**NAME:**

**EE538 Digital Signal Processing I**                    **Fall 2014**
**Exam 1**                                    **Friday, Sept. 29, 2014**

## Cover Sheet

**Write your name on this and every page**
Test Duration: 60 minutes.
Coverage: Chapters 1-5.
Open Book but Closed Notes.
Calculators NOT allowed.
This test contains **three** problems.
Show your work in the space provided for each problem.
You must show all work for each problem to receive full credit.
Always simplify your answers as much as possible.

| Prob. No. | Topic(s) | Points |
|---|---|---|
| 1. | Frequency Response and Interconnection of LTI Systems, Pole-Zero Diagrams | 35 |
| 2. | DT Autocorrelation, Cross-Correlation and their Related Properties | 30 |
| 3. | LTI Systems: Expressing Cross-Correlation in terms of Convolution | 35 |

# NAME:

**Problem 1.** [35 points]

(a) Consider System 1 with impulse response $h_1[n]$ below, where $p_1 = 0.8$ (in fractional form $p_1 = \frac{4}{5}$):

$$\text{System 1:} \quad h_1[n] = \frac{1}{p_1} \left\{ \delta[n] + (p_1^2 - 1)p_1^n u[n] \right\}$$

$$\text{x[n]} \xrightarrow{\quad} \boxed{h_1[n]} \xrightarrow{\quad} y_1[n]$$
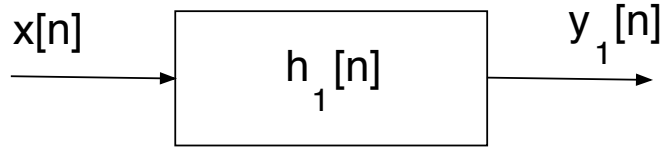
    (i) Draw a pole-zero diagram for this system.

    (ii) Plot the magnitude of the frequency response $|H_1(\omega)|$ over $-\pi < \omega < \pi$.

(b) Consider System 2 with impulse response $h_2[n]$ below, where $p_2 = -0.8$ (in fractional form $p_2 = -\frac{4}{5}$):

$$\text{System 2:} \quad h_2[n] = \frac{-1}{p_2} \left\{ \delta[n] + (p_2^2 - 1)p_2^n u[n] \right\}$$

$$\text{x[n]} \xrightarrow{\quad} \boxed{h_2[n]} \xrightarrow{\quad} y_2[n]$$
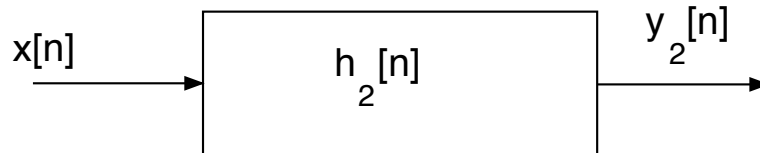
    (i) Draw a pole-zero diagram for this system.

    (ii) Plot the magnitude of the frequency response $|H_2(\omega)|$ over $-\pi < \omega < \pi$.

(c) Consider Systems 1 and 2 to be connected in parallel. Plot the magnitude of the frequency response $|H(\omega)|$ of the parallel combination of System 1 and System 2 over $-\pi < \omega < \pi$.

(d) Determine the overall output $y[n]$ of the parallel combination of System 1 and System 2 when the common input is:

$$x[n] = 3 + 2\cos(\pi n)$$

# NAME:
**Problem 2.** [30 points]

(a) Determine the autocorrelation $r_{x_0 x_0}[\ell]$ of the length-4 sequence $x_0[n]$ below, which is written two different ways. Write your answer in sequence form indicating where the value for $\ell = 0$ is located.

$$x_0[n] = u[n] - u[n-4] = \{1, 1, 1, 1\}$$

(b) Determine the autocorrelation $r_{x_1 x_1}[\ell]$ of the length-4 sequence $x_1[n]$ below. Write your answer in sequence form indicating where the value for $\ell = 0$ is located.

$$x_1[n] = e^{j\left(\frac{\pi}{2}n + \frac{\pi}{\sqrt{2}}\right)} \{u[n] - u[n-4]\}$$

(c) Determine the autocorrelation $r_{x_2 x_2}[\ell]$ of the length-4 sequence $x_2[n]$ below. Write your answer in sequence form indicating where the value for $\ell = 0$ is located.

$$x_2[n] = e^{j\pi(n-2)} \{u[n-2] - u[n-6]\}$$

(d) Determine the autocorrelation $r_{x_3 x_3}[\ell]$ of the length-4 sequence $x_3[n]$ below. Write your answer in sequence form indicating where the value for $\ell = 0$ is located.

$$x_3[n] = e^{-j\left(\frac{\pi}{2}n + \frac{\pi}{\sqrt{2}}\right)} \{u[n] - u[n-4]\}$$

(e) Sum your answers to parts (a) thru (d) to form the sum below. Do a stem plot of $r_{xx}[\ell]$.

$$r_{xx}[\ell] = r_{x_0 x_0}[\ell] + r_{x_1 x_1}[\ell] + r_{x_2 x_2}[\ell] + r_{x_3 x_3}[\ell]$$

# NAME:
**Problem 3.** [35 points]

(a) Determine the autocorrelation $r_{x_1x_1}[\ell]$ of the length-4 sequence $x_1[n]$ below, which is written two different ways. Write your answer in sequence form indicating where the value for $\ell = 0$ is located.

$$x_1[n] = \{1, -1, 1, 1\} = \delta[n] - \delta[n-1] + \delta[n-2] + \delta[n-3]$$

(b) Determine the autocorrelation $r_{x_1x_1}[\ell]$ of the length-4 sequence $x_1[n]$ below, which is written two different ways. Write your answer in sequence form indicating where the value for $\ell = 0$ is located.

$$x_2[n] = \{-1, 1, 1, 1\} = -\delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]$$

(c) The sequence $x_1[n]$ defined above is input to the system described by the simple difference equation below. Do a stem plot of the cross-correlation $r_{y_1x_1}[\ell]$ between the output and input.
$$y_1[n] = x_1[n-4] + x_1[n-6]$$

(d) The sequence $x_2[n]$ defined above is input to the same system described by the simple difference equation below. Do a stem plot of the cross-correlation $r_{y_2x_2}[\ell]$ between the output and input.
$$y_2[n] = x_2[n-4] + x_2[n-6]$$

(e) Sum your answers to parts (c) thru (d) to form the sum below. Do a stem plot of $r_{yx}[\ell]$.
$$r_{yx}[\ell] = r_{y_1x_1}[\ell] + r_{y_2x_2}[\ell]$$

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