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Effect of Update Rate on Perceived Instability of Virtual Haptic Texture

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Abstract— **This study investigates the effect of update rate on the perceived quality of haptic virtual textures, focusing on one type of perceived instability called "buzzing." Buzzing refers to the high-frequency noises that may emanate from a force-feedback device during haptic texture rendering. We first present a simulation of a haptic texture rendering system that explicitly models the sampled-data nature of the system. The simulation shows that a slow haptic update rate can significantly increase the high-frequency noise in the reconstructed force command sent to the haptic interface. This noise may excite the structural resonance of the haptic interface and result in a high-frequency buzzing at the interaction tool that a user holds. A subsequent psychophysical experiment, conducted over a wide range of update rate (250 Hz - 40 kHz), has confirmed the simulation results. The results show a 278% increase in the maximum stiffness (0.325 – 0.904 N/mm over the update rates tested) that could be used for rendering perceptually "clean" virtual haptic textures. These results argue for haptic update rates that are much higher than the widely-accepted value of 1 kHz. Considering that an application requiring hard surface rendering needs a stiffness value of at least 0.8 – 1.0 N/mm, we recommend a haptic update rate in the range 5 – 10 kHz for perceptually stable haptic texture rendering.**

I. Introduction

This paper presents new findings from our ongoing investigation with regard to the effect of haptic update rate on the perceived instability of virtual haptic textures. *Perceived instability* refers to any unrealistic sensations (e.g., buzzing or aliveness) that cannot be attributed to the physical properties of the virtual haptic textured surfaces being rendered by a force-feedback haptic interface (see [1]–[4] for our previous studies on this topic). It occurs frequently with virtual haptic textures rendered with current techniques (for example, see [5], [6]; see also [2] for a review). Eliminating perceived instability from virtual haptic textures is an immediate challenge that we are facing in order to achieve realistic rendering of virtual haptic objects with textures.

Our research group is among the first to have systematically investigated the perceived instability of virtual haptic textures. Our previous studies have concentrated on the analysis of a widely-used haptic texture rendering system in terms of perceived instability. For virtual haptic textures rendered with a popular force-feedback haptic interface (PHANToM), we first conducted psychophysical experiments to measure the parameter space within which virtual textured surfaces were perceived to be stable and realistic [1]–[3]. The effects of many factors including texture model parameter, rendering method, collision detection algorithm, and exploration mode were examined. Our results indicated that the parameter spaces for perceptually stable texture rendering were very small such that the stiffest textured plane that could be rendered without perceived instability felt like soft corduroy.

From our earlier studies, we discovered three primary types of perceived instability: buzzing, aliveness, and ridge instability (see [1] for details). For each type of perceived instability, we characterized the proximal stimuli that caused its percept and identified the sources [1]–[4]. We discovered that each type of perceived instability originated from different sources of the haptic texture rendering system. Specifically, device instability was responsible for buzzing, inaccurate virtual environment modeling was responsible for aliveness and ridge instability, and the relatively poor kinesthetic sense of the human somatosensory system was partially to be blamed for the perception of aliveness. These findings underscore the complexity of the mechanisms underlying perceived instability.

Our current work examines the effect of update rate on haptic rendering of virtual textures. In general, better quality and user performance can be achieved through virtual haptic environments rendered at higher update rates [7], [8]. Using a high update rate, however, limits the complexity of virtual environment that can be simulated in real time. It follows that an optimal update rate depends on the properties of virtual objects and haptic interface used for rendering. At present, 1 kHz is typically accepted for rendering rigid objects as a good compromise between the quality of virtual objects and the environment complexity. Beyond that, there are few general guidelines on the minimum update rate needed for a specific haptic rendering system. It goes without saying that such guidelines are extremely important for haptic rendering.

Recently, we have succeeded in developing a haptic texture rendering system with a very fast update rate of 50 kHz (See Appendix for details). We observed a quite noticeable improvement in the perceived quality of virtual textures rendered at this rate. Our ability to render virtual haptic textures at up to 50 kHz has made it possible for us to systematically examine the effect of update rate on haptic texture rendering. This paper reports the maximum stiffness of textured surfaces that could be rendered without

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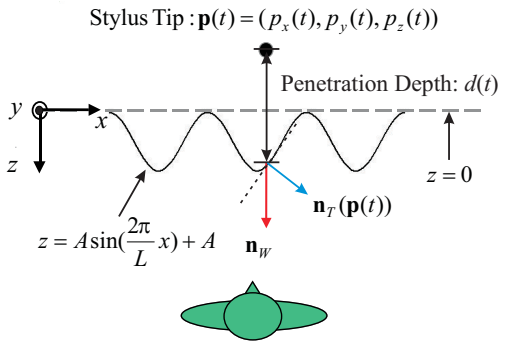

Fig. 1. An illustration of parameters used in haptic texture rendering.

