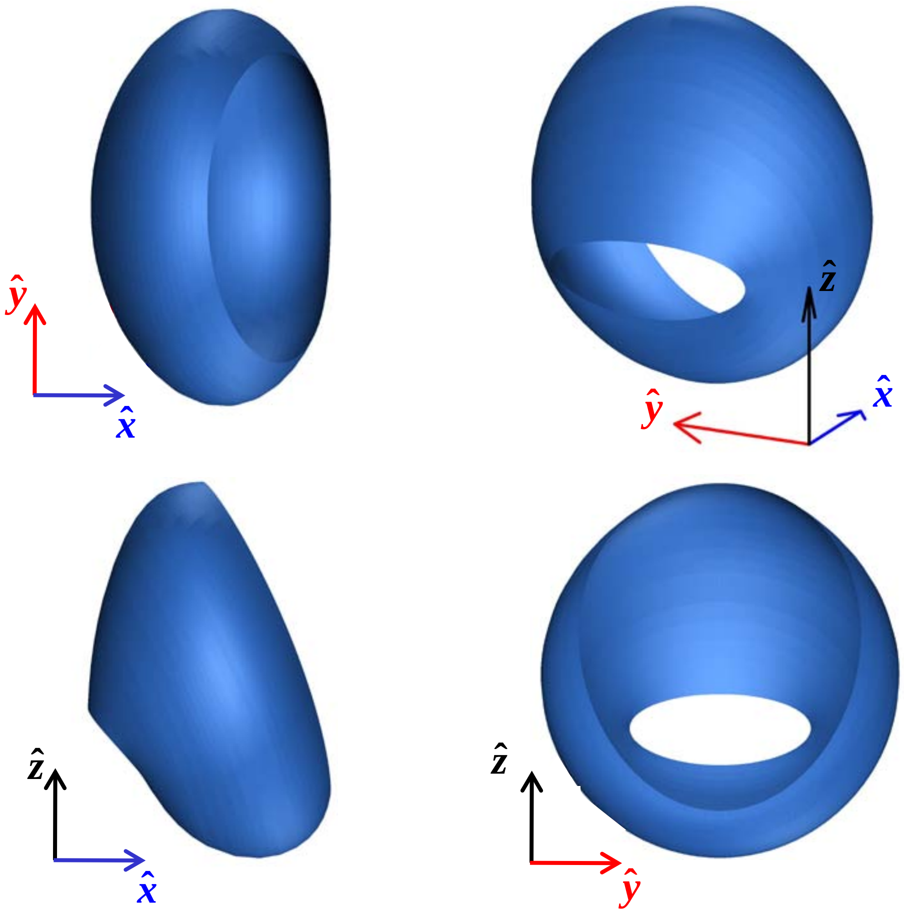
Formation Evolving Along Torus Near L_1
(**R**otating **L**ibration **P**oint Frame Centered at L_1)



Chief S/C Centered View
(RLP Frame)



Natural Solutions:
Halo Orbits (Associated with Modes 5 and 6)



Floquet Controller
(Remove Unstable + 2 Center Modes)

Find $\Delta\bar{v}$ that removes undesired response modes:

$$\sum_{j=1}^{6}\delta\bar{x}_j + \begin{bmatrix} 0_3 \\ I_3 \end{bmatrix}\Delta\bar{v} = \sum_{\substack{j=2,3,4 \\ \text{or} \\ j=2,5,6}} \left(1+\alpha_j\right)\delta\bar{x}_j$$

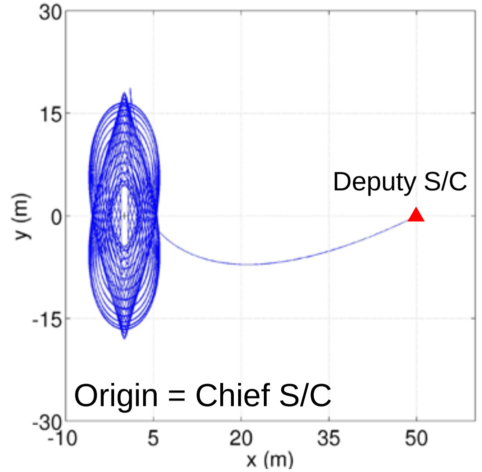
Remove Modes 1, 3, and 4:

$$\begin{bmatrix} \bar{\alpha} \\ \Delta\bar{v} \end{bmatrix} = \begin{bmatrix} \delta\bar{x}_{2\bar{r}} & \delta\bar{x}_{5\bar{r}} & \delta\bar{x}_{6\bar{r}} & 0_3 \\ \delta\bar{x}_{2\bar{v}} & \delta\bar{x}_{5\bar{v}} & \delta\bar{x}_{6\bar{v}} & -I_3 \end{bmatrix}^{-1} \left(\delta\bar{x}_1 + \delta\bar{x}_3 + \delta\bar{x}_4\right)$$

Remove Modes 1, 5, and 6:

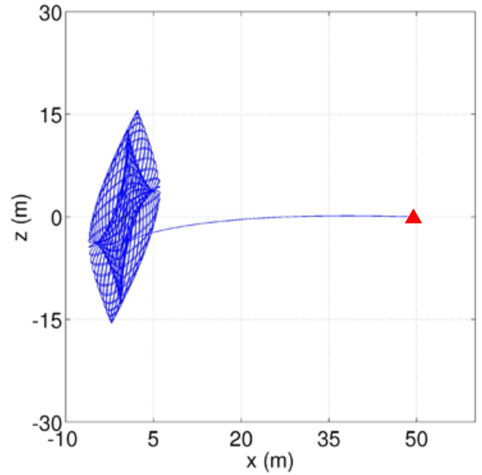
$$\begin{bmatrix} \bar{\alpha} \\ \Delta\bar{v} \end{bmatrix} = \begin{bmatrix} \delta\bar{x}_{2\bar{r}} & \delta\bar{x}_{3\bar{r}} & \delta\bar{x}_{4\bar{r}} & 0_3 \\ \delta\bar{x}_{2\bar{v}} & \delta\bar{x}_{3\bar{v}} & \delta\bar{x}_{4\bar{v}} & -I_3 \end{bmatrix}^{-1} \left(\delta\bar{x}_1 + \delta\bar{x}_5 + \delta\bar{x}_6\right)$$

