Constellation Design for Space-Based Situational Awareness Applications: An Analytical Approach

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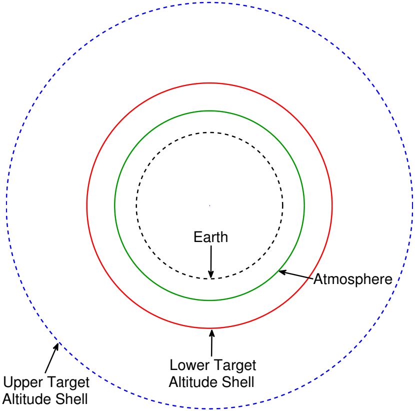
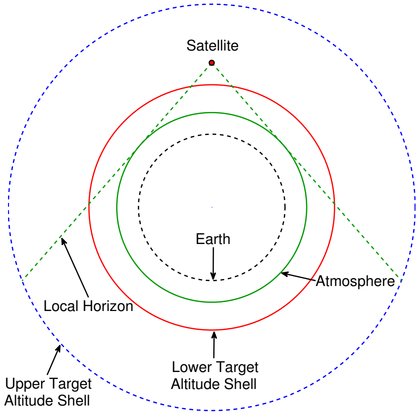
Dual-altitude Band ATH Coverage (1/4)



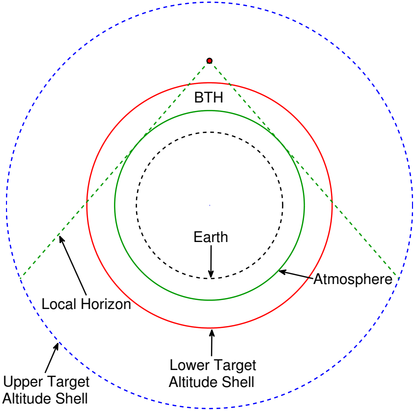
Reference boundaries

Dual-altitude Band ATH Coverage (2/4)



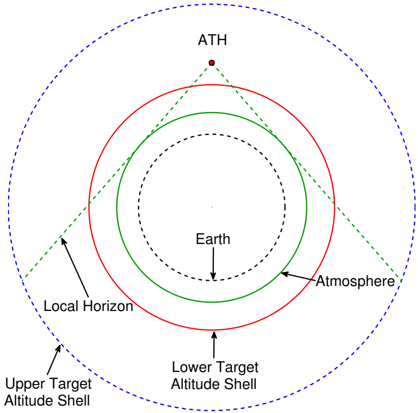
Single satellite & local horizon

Dual-altitude Band ATH Coverage (3/4)



Below-the-horizon coverage

Dual-altitude Band ATH Coverage (4/4)



Above-the-horizon coverage

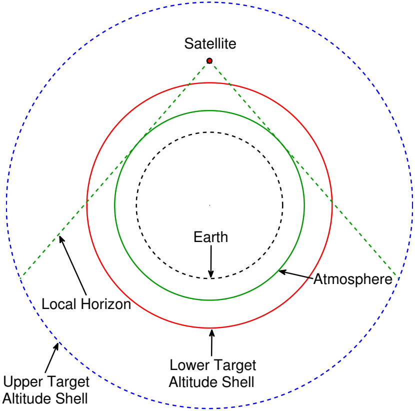
Past Studies

- Rider[1] used streets-of-coverage to determine min number of satellites for given coverage multiplicity
  - Limited in application
- Marchand & Kobel[2] used geometrical arguments to derive cost index for use with optimization
- Goal: Extend to constellations

[1] Rider, L., "Design of Low to Medium Altitude Surveillance Systems Providing Continuous Multiple Above-the-Horizon Viewing," Optical Engineering, Vol. 28, No. 1, Jan. 1989, pp. 25–29.

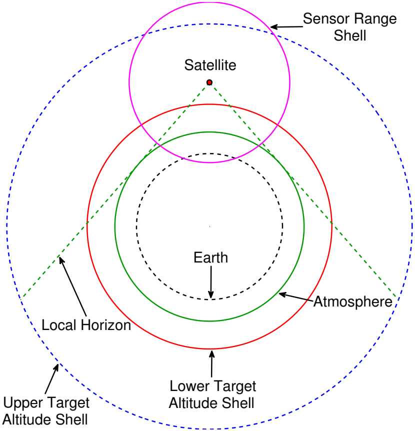
[2] Marchand, B. G. and Kobel, C. J., "Above the Horizon Satellite Coverage with Dual-Altitude Band Constraints," Journal of Spacecraft and Rockets, Vol. 46, No. 4, 2009, pp. 845–857.