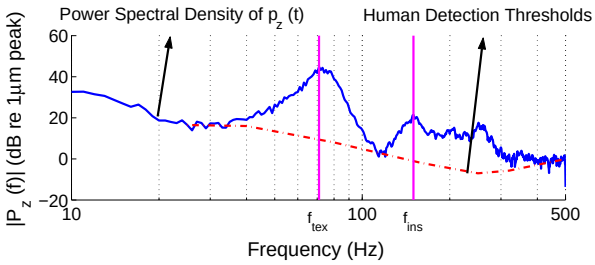

Fig. 2. Frequency domain characteristics of the signals responsible for buzzing.

perceived instability for a wide range of update rates (250 Hz – 40 kHz).

In the remainder of this paper, we define the texture rendering model and methods (Sec. II), introduce a typical perceived instability called "buzzing" (Sec. III), present simulation results on the expected effect of haptic update rate on buzzing (Sec. IV), summarize the main psychophysical experiment (Sec. V), and discuss the implications of our results (Sec. VI).

II. Virtual Haptic Texture

In this section, we introduce the haptic interface, the texture model and rendering methods used in our study. We used the PHANToM 1.0A with an encoder gimbal (SensAble Technologies; Woburn, MA, USA) for the simulation and experiment reported in this paper. The user faced a vertical virtual textured surface rendered with the PHANToM (see Fig. 1) and used the PHANToM stylus to feel the textured surface. The texture was modeled as

$$z = h(x) = A\sin\left(\frac{2\pi x}{L}\right) + A \tag{1}$$

in the PHANToM world coordinate frame, where A and L denote the amplitude and spatial wavelength of the sinusoidal textures, respectively. This texture model was superimposed on a 3D plane $z = 0$, forming the textured surface shown in Fig. 1.

Given the position of the PHANToM stylus tip $\mathbf{p}(t) = (p_x(t), p_y(t), p_z(t))$, the penetration depth $d(t)$ was defined as

$$d(t) = \begin{cases} 0 & \text{if } p_z(t) > h(p_x(t)) \\ h(p_x(t)) - p_z(t) & \text{if } p_z(t) \le h(p_x(t)) \end{cases} \tag{2}$$

We then computed forces using the following two methods:

$$\mathbf{F}_{mag}(t) = Kd(t)\mathbf{n}_W \tag{3}$$

$$\text{and } \mathbf{F}_{vec}(t) = Kd(t)\mathbf{n}_T(\mathbf{p}(t)), \tag{4}$$

where K was the surface stiffness, $\mathbf{n}_W$ was the normal vector of the underlying wall, and $\mathbf{n}_T(\mathbf{p}(t))$ was the normal of the textured surface at $\mathbf{p}(t)$. These methods were proposed in [5] and [6], respectively, and produce perceptually disparate virtual haptic textures even for the same model parameters. The constant-direction method $\mathbf{F}_{mag}(t)$ generates smooth textures while the varying-direction method $\mathbf{F}_{vec}(t)$ renders textures that feel rougher and sometimes sticky.

III. Perceived Instability: Buzzing

Our current work has focused on the relation between haptic rendering update rate and one class of perceived instability called "buzzing." Buzzing refers to high-frequency noise-like forces emanating from a force-feedback device during haptic texture rendering. This section summarizes the physical and perceptual characteristics of buzzing and discusses its source (see [1]–[3] for further details).

When a user strokes a virtual textured surface using a stylus, s/he receives texture information (e.g., the sinusoidal bumps in Fig. 1) in the form of a vibration originated from the stylus interacting with a textured surface. In addition to the ever-present vibration due to texture, the user sometimes perceives buzzing that appears to be at a higher frequency. Buzzing can occur with haptic textures rendered with the $\mathbf{F}_{mag}(t)$ or the $\mathbf{F}_{vec}(t)$ method, and is more apparent when the surface stiffness is relatively high. Buzzing is also more frequently observed than any other types of perceived instability (such as aliveness or ridge instability) and is usually more intense perceptually.