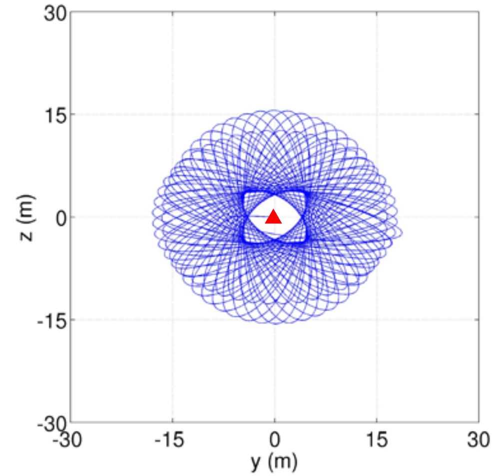
Deployment into Torus (Remove Modes 1, 5, and 6)



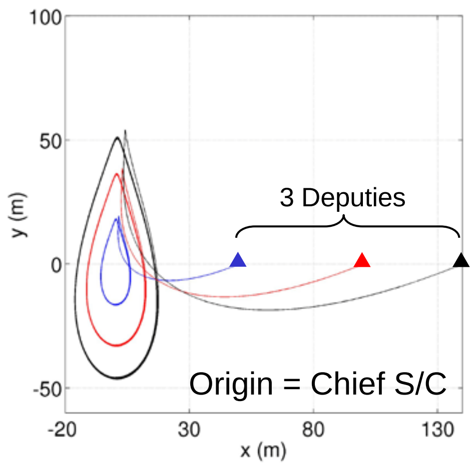
$$\bar{r}(0) = [5 \quad 00 \quad 0] \text{ m}$$

$$\dot{\bar{r}}(0) = [1 \quad -1 \quad 1] \text{ m/sec}$$



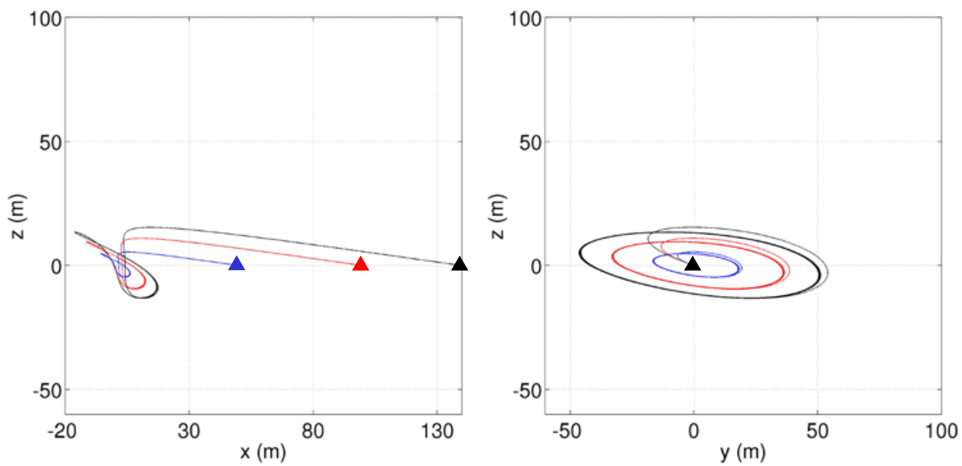


Deployment into Natural Orbits
(Remove Modes 1, 3, and 4)

