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Integrated Direct/Indirect Adaptive Robust Control of a Class of Nonlinear Systems Preceded by Unknown Dead-zone Nonlinearity

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***Abstract*— This paper presents an integrated direct/indirect adaptive robust control (DIARC) scheme for a class of nonlinear systems preceded by unknown non-symmetric, non-equal slope dead-zone nonlinearity. Due to the inherent nonlinear parametrizaton nature of the unknown dead-zone nonlinearity, existing robust adaptive control methods have been focusing on using various linearly parametrized models with on-line parameter adaptation for an approximate inversion of the unknown dead-zone nonlinearity. As a result, even in the absence of other uncertain nonlinearities and disturbances, asymptotic output tracking can never be achieved. Departing from those approximate dead-zone compensations, this paper makes full use of the fact that, though not being linearly parametrized globally, the unknown dead-zone nonlinearity can be linearly parametrized within certain known working ranges. Thus indirect parameter estimation algorithms with on-line condition monitoring can be employed for an accurate estimation of the unknown dead-zone nonlinearity. With these accurate estimates of dead-zone parameters, perfect dead-zone compensation is then constructed and utilized in the development of an integrated direct/indirect adaptive robust control algorithm for the overall system. Consequently, asymptotic output tracking is achieved even in the presence of unknown dead-zone nonlinearity. Furthermore, the proposed DIARC achieves certain guaranteed robust transient performance and final tracking accuracy even when the overall system may be subjected to other uncertain nonlinearities and time-varying disturbances. Comparative simulation results obtained validate the effectiveness of the proposed approach as well.**

I. INTRODUCTION

Dead-zone is an ubiquitous non-smooth nonlinearity arisen in hydraulic valves, DC motors and mechanical connection. This nonlinearity is a static input-output relationship which characterizes the insensitivity of the output to input values during certain working ranges. Actually the dead-zone output is often difficult to measure and the dead-zone parameters are seldom known completely or remain the same values over the entire life-span of a machine. Thus, how to effectively compensate the effect of dead-zone in the presence of parametric uncertainties without direct measurement of dead-zone output has always been a practically important problem. The problem is also quite difficult to solve as it is unclear if exact inversion of dead-zone for perfect dead-zone compensation is possible or not in such a case [1].

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During the past decades, significant amount of research has been devoted to solving the above problem in one way or another with quite a large number of publications appeared [2], [3], [4], [5], [6], [7], [8]. Specifically, an adaptive dead-zone inverse was first proposed in [2] under the unrealistic assumption of set certainty equivalence. Such an unrealistic assumption was subsequently removed in [3] but only bounded output tracking errors are achieved. In [4] and [5], fuzzy-logic and neural network were used to provide alternative interpretations to the basis functions needed for adaptive dead-zone inversions [3] but with no essential improvement on theoretically achievable results, i.e., only bounded output tracking errors are obtained. In [6], [8], the dead-zone was modeled as a combination of a linear input with either an unknown constant gain for symmetric dead-zones [6] or time-varying unknown gain for non-symmetric ones and a bounded disturbance-like term. With such a formulation, traditional robust adaptive control design techniques can be applied to achieve bounded output tracking errors without explicitly exploring the detailed dead-zone characteristics. In [7], smooth functions as opposed to the discontinuous ones in [3], [4] were explored to provide some approximate inversions of the dead-zone nonlinearity.

It is noted that, without assuming the dead-zone output being measured, none of the schemes proposed so far can achieve perfect adaptive compensation of unknown dead-zones for asymptotic output tracking. The reason for this might come from the fact that the hard dead-zone nonlinearity cannot be linearly parametrized globally. As the existing schemes are all based on direct adaptive control designs, without resorting to the unpractical discontinuous control inputs, to achieve asymptotic output tracking, all uncertain nonlinearities have to be linearly parametrized perfectly and globally, which is not possible for the unknown dead-zone nonlinearity. Departing from these approximate dead-zone compensations, in this paper, we will make full use of the fact that, though not being linearly parametrized globally, the unknown dead-zone nonlinearity can be linearly parametrized during most of its working ranges. As such, through on-line condition monitoring and the employment of indirect parameter estimation algorithms, accurate estimations of all the unknown dead-zone nonlinearity parameters are possible and perfect adaptive compensation of unknown dead-zones for asymptotic output tracking is achievable.

