Actuator Constrained Optimal Control of Formations Near the Libration Points

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Outline

Formation Limitations for Deep-Space Imaging Formations

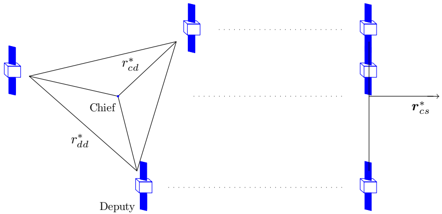


Figure: Formation Pointing

- Fixed size, shape, and orientation of the formation
- Fixed orientation of each member of the formation (deputy spacecraft)

Dynamical Sensitivities Near the Libration Points

- Previous investigations have focused on unconstrained continuous control solutions
  - Linear and nonlinear; feasible and optimal solutions
  - Non-natural formations require extremely precise control ($<$ nm/s^2 accelerations)
- These controls are impossible to implement with existing actuator technology

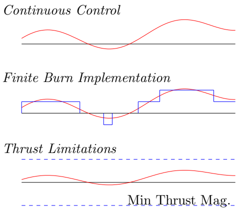


Figure: Implementing a Continuous Control Solution

- Cannot reproduce the fidelity of continuous control
- Continuous control may even be *smaller* than minimum thrust bound

Control Limitations for Deep-Space Imaging Formations

- Fixed thruster location on each spacecraft body
- Specified thrust acceleration magnitude
  - Based on actuator performance capability

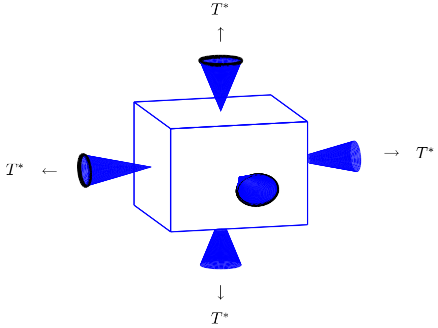


Figure: Spacecraft Body

Transcription Methods for Highly Constrained Problems

- The libration point formation problem motivates a unique solution method
  - Direct optimization methods serve as the foundation
  - Modifications allow for creative treatment of difficult constraints

Solution methods are generalized for any number of dynamical models.

Optimization via Direct Transcription

- Define a parameter vector consisting of state and control values at *nodes* (discrete points in time)

$$\boldsymbol{x} = [\cdots\ \boldsymbol{y}^T(t_j)\ \cdots\ \ \cdots\ \boldsymbol{u}^T(t_j)\ \cdots\ \ t_0\ t_f]^T$$

- Convert the Optimal Control Problem into a Parameter Optimization Problem

Minimize

$$J = \phi(t_0, \boldsymbol{y}_0, t_f, \boldsymbol{y}_f) + \int_{t_0}^{t_f} L(t, \boldsymbol{y}, \boldsymbol{u})\, dt \quad \Rightarrow$$

subject to

$$\begin{aligned} \dot{\boldsymbol{y}} &= \boldsymbol{f}(t, \boldsymbol{y}, \boldsymbol{u}) \\ \boldsymbol{0} &= \boldsymbol{\psi}_0(t_0, \boldsymbol{y}_0) \\ \boldsymbol{0} &= \boldsymbol{\psi}_f(t_f, \boldsymbol{y}_f) \\ \boldsymbol{0} &= \boldsymbol{\beta}(t, \boldsymbol{y}, \boldsymbol{u}) \end{aligned}$$

Minimize

$$F(\boldsymbol{x})$$

subject to

$$\Rightarrow \quad \boldsymbol{c}(\boldsymbol{x}) = \left[\boldsymbol{c}_{\psi_0}^T(\boldsymbol{x})\ \boldsymbol{c}_{\psi_f}^T(\boldsymbol{x})\ \boldsymbol{c}_{\beta}^T(\boldsymbol{x})\ \boldsymbol{c}_{\dot{y}}^T(\boldsymbol{x})\right]^T = \boldsymbol{0}$$