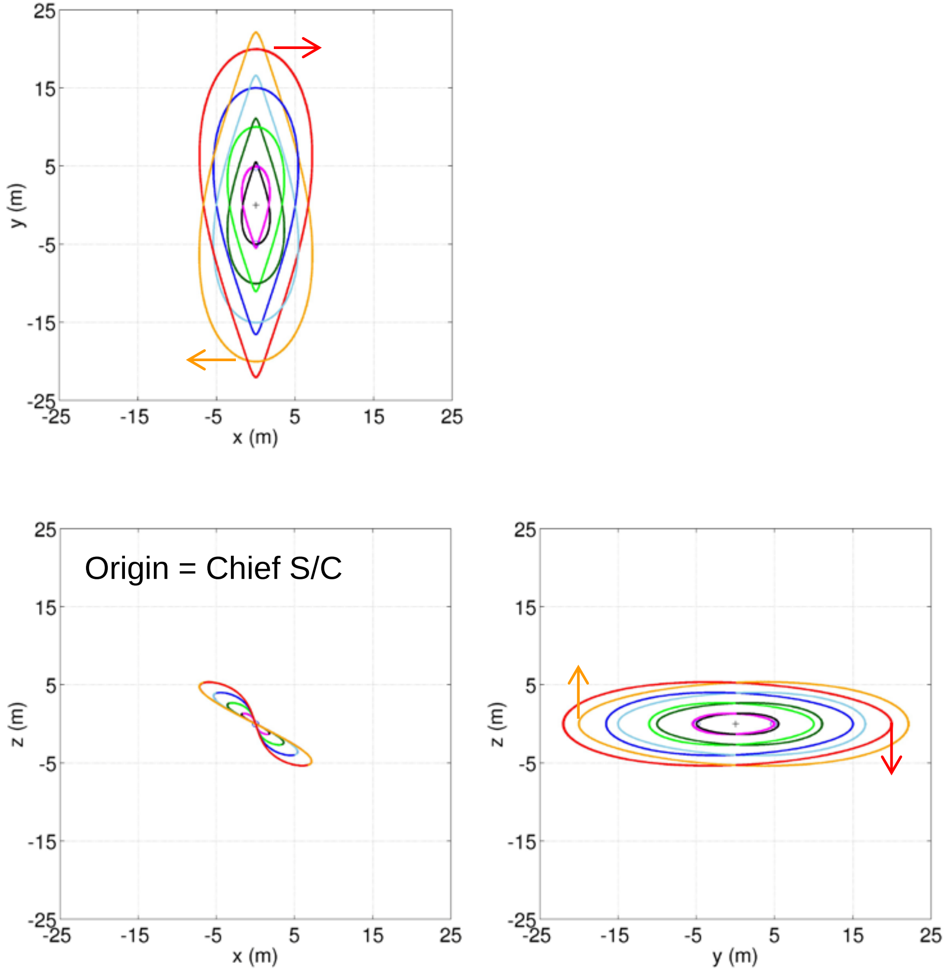


$$\bar{r}(0) = [r_0 \quad 0 \quad 0] \text{ m}$$

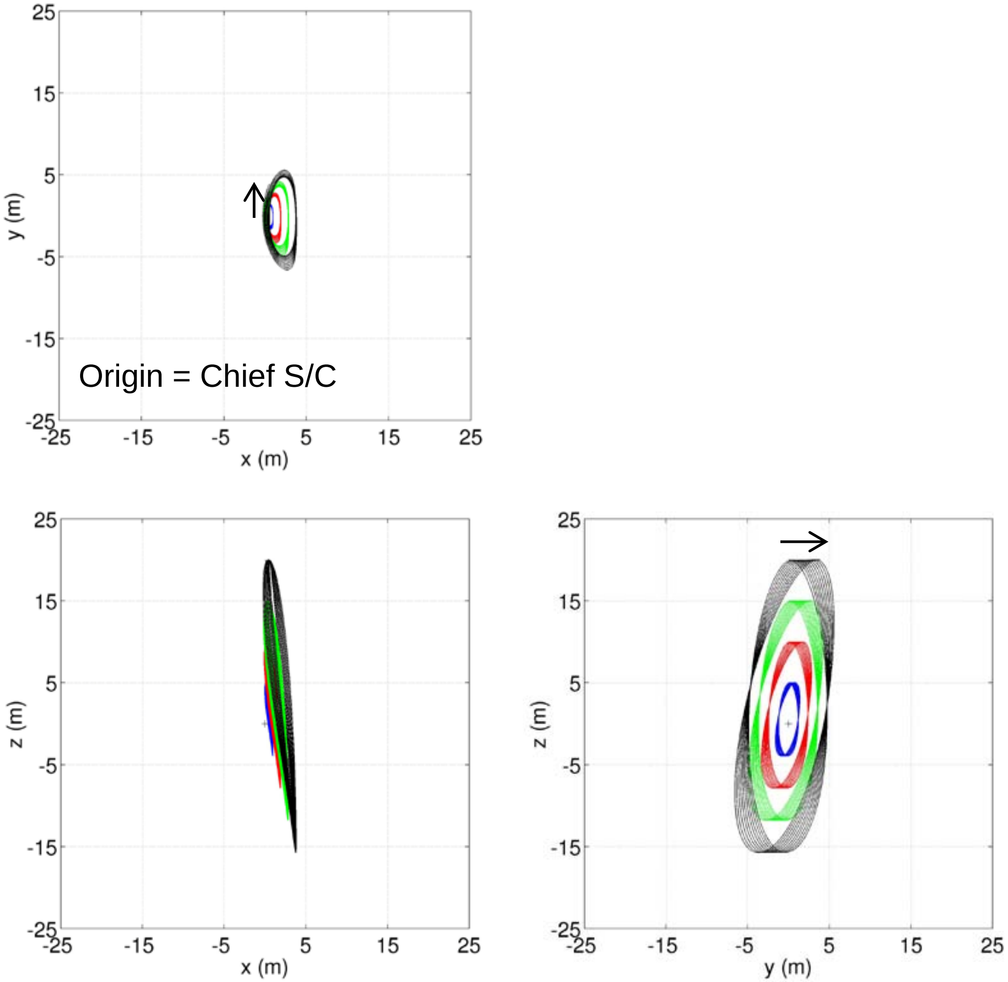
$$\dot{\bar{r}}(0) = [1 \quad -1 \quad 1] \text{ m/sec}$$



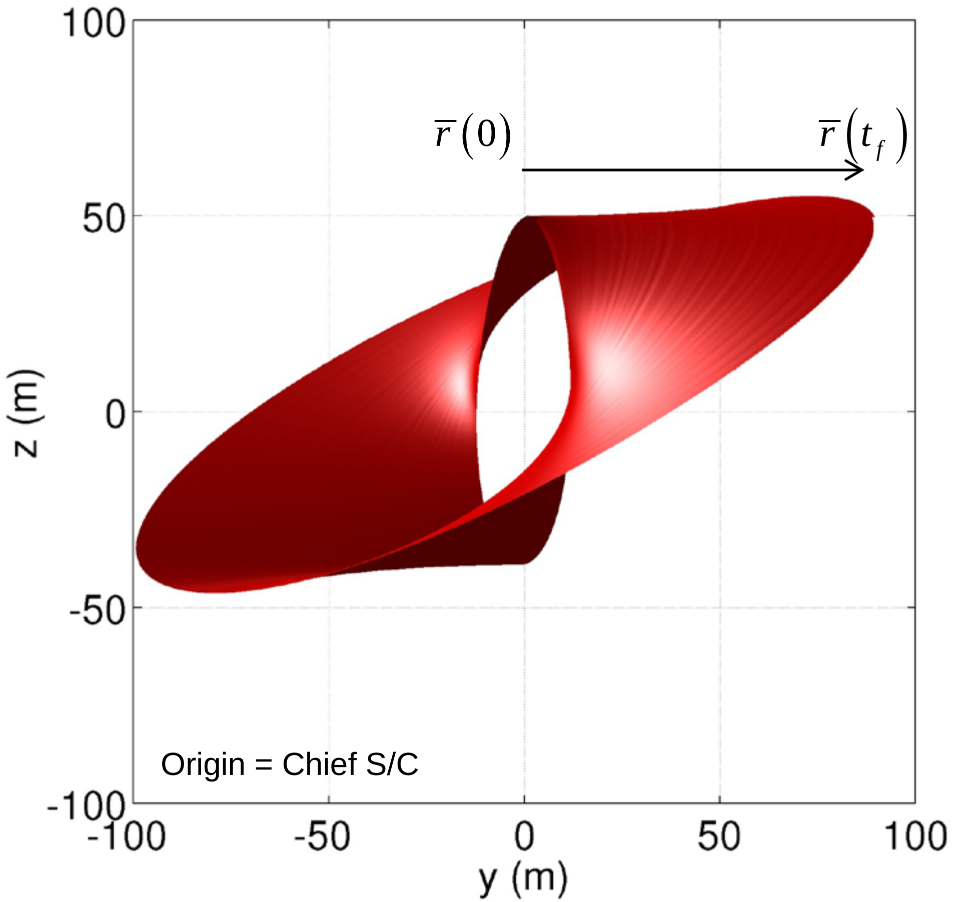
Nearly Periodic Formations



Nearly Vertical Formations



Evolution of Nearly Vertical Orbit Over 100 Orbital Periods



Conclusions

- Discrete Control of Natural Formations
  - Floquet controller
    - Effective in identifying nominal formations + deployment
    - May lead to feasible control strategies → non-natural formations
- Continuous Control of Non-Natural Formations
  - IFL/OFL effective; LQR computationally inefficient
  - OFL → spherical configurations + unnatural rates
  - Low acceleration levels → Implementation Issues
- Discrete Control of Non-Natural Formations
  - Small Formations → Good accuracy
  - Extremely Small ΔV's (10^{-5} m/sec)