In [9], an integrated direct/indirect adaptive robust control (DIARC) scheme was presented, in which the construction of parameter adaptation law can be totally independent from the design of underline robust control law. In this paper, the flexible parameter estimation process of the DIARC strategy and the dead-zone property of being linearly parametrized locally will be fully utilized to develop an integrated DIARC control algorithm for systems with unknown dead-zones. It is shown that the proposed DIARC is able to achieve accurate estimates of all dead-zone parameters when certain persistent excitation conditions are satisfied. Consequently, perfect dead-zone compensation will be achieved and asymptotic output tracking without assuming the dead-zone output being measured is proven theoretically. Furthermore, even when the persistent exciting conditions are not met and the overall system may be subjected to other uncertain nonlinearities and time-varying disturbances, it is theoretically shown that the proposed DIARC algorithm still achieves certain guaranteed robust transient performance and final tracking accuracy, namely, the bounds of the output tracking errors are directly related to the controller parameters in a known form, not only in L_2 norm as in the traditional robust adaptive controls [7] but also in practically more meaningful L_∞ norm. Comparative simulation results obtained validate the effectiveness of the proposed approach as well.

II. PROBLEM STATEMENT

A. System Model

This paper considers the same class of nonlinear systems as those commonly studied in recent literatures [6], [7]:

$$\begin{array}{l} x^{(n)} = \sum_{i=1}^{p} a_i Y_i(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)) + bw(t) + f_u \\ y = x(t), \quad w(t) = D(v(t)) \end{array} \tag{1}$$

where Y_i, $i = 1, 2, ..., p$, are some known continuous nonlinear functions, parameters a_i and b represent unknown constants. $v(t)$ is the output from the controller, $w(t)$ is the actual input to the plant, and $y(t)$ is the output from the plant. f_u represents the lumped uncertain nonlinearities including external disturbances. The actuator nonlinearity $D(v(t))$ is described as a dead-zone characteristic shown in (1) of Fig. 1. With input $v(t)$ and output $w(t)$, the dead-zone can be represented as in [3] as follows:

$$w(t) = D(v(t)) = \begin{cases} m_r v(t) - m_r b_r & for\ \ v(t) \geq b_r \\ 0 & for\ \ b_l < v(t) < b_r \\ m_l v(t) - m_l b_l & for\ \ v(t) \leq b_l \end{cases} \tag{2}$$

where the constant parameters m_r, m_l, b_r, and b_l stand for the right slope, left slope, right break-point and left break-point of the dead-zone respectively.

Let $b = 1$ for that its effect can be considered in the unknown slope m_r and m_l. For notation simplicity, let $\theta_b \in R^{p+4}$ be the vector of all unknown constant parameters, i.e., $\theta_b = [a_1, a_2, ..., a_p, m_r, m_r b_r, m_l, m_l b_l]^T$. $X = [x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)]$ is the state vector and $X_d = [x_d(t), \dot{x}_d(t), \ldots, x_d^{(n-1)}(t)]$ the desired trajectory vector. The control objective is to design a control law $v(t)$ to ensure that all closed-loop signals are bounded and X tracks X_d asymptotically with certain guaranteed transient responses under the following practical assumptions [1]

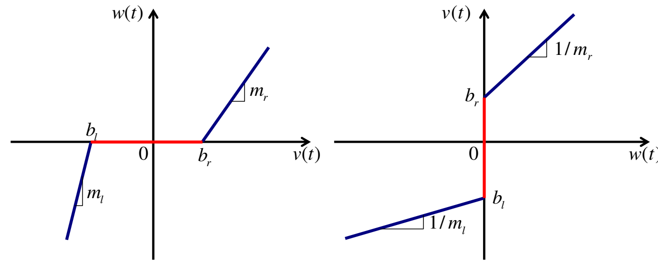

Fig. 1. (1) A dead-zone model (2) A particular perfect dead-zone inverse

Assumption 1: The dead-zone output $w(t)$ is not available for measurement and the dead-zone parameters are unknown, but their signs are known as $m_r > 0$, $m_l > 0$, $b_r > 0$, $b_l < 0$.