Considerations & Assumptions (1/6)



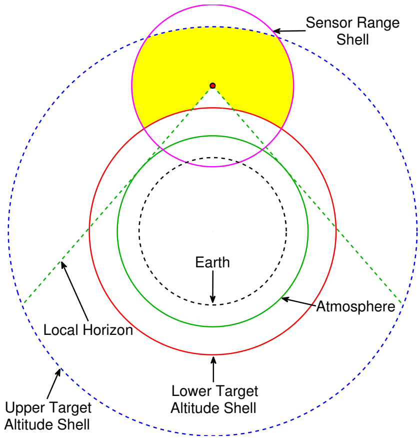
Single satellite in **circular orbit**

Considerations & Assumptions (2/6)



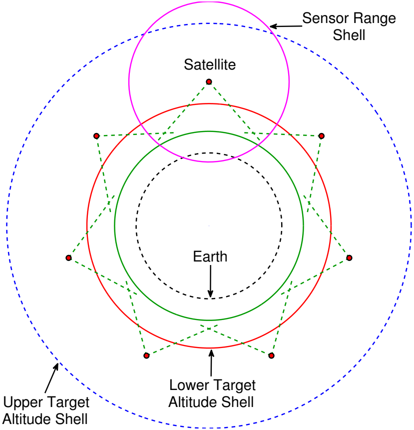
Omnidirectional sensor

Considerations & Assumptions (3/6)



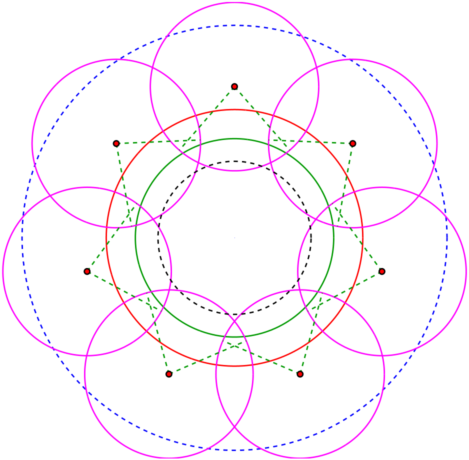
Coverage area for region of interest

Considerations & Assumptions (4/6)



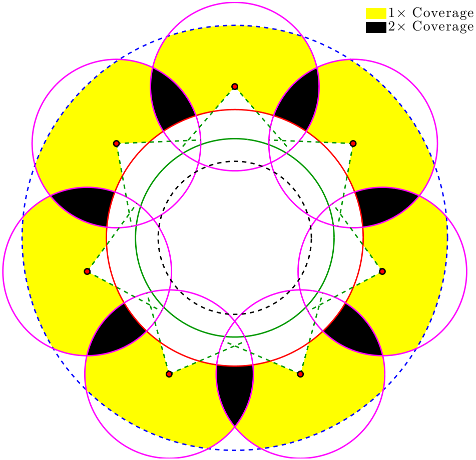
Satellites **equally spaced** along circular orbit

Considerations & Assumptions (5/6)



Equal omnidirectional sensors

Considerations & Assumptions (6/6)



Up to 2-fold coverage between adjacent satellites

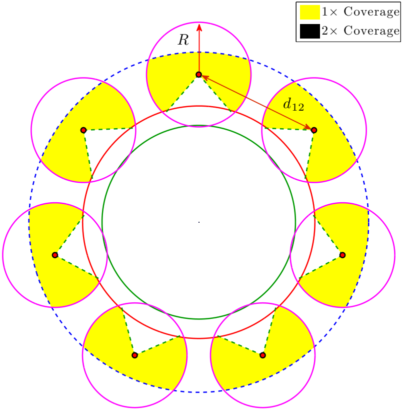
Goals

- Extend Marchand and Kobel's work to model constellation in a circular orbit
  - Make additional assumptions
  - Use geometrical arguments
  - Account for coverage multiplicities
- Demonstrate consistency in results with numerical approach proposed by Takano[3]

[3] Takano, A., "Numerical Analysis and Design of Satellite Constellations for Above the Horizon Coverage", Masters thesis, The University of Texas at Austin, December 2010.