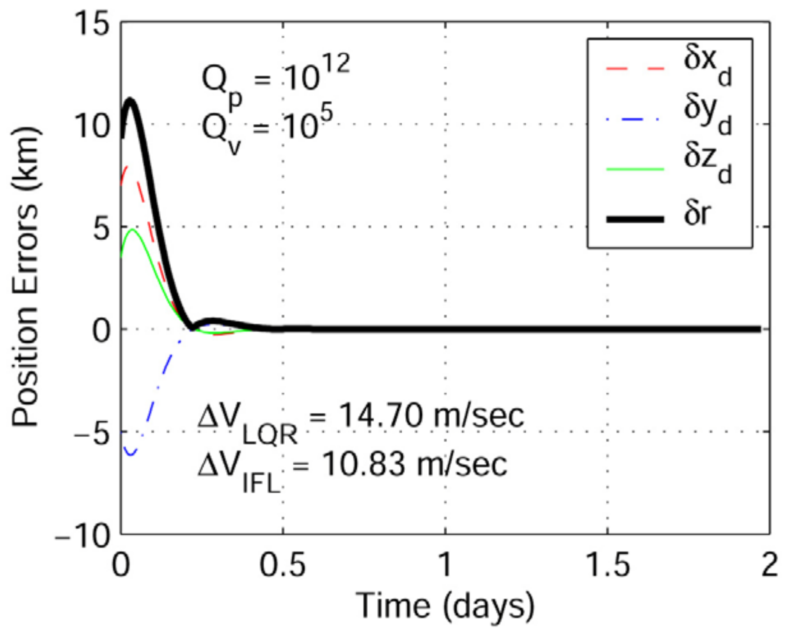
Backup Slides

LQR vs. IFL (CR3BP) Dynamic Response to Injection Error

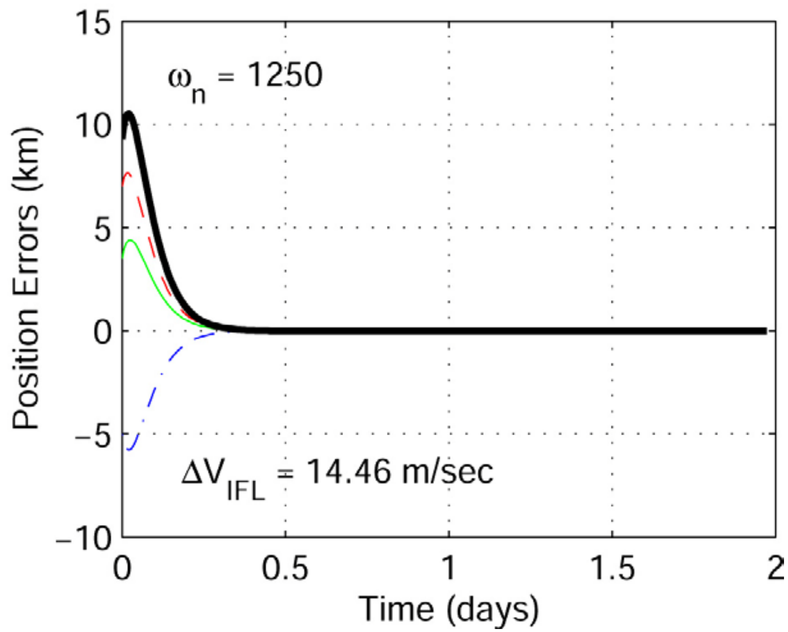
$$\rho = 5000 \text{ km}, \ \xi = 90^\circ, \ \beta = 0^\circ$$

$$\delta\bar{x}(0) = [7 \text{ km} \quad -5 \text{ km} \quad 3.5 \text{ km} \quad 1 \text{ mps} \quad -1 \text{ mps} \quad 1 \text{ mps}]^T$$