- Solve the resulting optimization problem with a standard Nonlinear Programming (NLP) algorithm

Multiple Segment Formulations

- Account for state or control discontinuities by dividing the problem into segments
  - Ideal treatment for finite burn control solutions
- Enforce appropriate constraints at the *knots* (segment boundaries)
- Include knot times or segment durations in parameter vector

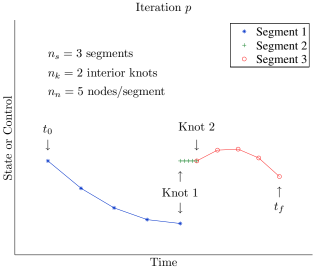


Figure: An Example of Segments and Knots

Impacts of Fixed Spacecraft Orientation

- A traditional finite burn formulation specifies thrust (acceleration) magnitude, but not direction
  - Assumes spacecraft can re-orient to deliver required thrust vector
  - Control space $\mathcal{U}_1$: $\boldsymbol{u}^T\boldsymbol{u} = (T^*)^2$
- If spacecraft orientation is predetermined (according to other mission requirements)
  - Actuator configuration must provide 3-axis maneuverability
    - Assume thrusters are located on principal axes of body frame $\mathcal{B} \equiv \{\hat{\boldsymbol{x}}_\mathcal{B}, \hat{\boldsymbol{y}}_\mathcal{B}, \hat{\boldsymbol{z}}_\mathcal{B}\}$
  - Control space $\mathcal{U}_2$: $u_i(u_i - T^*)(u_i + T^*) = 0, i = \hat{\boldsymbol{x}}_\mathcal{B}, \ldots, \hat{\boldsymbol{z}}_\mathcal{B}$

Fixed spacecraft orientation leads to discrete optimization, which gradient-type NLP algorithms cannot support.

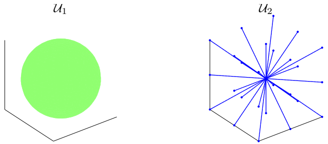


Figure: Control Spaces (a) $\mathcal{U}_1$ (Orientation Free), and (b) $\mathcal{U}_2$ (Orientation Fixed)

Managing Fixed Spacecraft Orientation

- Instead of optimizing control values (i.e. $-T^*, 0, T^*$), …

Prespecify control values by segment and optimize switching times

  - Knots are used to designate switching times in each control axis
  - Segments are bounded by switches in any control
  - The chronological ordering of knots changes at each iteration of the optimization

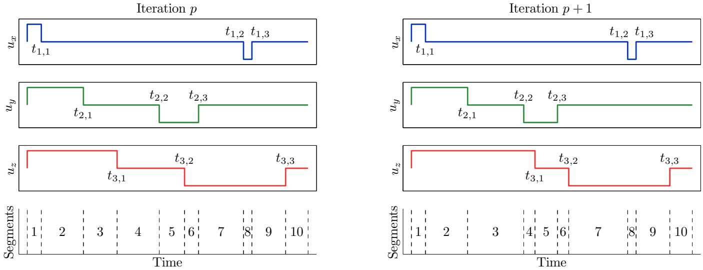


Figure: Conceptual Control Profile with Segment Divisions

Costs and Constraints

- Constraints
  - Initial time and states specified
  - Final time and formation size and plane specified
    - $r^*_{cd} = 1$ km distance between chief and deputy, $r^*_{dd} = 1.73$ km distance between deputies
    - Specified pointing $\boldsymbol{r}^{*\mathcal{I}}_{cs} = [1\ 0\ 0]$
  - State continuity (differential constraints) by segment
  - State equality across segments (at knots)
- Weighted Costs
  - Minimize thrust
  - Minimize formation size deviations along trajectory
  - Minimize formation plane deviations along trajectory

$$
\begin{aligned}
J &= w_1 J_1 + w_2 J_2 + w_3 J_3 \\
F(\boldsymbol{x}) &= w_1 F_1(\boldsymbol{x}) + w_2 F_2(\boldsymbol{x}) + w_3 F_3(\boldsymbol{x})
\end{aligned}
$$