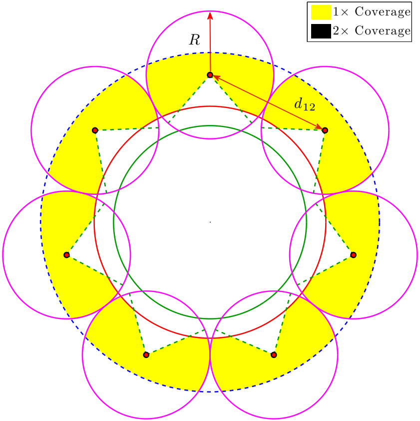
Coverage Multiplicity (1/3)

Only 1-fold coverage: $d_{12} > 2R$
2-fold coverage does not exist



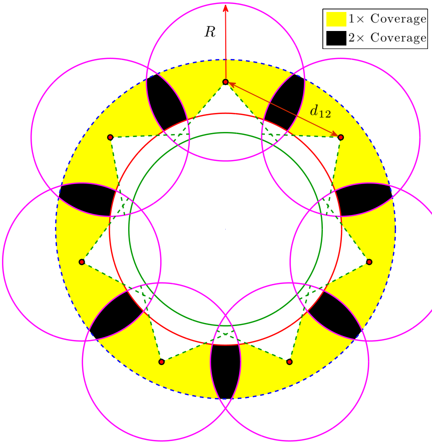
Coverage Multiplicity (2/3)

Upper limit of 1-fold coverage: $d_{12} = 2R$
2-fold coverage does not yet exist

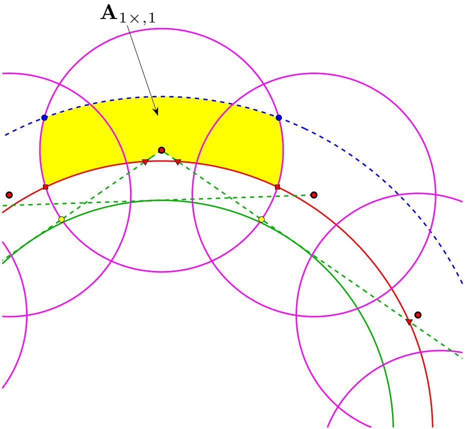


Coverage Multiplicity (3/3)

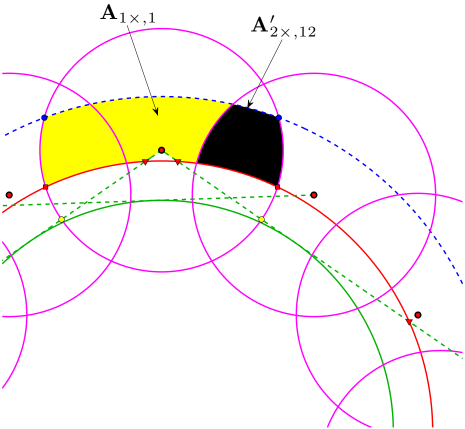
2-fold coverage created: $d_{12} < 2R$ (necessary condition)
2-fold and 1-fold coverage exist



Notation



Notation



General Formula

Consider summing all $\mathbf{A}_{1\times,i}$

- Total overlap area $\mathbf{A}'_{p\times}$ would be counted p times
- Thus, $\mathbf{A}'_{p\times}$ must be subtracted $p-1$ times from the sum

Total coverage area:

$$\mathbf{A}_{1\times} = \sum_{i=1}^{n} \mathbf{A}_{1\times,i} - \sum_{p=2}^{p_{max}} (p-1)\mathbf{A}'_{p\times}$$

Simplifications

- Maximum 2-fold coverage is considered
- 2-fold coverage considered only for adjacent satellites
- Due to symmetry, overlap area is equal for each pair of adjacent satellites

Final Result

For 2-fold coverage, the total coverage area reduces to

$$\mathbf{A}_{1\times} = n \left(\underbrace{\mathbf{A}_{1\times,1}}_{\substack{\text{area covered}\\ \text{by single sat}}} - \underbrace{\mathbf{A}'_{2\times,12}}_{\substack{\text{overlap area for}\\ \text{2 adjacent sats}}} \right)$$

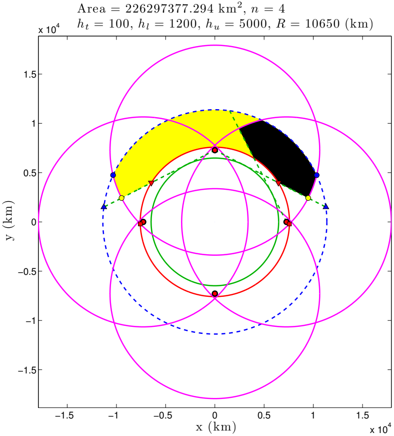
Notes:

- Equation gives the total coverage area
- $\mathbf{A}_{1\times,1}$ computed from results of Marchand and Kobel
- Computing $\mathbf{A}'_{2\times,12}$ becomes primary focus

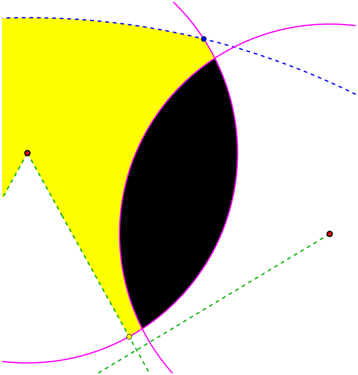
Overlap Area Components

⟶ Express complex shapes in terms of fundamental ones:

- Triangles
- Circular segments
- Quadrilaterals
- Combinations

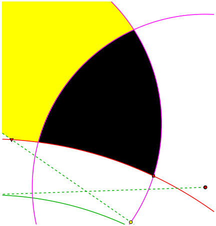


Example Overlap Area Shapes: 2 Vertices

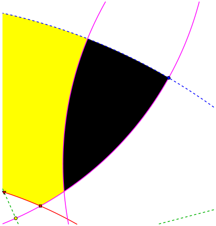


2

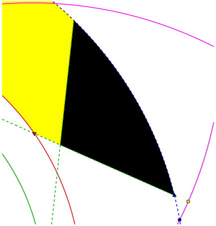
Example Overlap Area Shapes: 3 Vertices



3.i



3.ii



3.iii.b

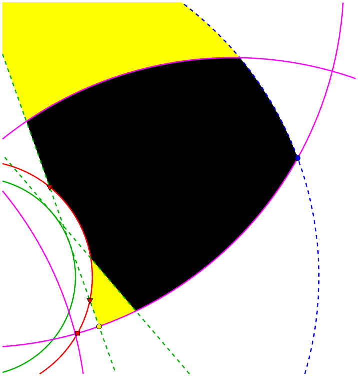
Taxonomy of Overlap Areas

Table: Relation between the Number of Vertices and Number of Unique Shapes

Number of Vertices	Number of Unique Shapes
2	1
3	3
4	3
5	3
6	3
7	2
8	1

- 16 unique shapes
- 22 cases

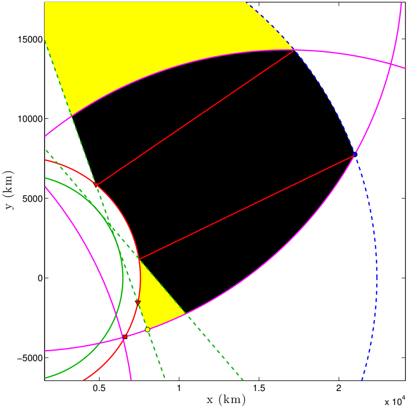
Example Calculation: Case 6.i.a (1/5)



overlap area =

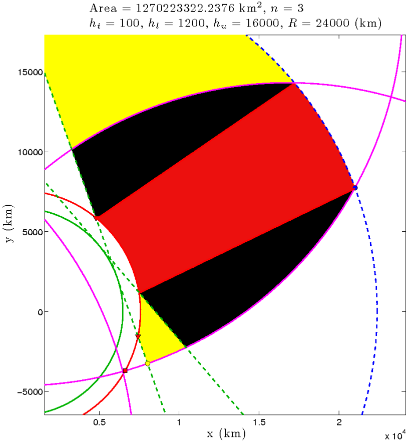
Example Calculation: Case 6.i.a (2/5)

Area = 1270223322.2376 km^2, n = 3
h_t = 100, h_l = 1200, h_u = 16000, R = 24000 (km)



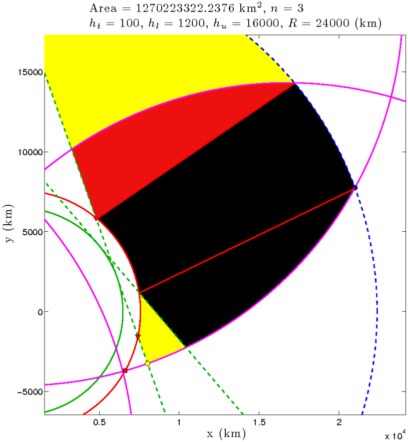
sum of three fundamental areas =

Example Calculation: Case 6.i.a (3/5)