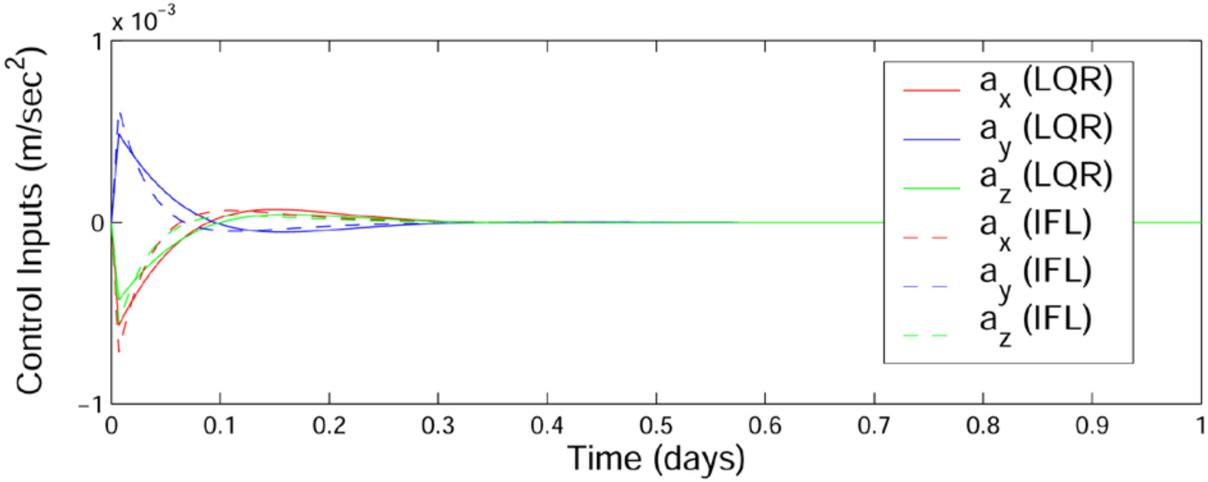
LQR Controller

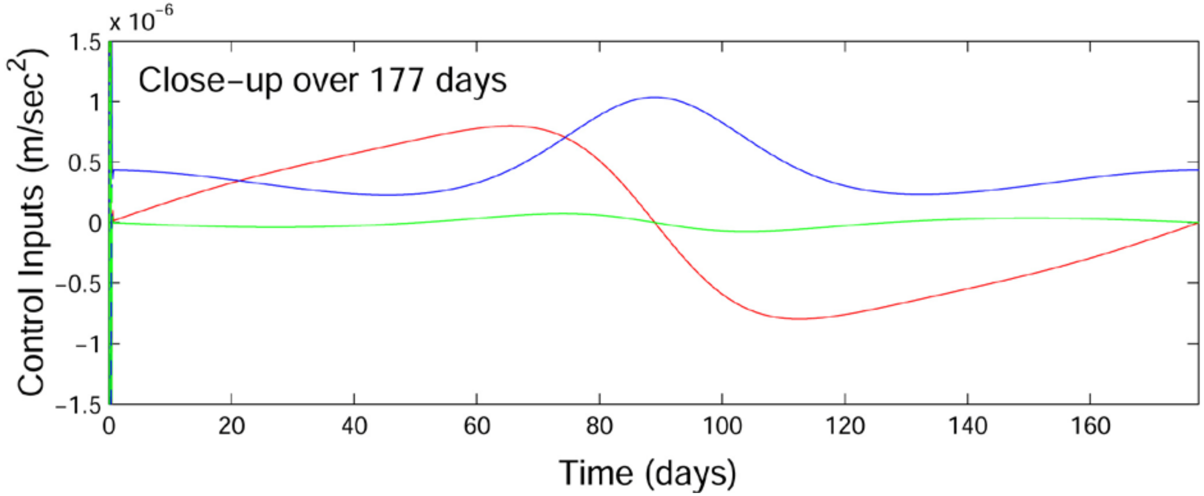


IFL Controller



LQR vs. IFL (CR3BP) Control Accelerations





Output Feedback Linearization (Radial Distance Control)

Formation Dynamics

$$\ddot{\bar{r}} = \Delta\bar{f}\left(\bar{r}\right) + \bar{u}\left(t\right) \quad \rightarrow \text{ Generalized Relative EOMs}$$

$$y = l\left(\bar{r}, \dot{\bar{r}}\right) \quad \rightarrow \text{ Measured Output}\left(\text{e.g. } y = r,\ y = r^2, y = r^{-1}\right)$$

Measured Output Response

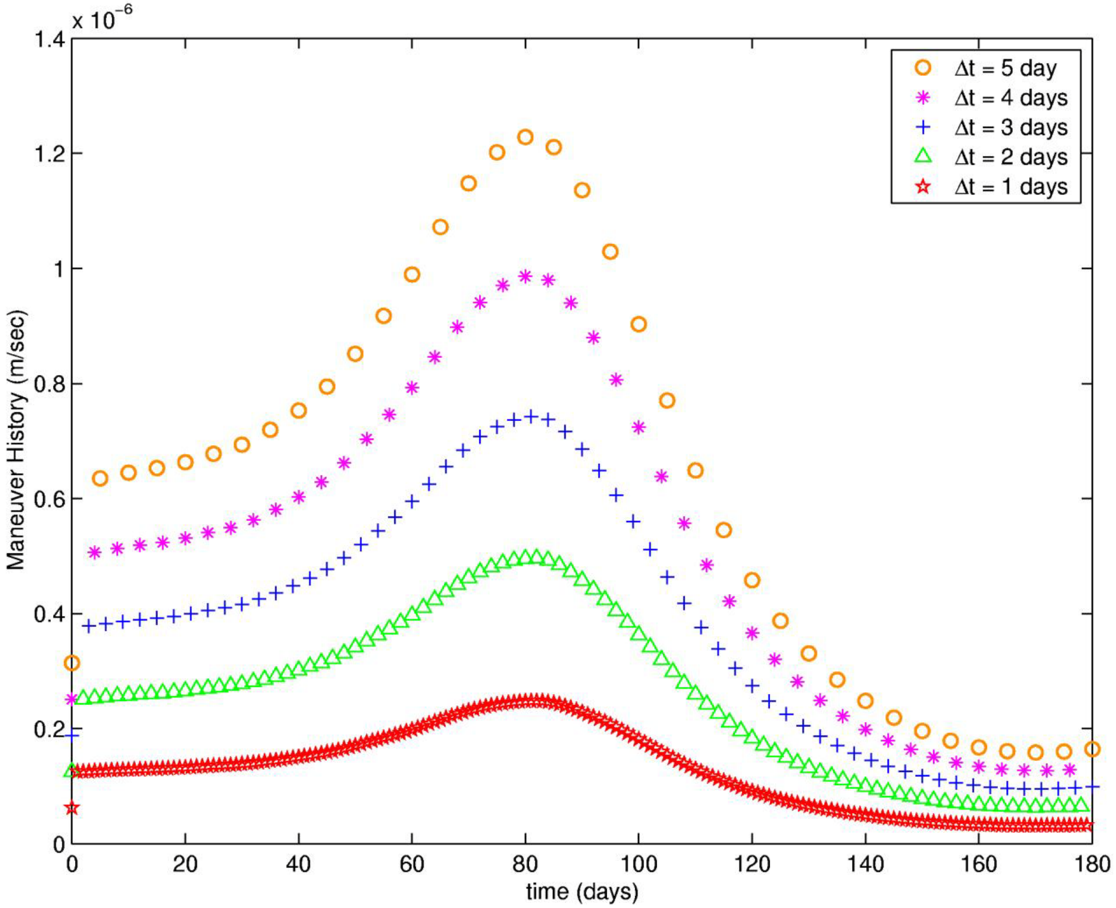
$$\ddot{y} = \frac{d^2 l}{dt^2} = \overbrace{p\left(\bar{r}, \dot{\bar{r}}\right) + q\left(\bar{r}, \dot{\bar{r}}\right)\bar{u}^T\bar{r}}^{\text{Actual Response}} = \overbrace{g\left(\bar{r}, \dot{\bar{r}}\right)}^{\text{Desired Response}}$$