Assumption 2: The unknown parameter vector θ_b is within a known bounded convex set Ω_{θ_b}. It is assumed that $\forall \theta_b \in \Omega_{\theta_b}$, $a_i \in [a_{imin}, a_{imax}]$, $i = 1, 2, ..., p$, and $0 < m_{rmin} \leq m_r \leq m_{rmax}$, $0 < m_{lmin} \leq m_l \leq m_{lmax}$, $0 < (m_r b_r)_{min} \leq m_r b_r \leq (m_r b_r)_{max}$, $(m_l b_l)_{min} \leq m_l b_l \leq (m_l b_l)_{max} < 0$, where a_{imin}, a_{imax}, m_{rmin}, m_{rmax}, m_{lmin}, m_{lmax}, $(m_r b_r)_{min}$, $(m_r b_r)_{max}$, $(m_l b_l)_{min}$, and $(m_l b_l)_{max}$ are all known constants.

Assumption 3: The uncertain nonlinearity f_u can be bounded by

$$|f_u| \leq \delta(X) f_d(t) \tag{3}$$

where $\delta(X)$ is a known function and $f_d(t)$ is an unknown but bounded time-varying function.

B. Dead-zone Compensation

The essence of compensating dead-zone effect is to employ a perfect dead-zone inverse function $v(t) = D_I(w(t))$ shown in (2) of Fig. 1 such that $D(D_I(w(t))) = w(t), \forall w(t)$. The following dead-zone inverse will be used [11]

$$v(t) = \kappa_+(w_d) \frac{w_d(t) + \widehat{(m_r b_r)}}{\widehat{m_r}} + \kappa_-(w_d) \frac{w_d(t) + \widehat{(m_l b_l)}}{\widehat{m_l}} \tag{4}$$

where $\widehat{m_r}$, $\widehat{(m_r b_r)}$, $\widehat{m_l}$, $\widehat{(m_l b_l)}$ are the estimates of m_r, $m_r b_r$, m_l, $m_l b_l$ respectively, w_d is the desired control signal which would achieve the stated control objective when there is no dead-zone effect. And $\kappa_+(w_d)$ and $\kappa_-(w_d)$ are defined by

$$\kappa_+(w_d)(t) = \begin{cases} 1 & if\ \ (w_d(t) > 0\ \ or\ (w_d(t) = 0\& \\ & |v(t_-) - \widehat{b}_l| \geq |v(t_-) - \widehat{b}_r|)) \\ 0 & else \end{cases} \tag{5}$$

$$\kappa_-(w_d)(t) = \begin{cases} 1 & if\ \ (w_d(t) < 0\ \ or\ (w_d(t) = 0\& \\ & |v(t_-) - \widehat{b}_l| < |v(t_-) - \widehat{b}_r|)) \\ 0 & else \end{cases} \tag{6}$$

where $\widehat{b_r} = \frac{\widehat{(m_r b_r)}}{\widehat{m_r}}$ and $\widehat{b_l} = \frac{\widehat{(m_l b_l)}}{\widehat{m_l}}$. The proposed DIARC in section III will use projection type adaptation law for all parameter estimates. With those types of parameter adaptation

[1]The following nomenclature is used throughout this paper: $\bullet_{min}$ and $\bullet_{max}$are the minimum value and maximum value of $\bullet(t)$ for all t respectively, $\widehat{\bullet}$ denotes the estimate of $\bullet$, $\widetilde{\bullet} = \widehat{\bullet} - \bullet$ denotes the estimation error, e.g., $\widetilde{\theta} = \widehat{\theta} - \theta$, $\bullet_i$ is the i^{th} component of the vector $\bullet$.

