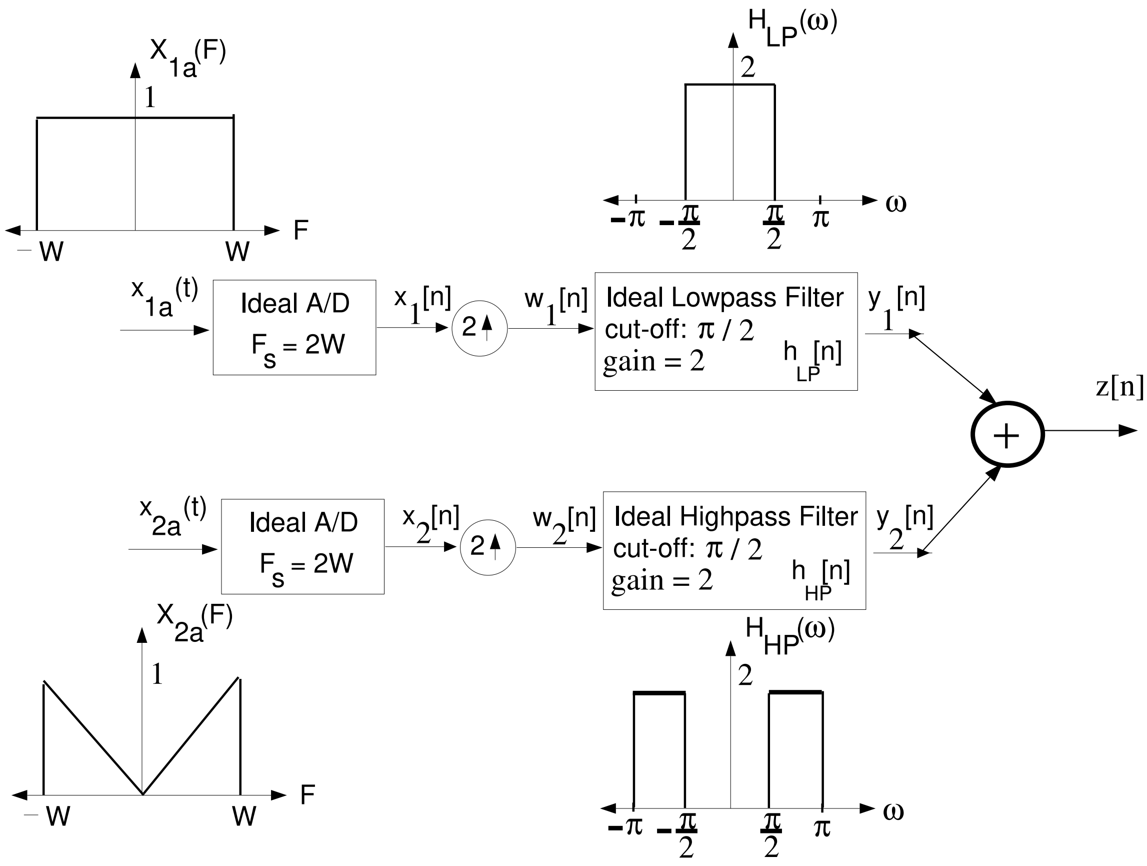
Digital Signal Processing I — Exam 2 — Fall 1998
Session 16 — Live: 20 Oct. 1998

Cover Sheet

Test Duration: 75 minutes.
Open Book but Closed Notes.
Calculators allowed.
This test contains **four** problems.
All work should be done in the blue books provided.
Do **not** return this test sheet, just return the blue books.

Prob. No.	**Topic of Problem**	**Points**
1.	Digital Subbanding	30
2.	Fractional Sampling Rate Conversion	20
3.	Multi-Stage Upsampling/Interpolation	20
4.	IIR Filter Design Via Bilinear Transform	30

Problem 1. [30 points]



The analog signals $x_{ia}(t)$ with CTFT $X_{ia}(F)$, $i = 1, 2$, shown are input to the system above, where $x_i[n] = x_{ia}(n/F_s)$, $h_{HP}[n] = (-1)^n h_{LP}[n]$, where

$$h_{LP}[n] = \frac{\sin(\frac{\pi}{2}n)}{\frac{\pi}{2}n}, \qquad -\infty < n < \infty,$$

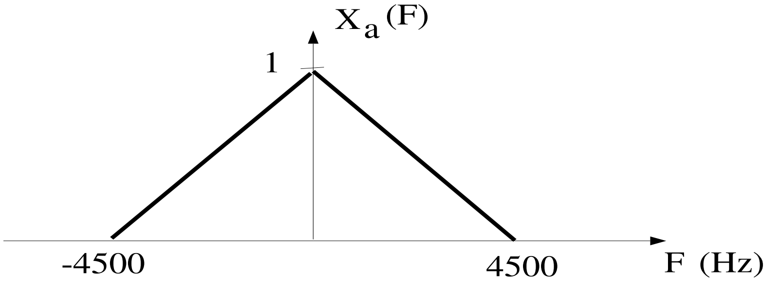
and

$$w_i[n] = \begin{cases} x_i(\frac{n}{2}), & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