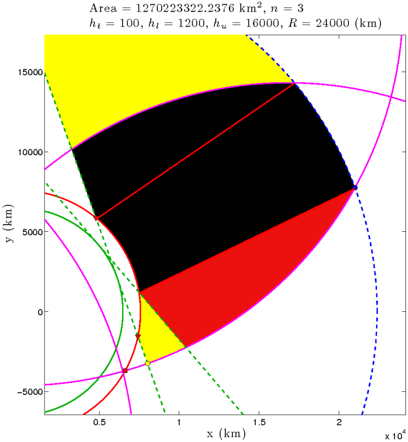
composite quadrilateral +

Example Calculation: Case 6.i.a (4/5)



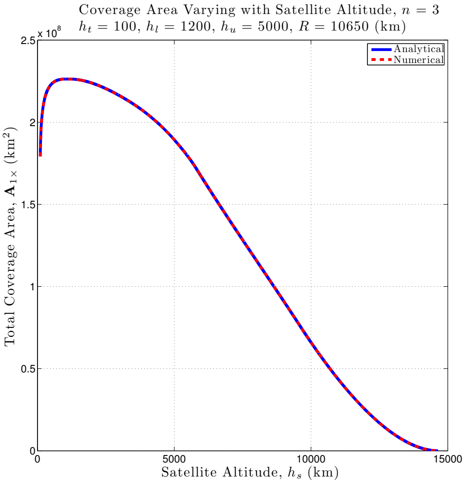
composite triangle +

Example Calculation: Case 6.i.a (5/5)



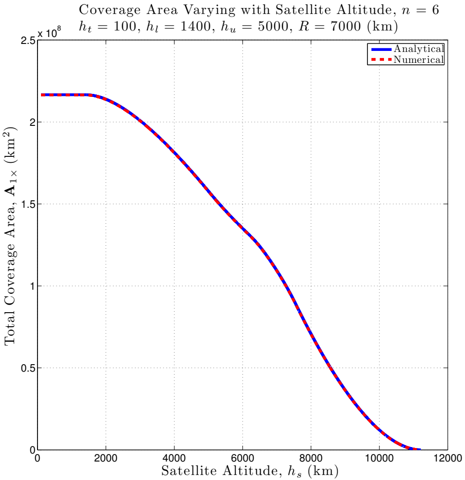
composite triangle

Consistency with Numerical Results (1/3)



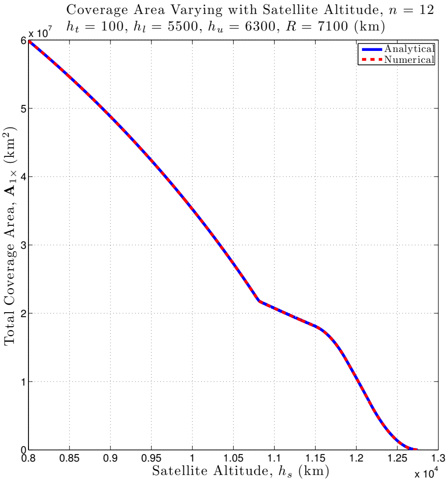
Covers 5 cases

Consistency with Numerical Results (2/3)



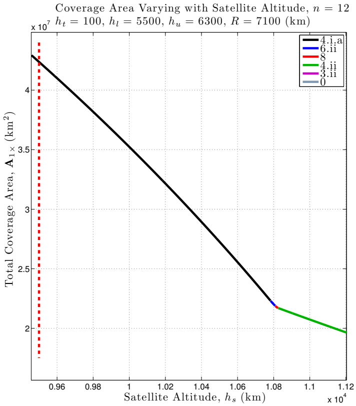
Covers 5 cases

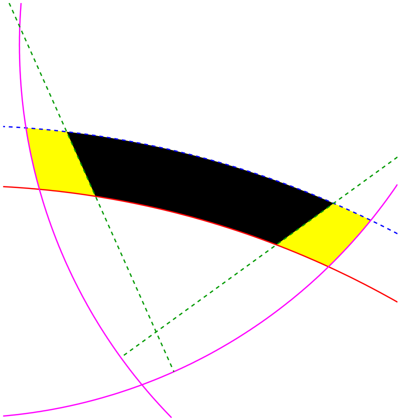
Consistency with Numerical Results (3/3)