The solid line in Fig. 2 shows a typical power spectral density of $p_z(t)$ (the position of the PHANToM stylus tip along the normal direction of the underlying wall, as shown in Fig. 1) recorded during buzzing. The dash-dotted line indicates human detection thresholds at the corresponding frequencies reproduced from [9] (see [2] for details). The first spectral peak at $f_{tex}(= 71$ Hz) corresponds to the texture-related vibration. The location of f_{tex} can be predicted based on the user's stroking velocity and the spatial wavelength of the sinusoidal texture model (again, see [2] for details). The user perceived the surface texture through the vibration at this frequency.

From Fig. 2, we can also observe a significant amount of energy in the high-frequency range starting from $f_{ins}(=$ 150 Hz). The spectral components at f_{tex} and at $\ge f_{ins}$ are well separated in frequency, and are well above the corresponding human detection thresholds. As is well known in the haptic psychophysics literature [10], vibrations occurring in the two spectral peak regions give rise to intense and distinctive perception. The user perceives the spectral component at f_{tex} as texture information, and regards the high-frequency vibrations starting from f_{ins} as an indication of perceived instability.

We found that the high-frequency buzzing noise was most likely caused by the mechanical resonance of the PHANToM based on a measured frequency response of

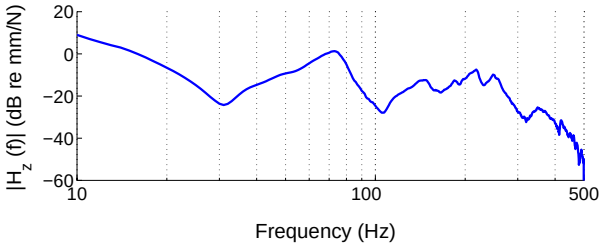

Fig. 3. A z-axis open-loop frequency response of the PHANToM 1.0A.

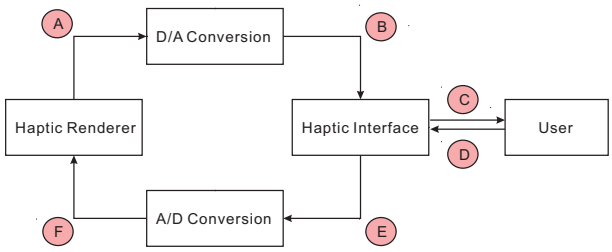

Fig. 4. Structure of the haptic rendering system used in simulation.

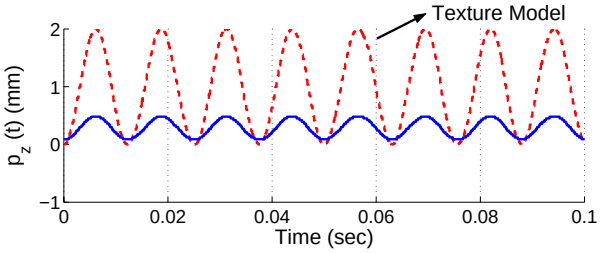

Fig. 5. Position in the z-direction (signal F in Fig. 4) with the corresponding sinusoidal texture model.

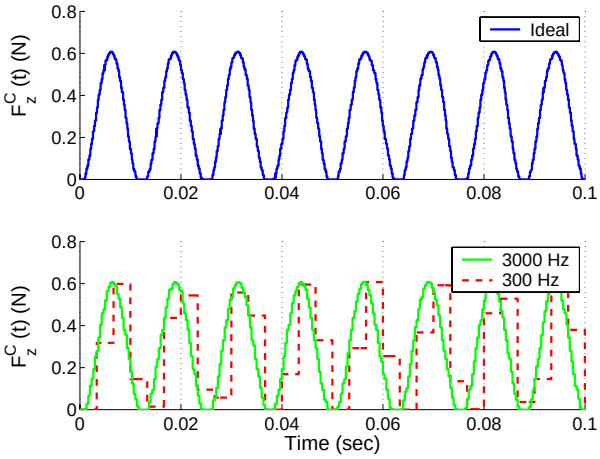

Fig. 6. Force commands in the z-direction (signal B in Fig. 4).