Scalar Nonlinear Functions of $\bar{r}$ and $\dot{\bar{r}}$

Scalar Nonlinear Constraint on Control Inputs

$$h\left(\bar{r}\left(t\right), \dot{\bar{r}}\left(t\right)\right) - \bar{u}\left(t\right)^T \bar{r}\left(t\right) = 0$$

Targeter Maneuver Schedule



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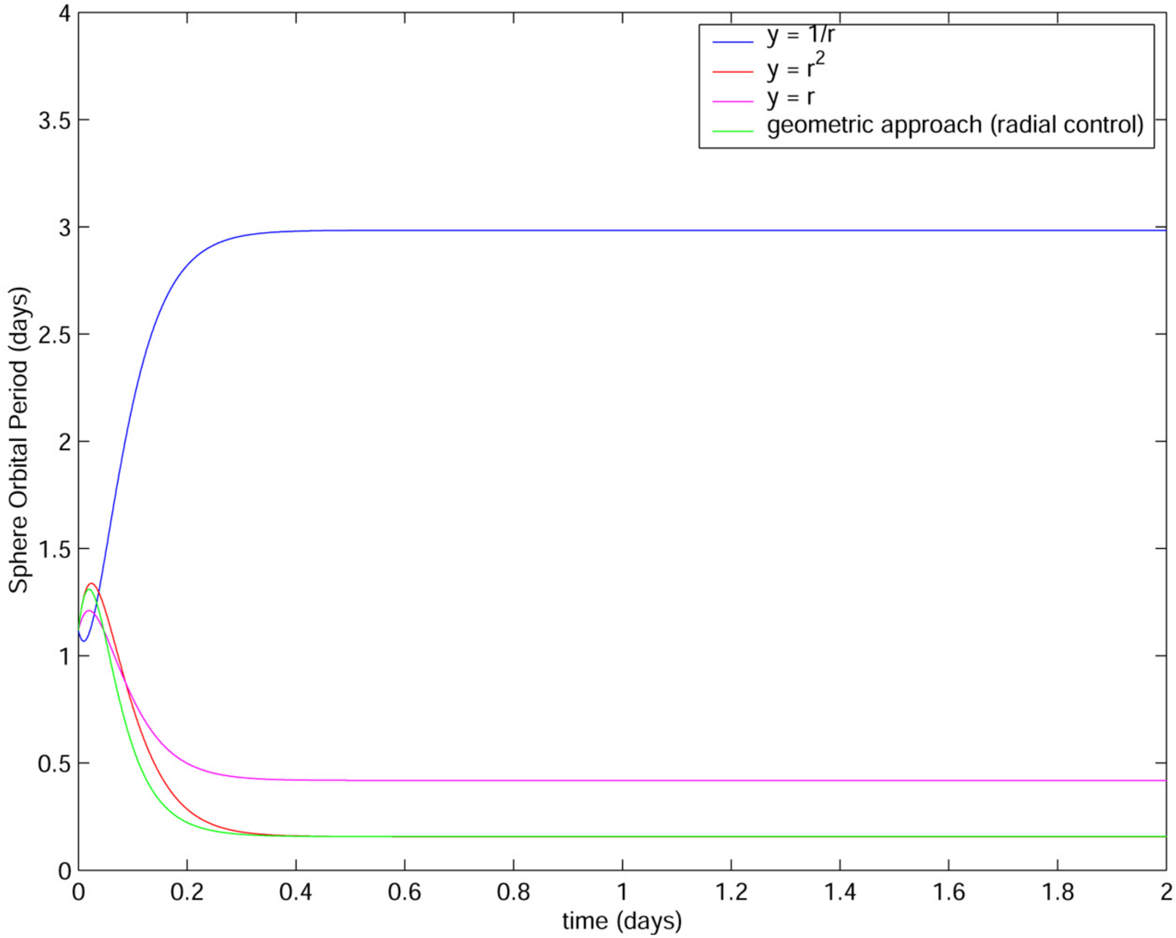
Converged Period for OFL Controlled Paths

