EE538 Digital Signal Processing I Exam 3 Session 41

Test Duration: 50 minutes. Open Book but Closed Notes.

Do all work in blue books provided. Only return the blue books.

Problem 1. [50 points]

Note that most of the quantities asked for can be computed at least two different ways. Either way you choose is fine. Keep in mind then that there are means to check your answers (which may affect my leniency relative to numerical mistakes.)

The autocorrelation values for an autoregressive process $x[n]$ of order $p = 2$ (AR(2) process) for lag values $m = 0$, $m = 1$, and $m = 2$ are

$$r_{xx}[0] = 4; \quad r_{xx}[1] = 2; \quad r_{xx}[2] = \frac{1}{2}$$

(a) Consider a first-order predictor

$$\hat{x}[n] = -a_1(1)x[n-1]$$

Determine the numerical value of the optimum predictor coefficient $a_1(1)$ and the numerical value of the corresponding minimum mean-square error $\mathcal{E}_1$.

(b) Consider a second-order predictor

$$\hat{x}[n] = -a_2(1)x[n-1] - a_2(2)x[n-2]$$

Determine the numerical values of the optimum predictor coefficients $a_2(1)$ and $a_2(2)$, and the numerical value of the corresponding minimum mean-square error $\mathcal{E}_2$.

(c) Consider a third-order predictor

$$\hat{x}[n] = -a_3(1)x[n-1] - a_3(2)x[n-2] - a_3(3)x[n-3]$$

Determine the numerical values of the optimum predictor coefficients $a_3(1)$, $a_3(2)$, and $a_3(3)$, and the numerical value of the corresponding minimum mean-square error $\mathcal{E}_3$.

(d) Determine the AR model parameters a_1 and a_2 and the power, σ_w^2, of the input white noise process that generated the AR process.

(e) Determine the numerical value of $r_{xx}[3]$.

(f) Determine the DTFT of $r_{xx}[m]$ defined as

$$S_{xx}(\omega) = \sum_{m=-\infty}^{\infty} r_{xx}(m)e^{-j\omega m}$$

Note: There is no need to compute a DTFT here; just determine $S_{xx}(\omega)$ but note infinite limits, not finite limits in the sum.

Problem 2. [50 points]

Let $x[n]$ be a discrete-time random process containing one real-valued sinewave as described by

$$x[n] = A\cos(\omega_0 n + \Theta)$$

where the amplitude, A, and frequency, ω_0 , of the sinusoid are each deterministic but unknown constants and Θ is a random variable uniformly distributed over a 2π interval. You are given the following three values of the true autocorrelation sequence $r_{xx}[m] = E\{x[n]x^*[n-m]\}$:

$$r_{xx}[0] = \sqrt{2}; \qquad r_{xx}[1] = 1; \qquad r_{xx}[2] = 0$$

(a) Consider a first-order predictor

$$\hat{x}[n] = -a_1(1)x[n-1]$$

Determine the numerical value of the optimum predictor coefficient $a_1(1)$ and the numerical value of the corresponding minimum mean-square error $\mathcal{E}_1$.

(b) Consider a second-order predictor

$$\hat{x}[n] = -a_2(1)x[n-1] - a_2(2)x[n-2]$$

Determine the numerical values of the optimum predictor coefficients $a_2(1)$ and $a_2(2)$, and the numerical value of the corresponding minimum mean-square error $\mathcal{E}_2$.

(c) Consider a third-order predictor

$$\hat{x}[n] = -a_3(1)x[n-1] - a_3(2)x[n-2] - a_3(3)x[n-3]$$

Determine the numerical values of the optimum predictor coefficients $a_3(1)$, $a_3(2)$, and $a_3(3)$, and the numerical value of the corresponding minimum mean-square error $\mathcal{E}_3$.

(d) Determine the numerical value of the frequency of the sinewave, ω_0.

(e) Determine the numerical value of $r_{xx}[3]$.

(f) Plot the DTFT of $r_{xx}[m]$ defined as

$$S_{xx}(\omega) = \sum_{m=-\infty}^{\infty} r_{xx}(m)e^{-j\omega m}$$

Note: Note infinite limits, not finite limits in the sum. Plot $S_{xx}(\omega)$ over $-\pi < \omega < \pi$.