the PHANToM. Fig. 3 shows the magnitude of a z-axis open-loop frequency response of the PHANToM (model 1.0A) that was measured with the stylus tip resting at the origin of the world coordinate frame and pointing to the z-direction. The frequency response is defined as $H_z(f) = P_z(f)/\widetilde{F}_z^C(f)$, where $\widetilde{F}_z^C(f)$ is the Fourier transform of the z-axis force command to the PHANToM $(F_z^C(t))$ and $P_z(f)$ is that of the z-axis position of the stylus tip $(p_z(t))$. This magnitude response plot exhibits unstable modes at frequencies above 150 Hz. Similar structural resonances have been reported for PHANToM model 1.5 [11]. A comparison of Figs. 2 and 3 suggests that the unstable modes in Fig. 3 contributed to the buzzing noise in Fig. 2.

IV. Simulation

In our preliminary testing, we observed that using a very high update rate for haptic texture rendering greatly improved the perceived quality of the resulting textures, particularly in terms of buzzing. This phenomenon was supported by simulation results where it was assumed that a user stroked a textured surface rendered at various update rates. In this section, we discuss the expected effect of haptic update rate on buzzing with an example of the simulation results.

Fig. 4 shows the structure of haptic texture rendering used in the simulation, with the sampled-data nature explicitly considered. The input to the Haptic Renderer (i.e., signal F in Fig. 4) was the position trajectory of the PHANToM stylus tip along the normal direction to the textured wall $(p_z(t))$. An example of signal F is provided in Fig. 5 where the stylus trajectory $p_z(t)$ is shown in solid line, and the textured surface height $h(p_x(t))$ is shown in dashed line. These signals were designed to resemble the typical stylus trajectories that were measured when a user stroked virtual textured surfaces [12]. Specifically, the amplitude (A) and wavelength (L) of the texture model were set to 1 mm and 2 mm, respectively, and the average stroking velocity $(\overline{v}_x)$ was assumed to be 160 mm/s. The fundamental frequency of the input signal F was thus 80 Hz $(= \overline{v}_x/L$; see [2] for details). The quantization level was set to the nominal position-sensing resolution of the PHANToM 1.0A $(= 0.03$ N/mm).

The input to the Haptic Interface (force command signal B in Fig. 4), denoted by $F_z^C(t)$, was computed using the rendering method $\mathbf{F}_{mag}(t)$ in Eqn. 3 with stiffness $K = 0.4$ N/mm.[1] Fig. 6 shows the force commands under three digital-to-analog (D/A) conversion conditions. The upper panel corresponds to the force command computed using the ideal noncausal reconstruction filter.[2] The lower panel shows two force commands reconstructed with zero-order hold (ZOH) at an update rate of 300 and 3000 Hz, respectively. It is apparent that the force command updated at 3000 Hz is quite smooth, but the force command updated at 300 Hz contains large step changes. This is to be expected for a force signal with a fundamental frequency of 80 Hz. The frequency domain representation of the three force commands are provided in Fig. 7. As expected, all three power spectra show a prominent peak at 80 Hz. The spectral density of the force command updated at 300 Hz

[1]Although both $\mathbf{F}_{mag}(t)$ and $\mathbf{F}_{vec}(t)$ can cause buzzing, we used $\mathbf{F}_{mag}(t)$ in the simulation for simplicity.

[2]For simulation, very large update rate was used to generate this force command.

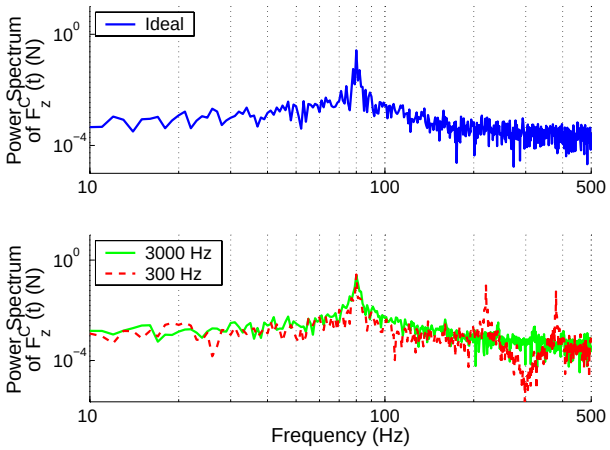

Fig. 7. Power spectral densities of the three force commands shown in Fig. 6.