law and the Assumption 2, the dead-zone parameter estimates $\widehat{m_r}$, $\widehat{(m_r b_r)}$, $\widehat{m_l}$, and $\widehat{(m_l b_l)}$ are guaranteed to be within their known bounded regions all the time. Thus, in the following, it is implicitly assumed that $\widehat{b}_r(t) > 0$ and $\widehat{b}_l(t) < 0, \forall t$. With these facts in mind, the following lemma can be obtained.

Lemma 1: [11] With the dead-zone inverse (4), the error between the actual dead-zone output w by (2) and the desired output w_d can be parametrized as

$$\begin{aligned} w - w_d = &\widetilde{(m_r b_r)}\kappa_+(w_d) + \widetilde{(m_l b_l)}\kappa_-(w_d) \\ &- \tfrac{w_d + \widehat{(m_r b_r)}}{\widehat{m_r}}\widetilde{m_r}\kappa_+(w_d) - \tfrac{w_d + \widehat{(m_l b_l)}}{\widehat{m_l}}\widetilde{m_l}\kappa_-(w_d) + d(t) \end{aligned} \tag{7}$$

where $d(t)$ is a function defined as

$$\begin{cases} 0 & if\ \kappa_+(w_d) = 1\ \&\ v(t) \geq b_r \\ m_r b_r - \frac{w_d + \widehat{(m_r b_r)}}{\widehat{m_r}} m_r & if\ \kappa_+(w_d) = 1\ \&\ 0 < v < b_r \\ m_l b_l - \frac{w_d + \widehat{(m_l b_l)}}{\widehat{m_l}} m_l & if\ \kappa_-(w_d) = 1\ \&\ b_l < v < 0 \\ 0 & if\ \kappa_-(w_d) = 1\ \&\ v(t) \leq b_l \end{cases} \tag{8}$$

and bounded above by

$$\begin{cases} \left| (m_r b_r)_{max} - \frac{m_{rmin}(m_r b_r)_{min}}{m_{rmax}} \right| & if\ \kappa_+(w_d) = 1 \\ \left| (m_l b_l)_{min} - \frac{m_{lmin}(m_l b_l)_{max}}{m_{lmax}} \right| & if\ \kappa_-(w_d) = 1 \end{cases} \tag{9}$$

In addition, $d(t) \in L_2[0,\infty)$ when $\widetilde{\theta}_b \in L_2[0,\infty)$ and $d(t) \longrightarrow 0$ when $\widetilde{\theta}_b(t) \longrightarrow 0$ as $t \longrightarrow \infty$.

III. Integrated Direct/Indirect Adaptive Robust Control (DIARC)

In this section, the integrated DIARC [9] will be used to synthesize $w_d(t)$ and an on-line parameter adaptation algorithm with conditional monitoring for the parameter estimates $\widehat{\theta}_b$ so that not only asymptotic output tracking can be achieved in spite of unknown dead-zone effect but also a guaranteed transient and steady-state output tracking performance is attained when other uncertain nonlinearities exist. As in [10], the first step is to use a projection type adaptation law structure to achieve a controlled learning or adaptation process as detailed in the following.

A. Projection Type Adaptation Law Structure

The widely used projection mapping $Proj_{\widehat{\theta}_b}$ [12] will be used to keep the parameter estimates within the known bounded set $\overline{\Omega}_{\theta_b}$, the closure of the set Ω_{θ_b} as in [10] as follows

$$\dot{\widehat{\theta}}_b = Proj_{\widehat{\theta}_b}(\Gamma\tau), \quad \widehat{\theta}_b(0) \in \Omega_{\theta_b} \tag{10}$$

where τ is an estimation function and $\Gamma(t) > 0$ is an adaptation rate matrix. With this adaptation law structure, the following desirable properties hold [10]:

(P1) The parameter estimates are always within the known bounded set $\overline{\Omega}_{\theta_b}$, i.e., $\widehat{\theta}_b \in \overline{\Omega}_{\theta_b}, \forall t$. Thus, from Assumption 2, $\forall t$, $a_{imin} \leq \widehat{a}_i(t) \leq a_{imax}$, $i = 1,2,...,p$, $0 < m_{rmin} \leq \widehat{m_r}(t) \leq m_{rmax}$, $0 < m_{lmin} \leq \widehat{m_l}(t) \leq m_{lmax}$, $0 \leq (m_r b_r)_{min} \leq \widehat{(m_r b_r)}(t) \leq (m_r b_r)_{max}$, and $(m_l b_l)_{min} \leq \widehat{(m_l b_l)}(t) \leq (m_l b_l)_{max} < 0$.

(P2)

$$\widetilde{\theta}_b^T(\Gamma^{-1}Proj_{\widehat{\theta}_b}(\Gamma\tau)) - \tau) \leq 0, \forall \tau \tag{11}$$

B. Integrated Direct/Indirect ARC Law

With the use of the projection type adaptation law structure (10), the parameter estimates are bounded with known bounds, regardless of the estimation function τ to be used. In the following, this property will be used to synthesize an integrated DIARC control law for the system (1) which achieves a guaranteed transient and steady-state output tracking accuracy in general. Define

$$s(t) = (\tfrac{d}{dt} + \lambda)^{n-1}\widetilde{x}(t) = \Lambda^T \widetilde{X}(t) \tag{12}$$

where $\lambda > 0$, $\Lambda^T = [\lambda^{n-1}, (n-1)\lambda^{n-2}, \ldots, 1]$, $\widetilde{X} = X - X_d$. Combining (1), (7) and (12), we get

$$\begin{aligned} \dot{s}(t) &= \Lambda_v^T \widetilde{X}(t) + \widetilde{x^n}(t) \\ &= \Lambda_v^T \widetilde{X}(t) - x_d^{(n)}(t) + \textstyle\sum_{i=1}^p a_i Y_i(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)) \\ &\quad + f_u + d(t) + w_d(t) + \widetilde{(m_r b_r)}\kappa_+(w_d) + \widetilde{(m_l b_l)}\kappa_-(w_d) \\ &\quad - \tfrac{w_d + \widehat{(m_r b_r)}}{\widehat{m_r}}\widetilde{m_r}\kappa_+(w_d) - \tfrac{w_d + \widehat{(m_l b_l)}}{\widehat{m_l}}\widetilde{m_l}\kappa_-(w_d) \end{aligned} \tag{13}$$

where $\Lambda_v^T = [0, \lambda^{n-1}, (n-1)\lambda^{n-2}, \ldots, (n-1)\lambda]$. The following DIARC function is proposed for w_d:

$$\begin{aligned} &w_d = w_{da} + w_{ds}, \quad w_{da} = w_{da1} + w_{da2}, \quad w_{ds} = w_{ds1} + w_{ds2}, \\ &w_{da1} = x_d^{(n)}(t) - \Lambda_v^T \widetilde{X}(t) - \textstyle\sum_{i=1}^p \widehat{a}_i Y_i(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)) \\ &w_{da2} = -\widehat{d}_c, \quad w_{ds1} = -k_{s1}s \end{aligned} \tag{14}$$

In (14), w_{da1} represents the usual model compensation with the physical parameter estimates $\widehat{a}_i$ updated using an on-line adaptation algorithm with condition monitoring to be detailed in subsection III-C. w_{da2} is a model compensation term similar to the fast dynamic compensation type model compensation used in the DARC designs [10], in which $\widehat{d}_c$ can be thought as the estimate of the low frequency component of the lumped model uncertainties. w_{ds} represents the robust control term in which w_{ds1} is a simple proportional feedback to stabilize the nominal system with a constant gain k_{s1} and w_{ds2} is a robust feedback term used to attenuate the effect of various model uncertainties for a guaranteed robust control performance in general. From (13) and (14),

