Applications of Artificial Potential Function Methods to Autonomous Space Flight

Sara K. Scarritt* and Belinda G. Marchand†

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*Graduate Student, Aerospace Engineering and Engineering Mechanics, The University of Texas at Austin, 210 E. 24th St., Austin, TX 78712.

†Assistant Professor, Aerospace Engineering and Engineering Mechanics, The University of Texas at Austin, 210 E. 24th St., Austin, TX 78712.

Artificial Potential Function (APF) Methods

- Extensive use in path planning applications
- Global minimum at goal, peaks at constraints
- Vehicle follows steepest descent of potential
- Extendable to more general trajectory planning

Modification for General Trajectory Design

- Discrete control parameter ($\Delta\mathbf{v}$) vs. continuous control
- Formation flight problem - time derivative of Φ to define switching time
- Minimum of $\Phi \rightarrow$ lowest available maneuver cost
- Potential as a function of velocity error [1] (i.e. Δv)
- Dynamical model to calculate desired velocity field

[1] Neubauer, J., *Controlling Swarms of Micro-Utility Spacecraft*, Ph.D. dissertation, Washington University in St. Louis, August 2002

Potential Function Construction

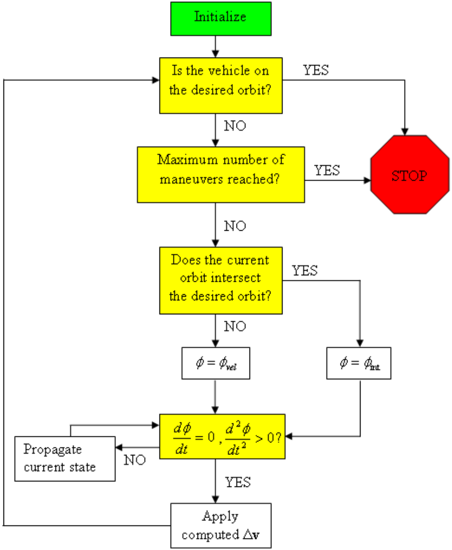
- Desired state/desired orbit (transfer time constraint)
- Two distinct cases: (1) intersecting orbits, and (2) non-intersecting orbits
- Initial orbit intersects target orbit → maneuver at intersection point:

$$\Phi_{int} = (\mathbf{r}_{intersect} - \mathbf{r}_0)^T (\mathbf{r}_{intersect} - \mathbf{r}_0),$$

- For non-intersecting orbits, $\Phi = \Phi(\Delta \mathrm{v})$:

$$\Phi_{vel} = \Delta \mathrm{v}^2,$$

APF Maneuver Planning



The Desired Velocity Field

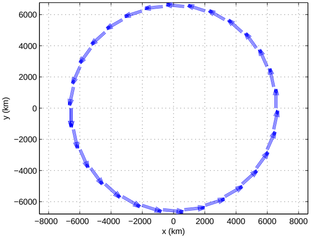
- Desired velocity depends on target point $\mathbf{r}_f$ along final orbit
- Target point assumed to be
  - apoapsis of the transfer orbit, if transferring to higher altitude
  - periapsis of the transfer orbit, if decreasing altitude
- Gives eccentricity vector direction, leads to desired velocity vector:

$$\mathbf{r}_f \rightarrow \hat{\mathbf{e}}_t \rightarrow e_t \rightarrow \mathbf{v}_t$$

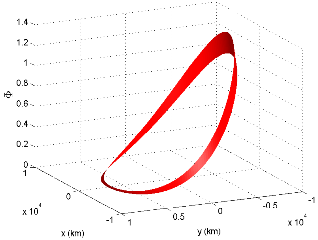
Coplanar Transfer: Desired Velocity Field

- Target point: 180° from current position

$$\hat{\mathbf{r}}_f = -\frac{\mathbf{r}_0}{r_0}.$$



(a) Velocity Field



(b) Resulting Potential

Figure: Coplanar Velocity Field and Potential Function

Coplanar Transfer

- Total Δv: 1.25 km/s
- Doubly cotangential 180° transfer - matches analytical optimal result

Table: Coplanar Transfer Initial and Target States

Parameter	Initial	Target
x (km)	-6478.145	0.000
y (km)	0.000	12587.983
z (km)	0.000	0.0000
v_x (km/s)	0.000	-4.708
v_y (km/s)	-7.844	0.000
v_z (km/s)	0.000	0.000