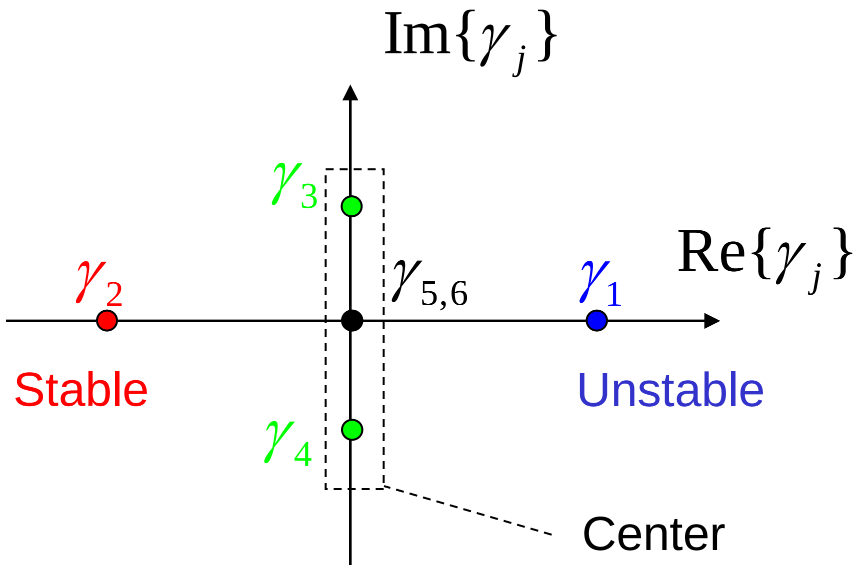


Stability of *T*-Periodic Orbits

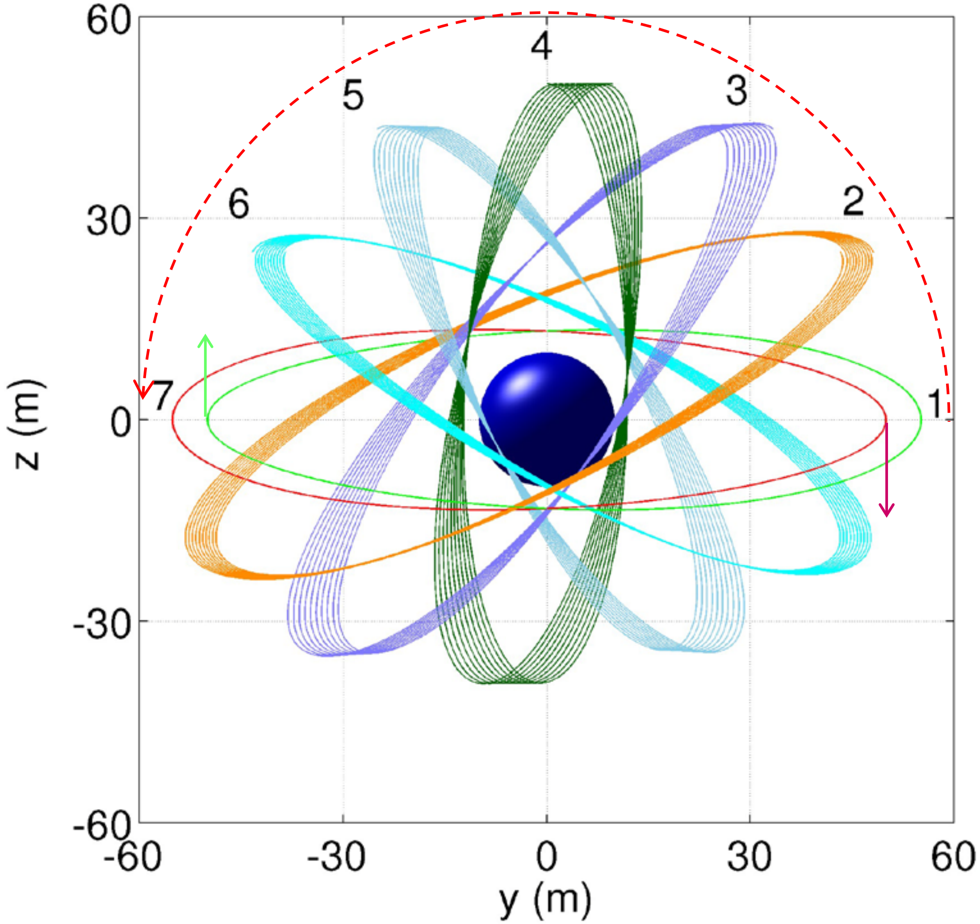
Linear Variational Equation:

$$\delta\overline{x}(t) = \Phi(t,0)\,\delta\overline{x}(0)$$

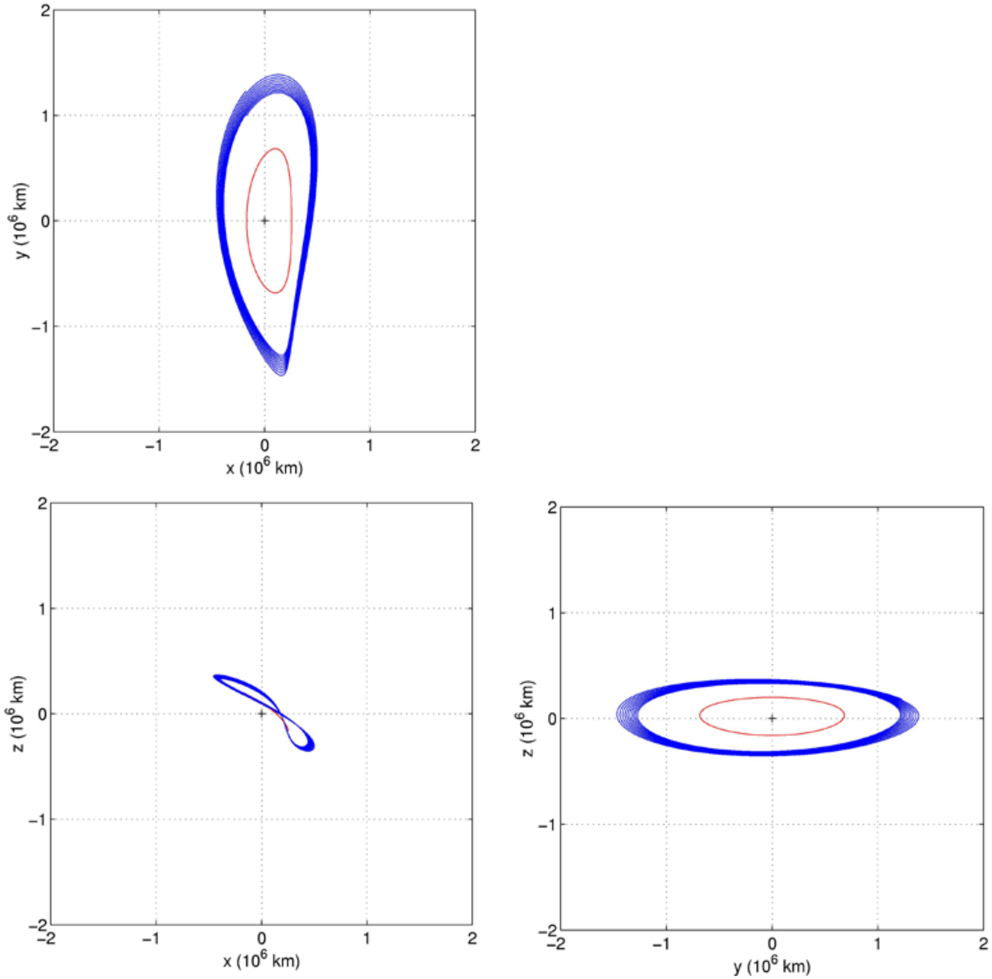
$\delta\overline{x}(t) \rightarrow$ measured relative to periodic orbit