Covers 5 cases

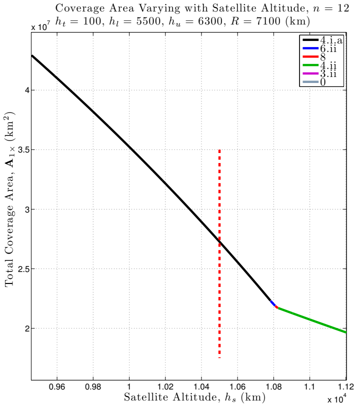
Explanation of Kink: Before, $r_s = 9,500$ km

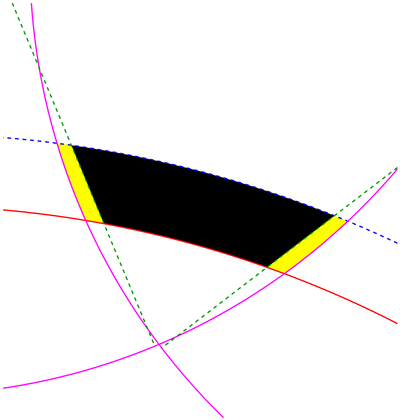




Overlap area shape: **4.i.a**

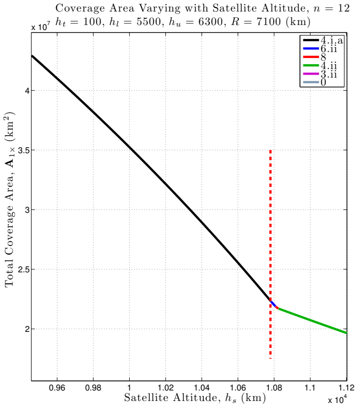
Explanation of Kink: Before, $r_s = 10,500$ km

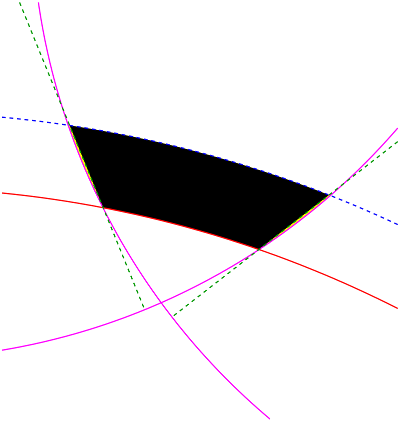




Overlap area shape: **4.i.a**

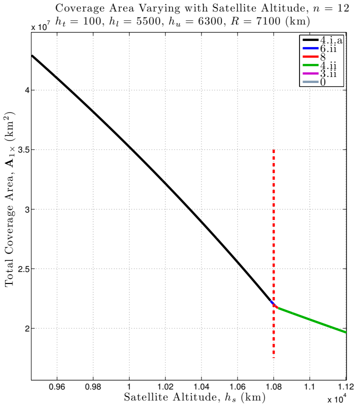
Explanation of Kink: During, $r_s = 10,780$ km

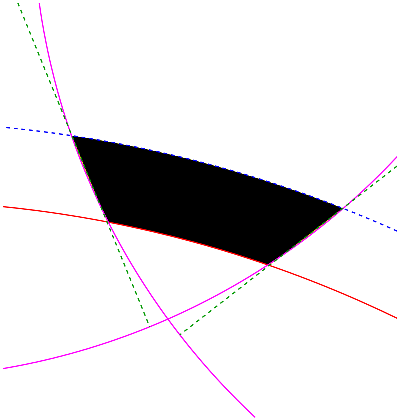




Overlap area shape: **4.i.a**

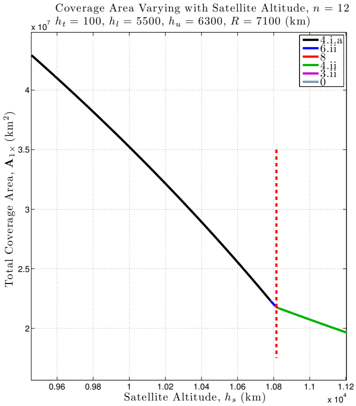
Explanation of Kink: During, $r_s = 10,800$ km