$$\begin{aligned} \dot{s}(t) = &f_u + d(t) - \textstyle\sum_{i=1}^p \widetilde{a}_i Y_i(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)) \\ &+ [\tfrac{m_r}{\widehat{m_r}}(w_{ds} + w_{da2}) - \tfrac{\widetilde{m_r}}{\widehat{m_r}} w_{da1} + \widetilde{(m_r b_r)} - \widetilde{m_r}\tfrac{\widehat{(m_r b_r)}}{\widehat{m_r}}]\kappa_+(w_d) \\ &+ [\tfrac{m_l}{\widehat{m_l}}(w_{ds} + w_{da2}) - \tfrac{\widetilde{m_l}}{\widehat{m_l}} w_{da1} + \widetilde{(m_l b_l)} - \widetilde{m_l}\tfrac{\widehat{(m_l b_l)}}{\widehat{m_l}}]\kappa_-(w_d) \end{aligned} \tag{15}$$

Define a constant d_c and time varying $\triangle^\star(t)$ such that

$$\begin{aligned} d_c + \triangle^\star(t) = &f_u + d(t) - \textstyle\sum_{i=1}^p \widetilde{a}_i Y_i(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)) \\ &-[\tfrac{\widetilde{m_r}}{\widehat{m_r}}\kappa_+(w_d) + \tfrac{\widetilde{m_l}}{\widehat{m_l}}\kappa_-(w_d)]w_{da1} + [\widetilde{(m_r b_r)} \\ &-\widetilde{m_r}\tfrac{\widehat{(m_r b_r)}}{\widehat{m_r}}]\kappa_+(w_d) + [\widetilde{(m_l b_l)} - \widetilde{m_l}\tfrac{\widehat{(m_l b_l)}}{\widehat{m_l}}]\kappa_-(w_d) \\ = &f_u + d(t) + \vartheta\psi(t) \end{aligned} \tag{16}$$

where $\vartheta = [\widetilde{a}_1, \widetilde{a}_2, \ldots, \widetilde{a}_p, \frac{\widetilde{m_r}}{\widehat{m_r}}\kappa_+(w_d) + \frac{\widetilde{m_l}}{\widehat{m_l}}\kappa_-(w_d), [\widetilde{(m_r b_r)} - \widetilde{m_r}\frac{\widehat{(m_r b_r)}}{\widehat{m_r}}]\kappa_+(w_d) + [\widetilde{(m_l b_l)} - \widetilde{m_l}\frac{\widehat{(m_l b_l)}}{\widehat{m_l}}]\kappa_-(w_d)], \psi(t) = -[Y_1, Y_2, \ldots, Y_p, w_{da1}, -1]^T$. Conceptually, (16) lumps the original system uncertain nonlinearity f_u with the model uncertainties due to physical parameter estimation errors, and divides it into the static component d_c (or low frequency component in reality) and the high frequency components $\triangle^\star(t)$. In the following, the low frequency component d_c will be compensated through fast adaptation similar to those in the DARC [10] and DIARC [9] as follows.

Let d_{cM} be any pre-set bound and use this bound to construct the projection type adaptation law for $\widehat{d_c}$:

$$\dot{\widehat{d_c}} = Proj_{\widehat{d_c}}(\gamma s) = \begin{cases} 0 & if \quad |\widehat{d_c}(t)| = d_{cM} \ \& \ \widehat{d_c}s > 0 \\ \gamma s & else \end{cases} \tag{17}$$

with $\gamma > 0$ and $\widehat{d_c}(0) = 0$. Such an adaptation law guarantees $|\widehat{d_c}(t)| \le d_{cM}$, $\forall t$. For notation simplicity, define

$$k^\star = \kappa_+(w_d)\frac{m_r}{\widehat{m_r}} + \kappa_-(w_d)\frac{m_l}{\widehat{m_l}} \tag{18}$$

which has a value of 1 in the absence of dead-zone slopes estimation error. Substituting (16) to (15) and noting (14), $\dot{s}(t)$ can be written as

$$\begin{aligned} \dot{s}(t) &= k^\star(w_{ds} + w_{da2}) + d_c + \triangle^\star(t) \\ &= k^\star w_{ds1} + k^\star w_{ds2} - \widetilde{d_c} + (1-k^\star)\widehat{d_c} + \triangle^\star(t) \end{aligned} \tag{19}$$