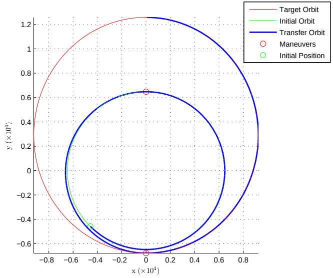
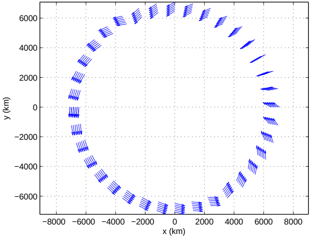


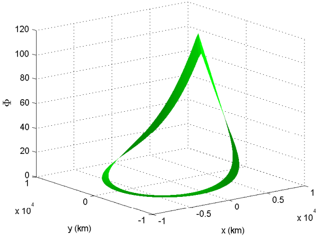
Figure: Coplanar Transfer

The Desired Velocity Field: Inclined Transfer

- Target point: intersection of initial and desired orbit planes
- Higher altitude solution to minimize cost of the plane change



(a) Velocity Field



(b) Resulting Potential

Figure: Non-Coplanar Velocity Field and Potential Function

Inclined Transfer

- Total Δv: 3.46 km/s $\rightarrow$ Compare to 6.11 km/s for Lambert solution
- Consider three-maneuver sequence to reduce plane change

Table: Inclined Transfer Initial and Target States

Parameter	Initial	Target
x (km)	-5610.238	190.512
y (km)	0.000	12396.744
z (km)	3239.073	2177.562
v_x (km/s)	0.000	-4.690
v_y (km/s)	-7.844	0.000
v_z (km/s)	0.000	0.410

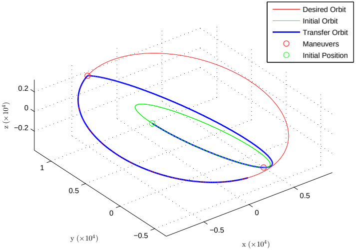


Figure: Non-Coplanar Transfer

Lunar Example (1/3)

- More complex test of APF algorithm
- Specific time needed at target state; not guaranteed by APF method as-is
- Offset targeting to do timing match, typically converges in 3-4 iterations

Table: Initial Conditions

Epoch	2-Aug-2018 17:16:06 TDT
x (km)	-1834.7155
y (km)	-66.2361
z (km)	-73.9653
v_x (km/s)	-0.0864
v_y (km/s)	0.8139
v_z (km/s)	1.4136

Table: Estimated Arrival Conditions

Epoch	7-Aug-2018 00:52:08 TDT
Geocentric Altitude (km)	121.92
Longitude (deg)	-134.5456
Geocentric Latitude	-19.20410
Geocentric Azimuth (deg)	13.9960
Geocentric Flight Path Angle (deg)	-5.8600

Lunar Example (2/3)

- Total Δv: 1.9483 km/s

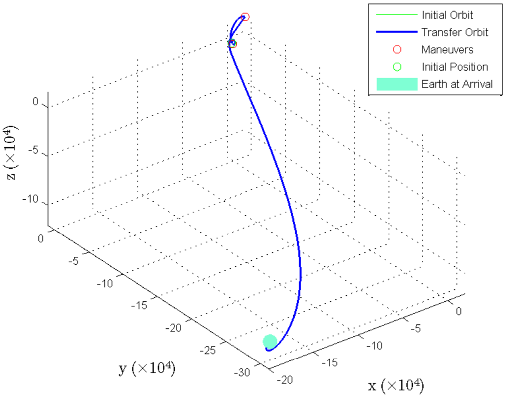


Figure: APF Lunar Return, 1.9483 km/s

Lunar Example (3/3)

- Investigate phasing effects on total cost
- Shift departure/arrival epoch by n revolutions

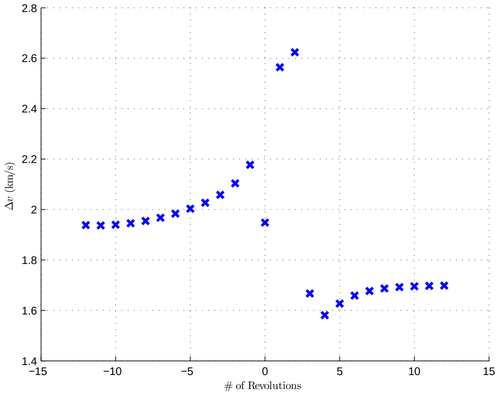


Figure: Δv of Time-Shifted Transfers vs. # of Revolutions

Conclusions

- Preliminary exploration of artificial potential function methods as a design tool for generating startup arcs
- Candidate potential function construction presented
- Method for calculating a desired velocity field developed based on two-body analysis
- APF trajectory design algorithm developed and tested
- APF method is promising, but room for improvement in design of potential field
- Future work will focus on constructing more complex potentials for use in multi-body regimes