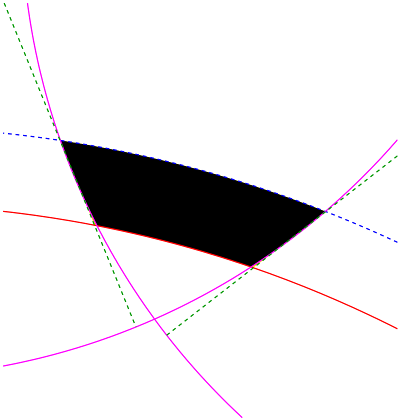




Overlap area shape: **6.ii**

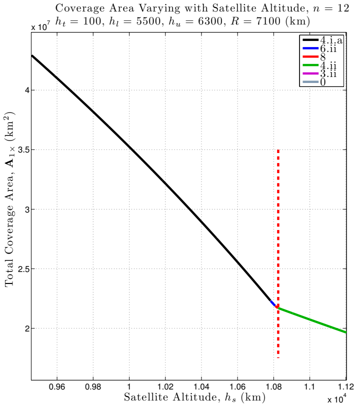
Explanation of Kink: During, $r_s = 10,815$ km

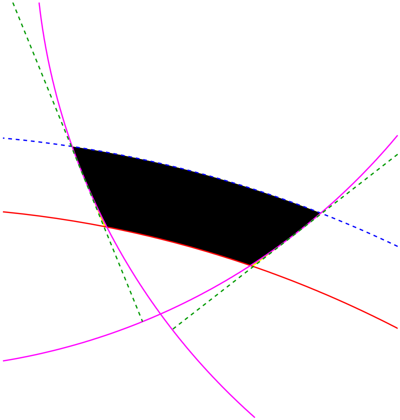




Overlap area shape: **8**

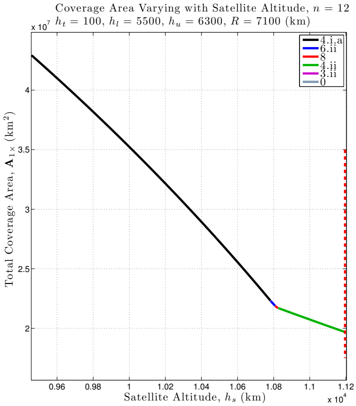
Explanation of Kink: During, $r_s = 10,825$ km

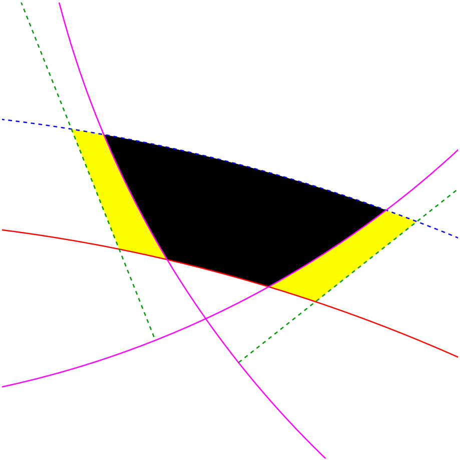




Overlap area shape: **4.ii**

Explanation of Kink: After, $r_s = 11,200$ km





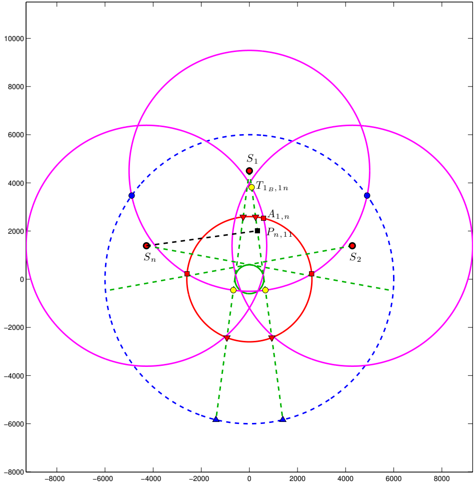
Overlap area shape: **4.ii**

Conclusion

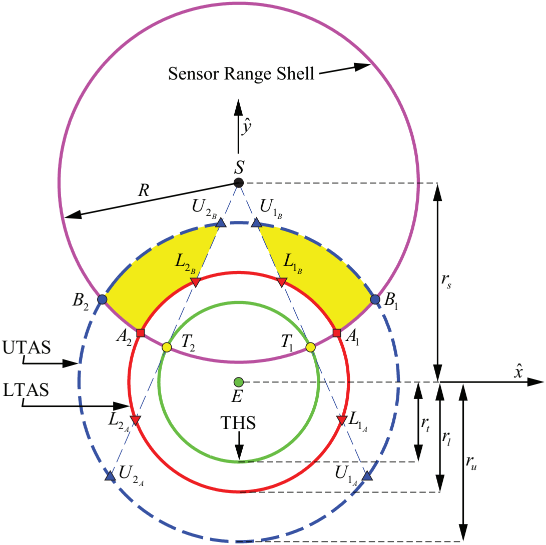
- Developed an exact expression for up to 2-fold coverage
- Exact objective function agrees with numerical model developed by Takano
- Objective function suitable for use in optimal constellation design