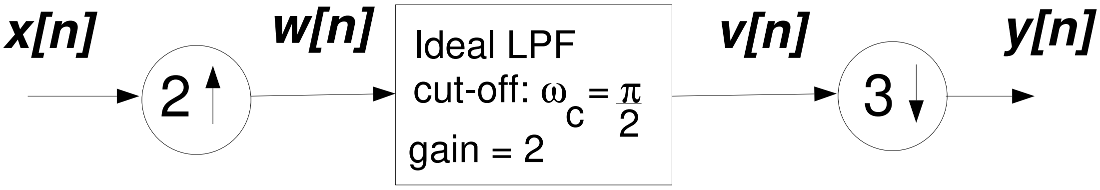
(15 pts) Plot the magnitude of the DTFT of $z[n]$ over $-\pi < \omega < \pi$ showing as much detail as possible. Note that it is not necessary to do a lot of mathematical analysis to obtain your answer. If you know what the system is doing, very little analysis is needed to plot $Z(\omega)$.

(15 pts) Draw a block diagram of a system that recovers $x_1[n]$ and $x_2[n]$ individually from the composite signal $z[n]$ using **only** decimators and the ideal lowpass and highpass filters employed in the block diagram above. You do **not** need to worry about computational efficiency.

Problem 2. [20 points] The signal $x[n] = x_a(n/F_s)$ is obtained by sampling the signal $x_a(t)$ with spectrum

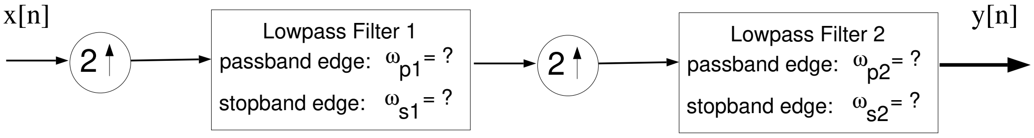


at a rate of $F_s = 9$ KHz $= 9000$ Hz. The resulting DT signal $x[n]$ is input to the following system:



where $y[n] = v[3n]$ and the impulse response for the ideal lowpass filter is $h[n] = \frac{\sin(\frac{\pi}{2}n)}{\frac{\pi}{2}n}$. Denoting the DTFT of the output $y[n]$ as $Y(\omega)$, plot $|Y(\omega)|$ over $-\pi < \omega < \pi$ showing as much detail as possible. Note that it is not necessary to do a lot of mathematical analysis to obtain your answer. If you know what the overall system is doing, very little mathematical analysis is needed to plot $Y(\omega)$ (be careful about aliasing, though).

Problem 3. [20 points] The signal $x[n] = x_a(n/F_s)$ is obtained by sampling an analog signal $x_a(t)$ having a bandwidth of $W = 15$ KHz at a rate of $F_s = 40$ KHz. It is desired to increase the sampling rate by a factor of $L = 4$ to $F_{s_{new}} = 160$ KHz in two stages via the system below, where ideally $y[n] = x_a(n/4F_s)$



(a) Determine the passband edge, ω_{p1}, and stopband edge, ω_{s1}, of the first lowpass filter.

(b) Determine the passband edge, ω_{p2}, and stopband edge, ω_{s2}, of the second lowpass filter.

Problem 4. [30 points]

An analog Butterworth filter of order $N = 2$ with a 3-dB cut-off at $\Omega_c = 1$ Hz has the following transfer function (Laplace Transform):

$$H_a(s) = \frac{1}{s^2 + \sqrt{2}s + 1} \tag{1}$$

The goal is to use the bilinear transform technique to design a digital lowpass filter having a 3 dB cut-off at $\omega_c = \frac{2\pi}{5} = 0.4\pi$ rads/sec.

(a) Determine the constant c so that the bilinear transformation

$$s = c\frac{z-1}{z+1} \tag{2}$$

maps $\Omega_c = 1$ Hz to $\omega_c = \frac{2\pi}{5}$ rads/sec through the transformation $\Omega = c\ \tan\{\omega/2\}$. **Note: For this problem, you are required to approximate** $\tan\{\frac{\pi}{5}\}$ **as** $\frac{1}{\sqrt{2}}$**. That is, use the approximation:**

$$\tan\left\{\frac{\pi}{5}\right\} \approx \frac{1}{\sqrt{2}}$$

(b) Determine the transfer function, $H(z)$, of the digital filter obtained by applying the bilinear transform in (2) to the analog filter described by (1) with the value of c determined in part (a). Simplify as much as possible.

(c) Determine the poles and zeros of the resulting digital filter, $H(z)$. Draw the pole-zero diagram. Is the resulting digital filter stable?

(d) Determine the difference equation for implementing the resulting digital lowpass filter.