Baseline Initial Guess

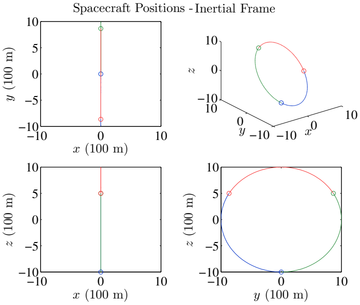
Trajectory Legend

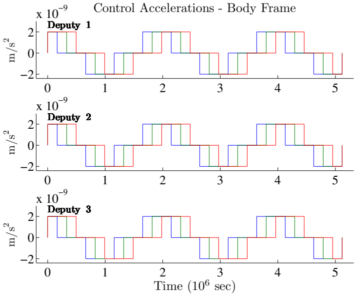
- Deputy 1 Trajectory
- Deputy 2 Trajectory
- Deputy 3 Trajectory

t_0 t_f

Control Legend

- Axis 1 Control (u_x)
- Axis 2 Control (u_y)
- Axis 3 Control (u_z)





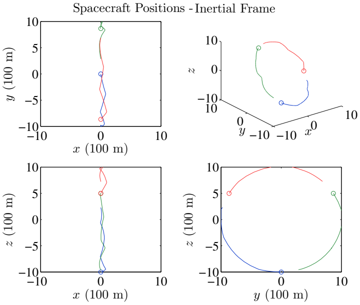
Baseline Solution

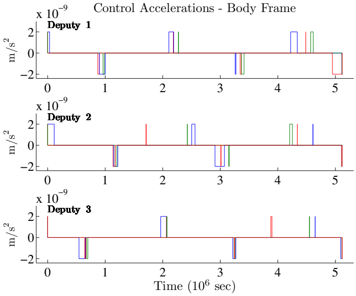
Trajectory Legend

Deputy 1 Trajectory
Deputy 2 Trajectory
Deputy 3 Trajectory
t_0 t_f

Control Legend

Axis 1 Control (u_x)
Axis 2 Control (u_y)
Axis 3 Control (u_z)





Conclusions

- A modified collocation method with a segment-time switching algorithm leads to highly constrained control solutions
- Generalized formulation allows users to input
  - formation configuration, size, orientation, and rotation rate
  - thruster capability and placement
  - dynamic model and reference trajectory
  - initial and terminal conditions
- Suited to aid in establishing requirements and capabilities for highly constrained formations

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Varying Parameters to Obtain Different Solutions

- Final time
- Number of nodes and knots
- Initial guess
- Thrust magnitude

Table: Comparison of Solutions with Various Parameters

	Baseline	t_f	n_k, t_f	Feasible Guess	Thrust
n_n	4	4	6	4	4
n_k	10	10	20	10	10
t_f (10^6 sec)	5.1183	10.2366	10.2366	5.1183	5.1183
Guess	Baseline	Baseline	Baseline	Feasible	Baseline
w_{Thrust}	$\frac{1}{400}$	$\frac{1}{400}$	$\frac{1}{400}$	$\frac{1}{400}$	$\frac{1}{1600}$
w_{Distance}	0.1	0.1	0.1	0.1	0.1
w_{Plane}	1	1	1	1	1
Thrust (km/s^2)	2.0e-12	2.0e-12	2.0e-12	2.0e-12	4.0e-12
n	2333	2333	6779	2333	2333
# Iterations	45	157	194	95	272
Computational Time (sec)	253.91	956.41	3000.35	575.88	1570.04
Weighted Thrust Cost	1.6233	5.8133	6.6838	10.9247	1.1324
Weighted Formation Cost	16.1818	634.3255	67.7092	35.5675	6.9286
Weighted Plane Cost	0.8591	84.6949	4.6852	4.9035	1.0632
Total Cost	18.6643	724.8338	79.0782	51.3956	9.1242