Questions

Alternative 2-fold Coverage (non-adjacent satellites)

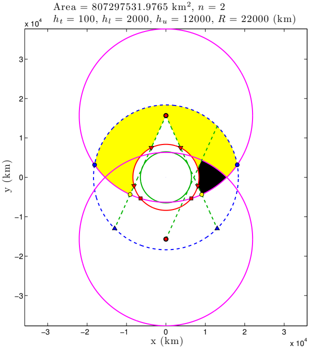


Space-Based Sensors: ATH Coverage



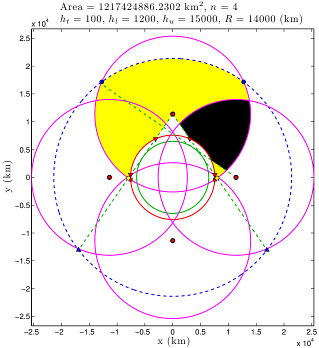
Definition of Overlap Area

$n = 2$



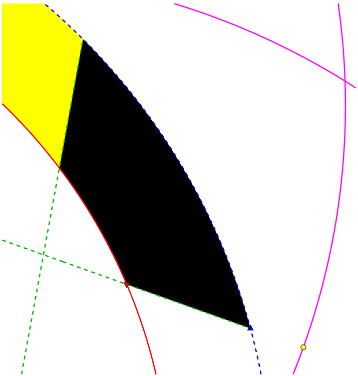
$$\mathbf{A}'_{2\times,12} = \frac{1}{2}\left(\mathbf{A}_{1\times,1} \cap \mathbf{A}_{1\times,2}\right)$$

$n > 2$

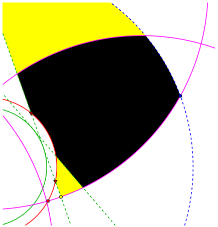


$$\mathbf{A}'_{2\times,12} = \mathbf{A}_{1\times,1} \cap \mathbf{A}_{1\times,2}$$

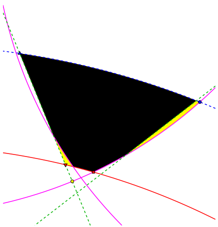
Example Calculation 2: Case 4.i.b



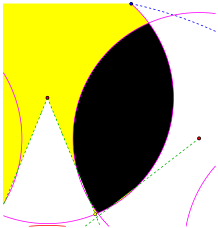
Example Overlap Area Shapes: 6 Vertices



6.i.a

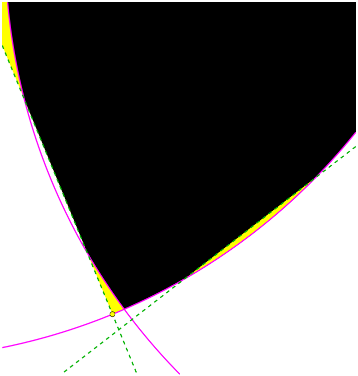


6.ii



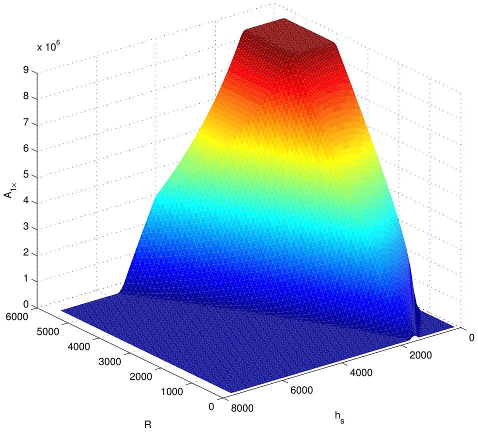
6.iii

Overlap Area Shape: Case 6.iii Close-up



6.iii

Larger Parameter Space: Fix r_u



Larger Parameter Space: Fix R