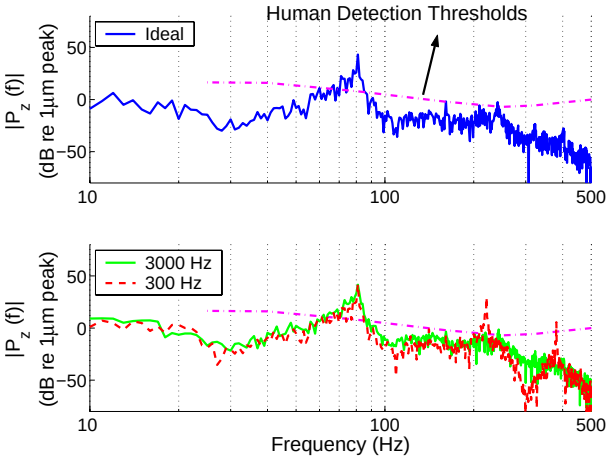

Fig. 8. Power spectral densities of the position outputs of the PHANToM along the z-axis (signal C in Fig. 4).

also exhibit significant spectral peaks at 220 and 380 Hz.

Finally, we calculated the output of the Haptic Interface, i.e., the input to the User (signal C in Fig. 4). We assumed that the PHANToM stylus moved around the origin of the PHANToM coordinate frame. We then approximated the dynamics of the PHANToM to the linearized frequency response whose magnitude response was shown earlier in Fig. 3. By multiplying this magnitude response with the power spectral densities of the force commands shown in Fig. 7, we obtained the spectral densities of the PHANToM position outputs.

The results are shown in Fig. 8 along with the human detection thresholds. In the upper panel showing the PHANToM position signal from the ideal force command, only one spectral peak at 80 Hz is well above the human detection threshold. Therefore, we predict that the user will perceive a "clean" texture from the vibration at 80 Hz. In the lower panel, the same can be said about the position signal generated from the force command updated with the ZOH at 3000 Hz. Note, however, that the position signal driven by the force command updated at 300 Hz contains an additional peak at 220 Hz that is well above the corresponding human detection threshold. We therefore conclude that when forces are updated at 300 Hz, the user will perceive not only the texture information from the vibration at 80 Hz, but also the perceived instability of buzzing from the vibration at 220 Hz. The buzzing noise is subsequently fed back to the Haptic Renderer (through the path D-E-F in Fig. 4). This closed-loop behavior usually increases the intensity of buzzing and widens its frequency range, thereby creating the closed-loop response similar to that shown earlier in Fig. 2.

In summary, the simulation results suggest that using a higher haptic update rate decreases the high-frequency signal content of the reconstructed force command to the force-feedback haptic interface, and consequently reduces the energy of the signal components that can cause the perception of buzzing. A relatively high haptic update rate can be particularly advantageous to force-feedback devices with structural resonances, such as the PHANToM.

V. Experiment

To verify the simulation results, we conducted a psychophysical experiment. The results confirmed that a higher haptic update rate could improve the perceived quality of virtual haptic textures. This section reports the design and results of the psychophysical experiment.

A. Methods

The PHANToM 1.0A model was used to render virtual haptic textures. The texture was modeled as one-dimensional sinusoidal gratings (Eq. 1). In all experimental conditions, the amplitude and spatial wavelength of the sinusoidal grating were set to 1 mm and 2 mm, respectively. The texture model was rendered using $\mathbf{F}_{vec}(t)$ (Eq. 4). Compared to those rendered with $\mathbf{F}_{mag}(t)$, virtual textures rendered with $\mathbf{F}_{vec}(t)$ exhibit buzzing more often and more intensely. The independent variable in the experiment was the haptic update rate for texture rendering. Eight update rates were tested: 250, 500, 1k, 2k, 5k, 10k, 20k, and 40 kHz. The dependent variable measured in each experimental condition was the maximum stiffness threshold K_T under which the textured surface was perceived to be stable without buzzing.

The method of limits, a well-established classical psychophysical method [13], was employed to estimate stiffness thresholds. The detailed procedure was essentially the same as that used in our previous experiments [1]–[3]. Each experimental condition consisted of 25 ascending series and 25 descending series. The minimum stiffness (K_{min}) and maximum stiffness (K_{max}) were set to 0.0 and 1.6 N/mm, respectively, based on preliminary testing. The increment of stiffness ΔK was 0.05 N/mm for all conditions. More details can be found in [2].