Noting Assumptions 2 and 3, and (P1), there exists a w_{ds2} such that the following two conditions are satisfied:

$$\begin{aligned} &(i) \quad s w_{ds2} \le 0 \\ &(ii) \quad s[k^\star w_{ds2} - \widetilde{d_c} + (1-k^\star)\widehat{d_c} + \triangle^\star(t)] \le \varepsilon + \varepsilon_d f_d^2 \end{aligned} \tag{20}$$

where ε and ε_d are design parameters which can be arbitrarily small. Essentially, (ii) of (20) shows that w_{ds2} is synthesized to dominate the model uncertainties coming from both parametric uncertainties and uncertain nonlinearities, and (i) of (20) is to make sure that w_{ds2} is dissipative in nature so that it does not interfere with the functionality of the adaptive control part w_{da}.

Remark 1: One example of w_{ds2} satisfying (20) can be found in the following way. Let h be any function satisfying

$$h \ge |k^\star_{max}||w_{da2}| + ||\vartheta_{max}||||\psi|| + |d(t)|_{max} \tag{21}$$

where $k^\star_{max} = max\{\frac{m_{rmax}}{m_{rmin}}, \frac{m_{lmax}}{m_{lmin}}\}$, $\vartheta_{1max} = a_{1max} - a_{1min}$, $\vartheta_{2max} = a_{2max} - a_{2min}, \ldots, \vartheta_{rmax} = a_{rmax} - a_{rmin}$, $(\vartheta_{r+1})_{max} = max\{\frac{m_{rmax}-m_{rmin}}{m_{rmin}}, \frac{m_{lmax}-m_{lmin}}{m_{lmin}}\}$, $(\vartheta_{r+2})_{max} = max\{(m_r b_r)_{max} - (m_r b_r)_{min} + (m_{rmax} - m_{rmin})\frac{(m_r b_r)_{max}}{m_{rmin}}, (m_l b_l)_{max} - (m_l b_l)_{min} - (m_{lmax} - m_{lmin})\frac{(m_l b_l)_{min}}{m_{lmin}}\}$, $|d(t)|_{max} = max\{|(m_r b_r)_{max} - m_{rmin}\frac{(m_r b_r)_{min}}{m_{rmax}}|, |(m_l b_l)_{min} - m_{lmin}\frac{(m_l b_l)_{max}}{m_{lmax}}|\}$. Then, w_{ds2} can be chosen as

$$w_{ds2} = -[\frac{1}{4\varepsilon k^\star_{min}}h^2 + \frac{1}{4\varepsilon_d k^\star_{min}}\delta^2(X)]s \tag{22}$$

where $k^\star_{min} = min\{\frac{m_{rmin}}{m_{rmax}}, \frac{m_{lmin}}{m_{lmax}}\}$. Using the same techniques as in [9], it is easy to show that the above choice of w_{ds2} does satisfy (20).

Theorem 1: [11] When the DIARC control law (14) with the dead-zone inverse (4) and the projection type adaptation law (10) is applied, regardless the estimation function τ to be used, in general, all signals in the resulting closed loop system are bounded and the output tracking is guaranteed to have a prescribed transient performance and final tracking accuracy in the sense that the tracking error index s is bounded above by

$$s(t)^2 \le e^{-\lambda_v t}s(0)^2 + \frac{2(\varepsilon + \varepsilon_d f_{dmax}^2)}{\lambda_v}[1 - e^{-\lambda_v t}] \tag{23}$$

where $\lambda_v = 2k^\star_{min}k_{s1}$ and f_{dmax} represents the L_∞ norm of the bounded time function $f_d(t)$.