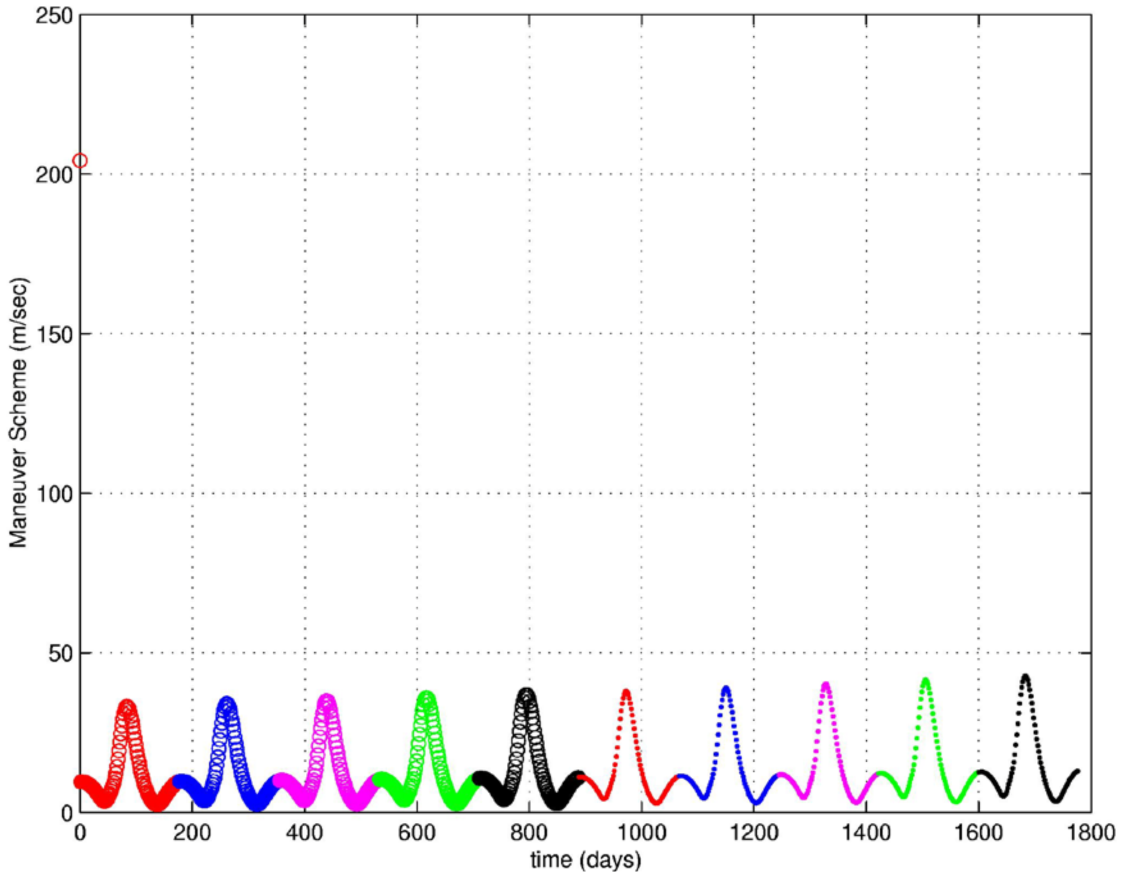
Evolution of Nearly Vertical Orbits Along the *yz*-Plane



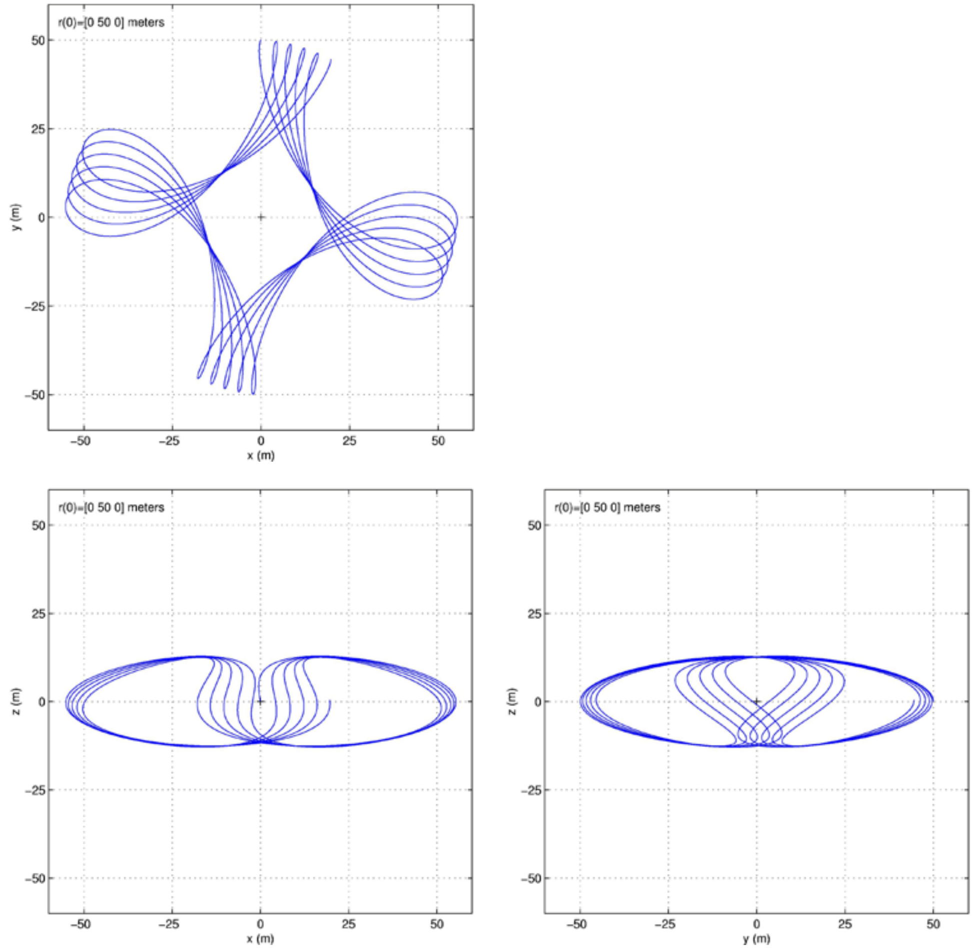
Floquet Control (Large Formations – Example 1)



Floquet Controller Maneuver Schedule (For Example 1)



Nearly Periodic Formations (Inertial Perspective)



Nearly Vertical Formations (Inertial Perspective)