Two subjects (one male and one female) participated in the psychophysical experiment. Both subjects had participated in our previous experiments [1]–[3]. Subject S1 was an experienced user of the PHANToM device. Subject S2 had not used any haptic interface prior to her participation in our previous experiments. The subjects are right-handed

and reported no known sensory or motor abnormalities with their upper extremities.

During the experiment, the subjects were required to stroke the textured surface with the PHANToM stylus, and to judge whether the surface contained any buzzing. They were also asked to maintain a constant stroking velocity to the best of their ability throughout the experiment.

B. Results

The maximum stiffness thresholds for perceptually stable haptic texture rendering are summarized in Fig. 9. The datum points for the two subjects are shown with a slight offset for clarity. Also shown are the error bars corresponding to the standard deviations of the stiffness thresholds. As the haptic update rate increased from 250 Hz to 40 kHz, the maximum stiffness for perceptually stable haptic texture rendering increased from 0.321 to 0.964 N/mm for S1, and from 0.329 to 0.843 N/mm for S2. An ANOVA analysis confirmed the main effect of the haptic update rate for both subjects [$F(7, 392) = 241.58$, $p < 0.0001$ for S1; $F(7, 392) = 131.34$, $p < 0.0001$ for S2]. A Tukey's HSD test confirmed that all stiffness thresholds of S1 were statistically different [$q(0.95; 7, 392) = 4.31$; minimum significant difference = 0.057 N/mm], except for the thresholds at 500 Hz and 1 kHz and those at 5 kHz and 10 kHz. For S2, all stiffness thresholds were statistically pairwise-different [$q(0.95; 7, 392) = 4.31$; minimum significant difference = 0.068 N/mm], except for the thresholds at 500 Hz and 2 kHz, those at 5 kHz and 20 kHz, and those at 10 kHz and 40 kHz.

The general trend exhibited in Fig. 9 was that the stiffness threshold increases with the haptic update rate. The change in stiffness threshold was roughly linear with the logarithm of haptic update rate, with S1's data showing a continuous upward trend while S2's data showing a plateau at high update rates (> 10 kHz). The average increase in stiffness threshold was 278% over the haptic update range 250 Hz - 40 kHz. This was a significant increase both numerically and perceptually. The haptic virtual textures that could be stably rendered at 250 Hz were quite soft, whereas the virtual textures rendered at 40 kHz felt like a hard metal blade with sharp teeth.

The stiffness thresholds measured at the update rate of 1 kHz were 0.479 and 0.437 N/mm for S1 and S2, respectively. Perceptually, textures rendered at these stiffness values felt rather soft. When the update rate increased to more than 10 kHz, the stiffness threshold nearly doubled (0.734 and 0.805 N/mm at 10 kHz for S1 and S2, respectively). Textures rendered at a stiffness of 0.8 – 1.0 N/mm with the PHANToM felt as hard as plastic or metal.[3] These results indicate that an update rate much faster than the conventional 1 kHz is needed in order to render hard virtual haptic textures that are perceptually "clean."

[3]Note that some other haptic interfaces may stably render a hard surface at 1 kHz with relatively higher stiffness values (for example, see [14], [15]).

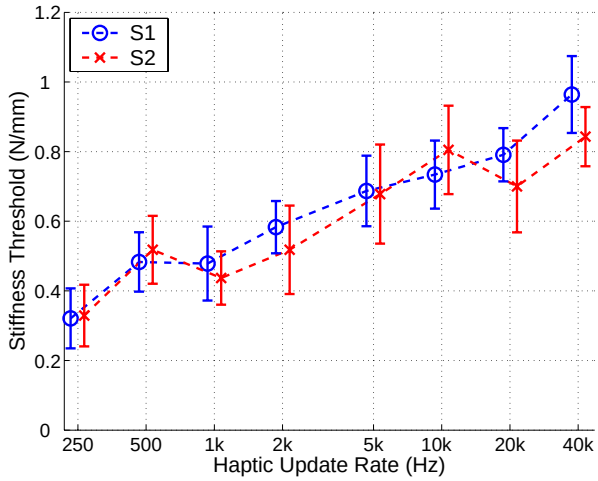

Fig. 9. Stiffness thresholds as a function of haptic update rates measured in the psychophysical experiment.

VI. Conclusions and Future Work

In this paper, we studied the effect of haptic update rate on perceived instability of virtual textures. We first described the physical and perceptual characteristics of buzzing, a type of perceived instability frequently observed in haptic virtual textures. We then presented the simulation results in the frequency domain suggesting that buzzing may be caused by an update rate that was too low for the texture model and stroking velocity under consideration. What followed was the design and results of the psychophysical experiment that systematically examined the effect of update rate on the maximum stiffness that could be used for perceptually stable haptic texture rendering. A wide range of haptic update rates from 250 Hz to 40 kHz was tested in the psychophysical experiment. The results showed a nearly linear increase in the stiffness threshold for the logarithm of haptic update rate. Both simulation and experimental results showed that haptic update rate is a critical factor in determining the perceived quality of virtual haptic textures. In particular, the experimental results indicated that using a high haptic update rate (e.g., > 10 kHz) can roughly eliminate the perceived instability of buzzing for most haptic texture rendering applications.

Our results can be interpreted from the viewpoint of digital control. A unique feature of haptic texture rendering is that the vibration frequency corresponding to texture is determined by both the spatial frequency of the texture model and the user's stroking velocity. A relatively fine surface texture combined with a typical stroking velocity can easily produce force commands with a fundamental frequency of a few hundred Hz. When a ZOH filter is used for D/A conversion, a sampling rate that is 10 – 20 times faster than the signal bandwidth is usually recommended [16]. This results in a preferred update rate in the 5 – 10 kHz range for haptic texture rendering. Therefore, the widely-accepted haptic-rendering update rate of 1 kHz may not be adequate for perceptually "clean" haptic texture rendering, as indicated by our experimental results.

Future research will be pursued in two directions. One

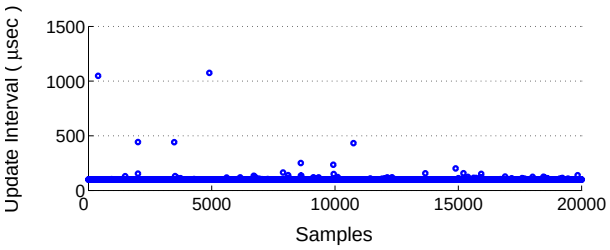

Fig. 10. Haptic update intervals measured with a high-resolution counter at an update rate of 10 kHz.

is to perform a complete analysis of the effect of haptic update rate on buzzing based on the idea outlined in Sec. IV. The other is to develop a multi-rate haptic rendering architecture for texture rendering with a very fast update rate (examples of multi-rate haptic rendering systems can be found in [17], [18]). Our goal is to develop a haptic texture rendering system with a very high update rate ($>$ 10 kHz) with enough computing power to be spared for other tasks such as collision detection.

Appendix: Implementation of a Very Fast Haptic Update Rate

In order to achieve a very fast haptic update rate, we used a high-resolution counter in a PC. The PC used in our experiment was a Dell Precision Workstation 620 (dual Pentium III Xeon 993 MHz and 512 MB RDRAM) running the Microsoft Windows XP operating system. The high-resolution counter in our PC has a time resolution of 1.007 ns, which is much faster than the 1 ms time resolution provided by the Microsoft Windows timer APIs.

The input/output (I/O) module used the GHOST programming library v4.0 for the PHANToM. We used the gstDeviceIO class of the GHOST SDK for a non-1-kHz update rate. We hasten to point out that we did not use the I/O functions of the gstDeviceIO class as recommended by SensAble Technologies, because we could not achieve an update rate faster than about 1.5 kHz with them. This limitation was due to the internal hardware-checking mechanism of the gstDeviceIO I/O functions [personal communication with Billy Chan at SensAble Technologies, 2004]. We therefore implemented our own I/O routines that bypassed the internal mechanism and accomplished a maximum update rate of 50 kHz for haptic texture rendering.

We also examined the timing accuracy of our haptic updating module and confirmed that the resulting update intervals were very consistent. An example of haptic update intervals measured using the high-resolution counter is provided in Fig. 10. In this case, the update rate was set to 10 kHz, and 20,000 update intervals were measured. Except for a few large outliers, most of the measured update intervals lay near 100 μs. The mean and standard deviation of the 20,000 datum points were 100 μs and 11 μs, respectively. The relatively large standard deviation was mainly due to the several large outliers as can be seen in the figure. We suspected that these outliers were caused by background system threads such as a virus vaccine program. We also tested the timing accuracy at many other update rates and obtained similar results.

Acknowledgement

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