C. Parameter adaptation with On-line Condition Monitoring

In the above subsection, as long as the parameter estimates are updated with the projection type adaptation law (10), a DIARC control law which can admit any estimation function has been constructed, and a guaranteed output tracking transient and steady-state performance is achieved. Thus, the reminder of this section is to construct suitable parameter adaptation algorithms so that an improved final tracking accuracy - asymptotic output tracking can be obtained in the absence of uncertain nonlinearities (i.e., assuming $f_u = 0$ in (1)) with an emphasis on having a good parameter estimation process as well. For this purpose, rather than using any transformed tracking error dynamics to build parameter estimation model as in the direct adaptive designs, the actual plant model (1) will be directly used to construct specific estimation functions as detailed below.

In practice, almost all nonlinear control laws such as the proposed DIARC will have to be implemented using microprocessor with certain digitization. As such, in the following, we will construct the parameter adaption algorithms directly in discrete domain. Specifically, let $t_k = kT$ represent the sampling instances where T is the sampling period and $k = 0, 1, 2, \ldots$. Integrating the plant dynamics (1) with $f_u = 0$ during the sampling period from t_{k-1} to t_k, we have

$$\begin{aligned} \int_{t_{k-1}}^{t_k} x^{(n)}dt = \sum_{i=1}^p a_i \int_{t_{k-1}}^{t_k} Y_i(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t))dt \\ + \int_{t_{k-1}}^{t_k} D(v(t))dt \end{aligned} \tag{24}$$

which leads to the following digitized plant dynamics:

$$\begin{aligned} &x^{(n-1)}(t_k) - x^{(n-1)}(t_{k-1}) = D(v(t_{k-1}))T \\ &+ \sum_{i=1}^p a_i Y_i(x(t_{k-1}), \dot{x}(t_{k-1}), \ldots, x^{(n-1)}(t_{k-1}))T \end{aligned} \tag{25}$$

in which the fact that $v(t) = v(t_{k-1}), \forall t \in [t_{k-1}, t_k)$ in digital implementation using zero-order hold and the approximation of the nonlinear function Y_i by its value at the sampling instance t_{k-1} are used.

Note that globally the dead-zone nonlinearity (2) cannot be linearly re-parametrized by its parameters m_r, m_l, b_r, and b_l with some known basis functions, which is why the model error $d(t)$ in (7) is non-zero during some working ranges shown in (8). In the following, we will make full use of the fact that the dead-zone can be perfectly linearly parametrized during the normal working regions of $v \ge b_r$ or $v \le b_l$ to bypass the problem.

Lemma 2: [11] Define a positive constant B as $B = max\{(m_r b_r)_{max} - (m_r b_r)_{min}, \ (m_l b_l)_{max} - (m_l b_l)_{min}\}$. Then, when $|w_d(t)| \ge B$, the dead-zone input $v(t)$ by the proposed dead-zone inverse (4) would satisfy $v(t) \ge b_r$ or $v(t) \le b_l$.

For notation simplicity, define

$$\chi_+(\bullet)=\begin{cases}1 & if \bullet>0\\ 0 & if\ else\end{cases},\quad \chi_-(\bullet)=\begin{cases}1 & if \bullet<0\\ 0 & if\ else\end{cases} \tag{26}$$

and $\chi(w_d(t_{k-1})) = \chi_+(w_d(t_{k-1})-B)+\chi_-(w_d(t_{k-1})+B)$. Then, noting Lemma 2 and multiplying the digitized plant dynamics (25) by $\chi(w_d(t_{k-1}))$, we have

$$\begin{aligned}\chi(w_d(t_{k-1}))&\left(x^{(n-1)}(t_k)-x^{(n-1)}(t_{k-1})\right)\\ &=\chi\sum_{i=1}^{p}a_iY_i(x(t_{k-1}),\dot{x}(t_{k-1}),\ldots,x^{(n-1)}(t_{k-1}))T\\ &\quad+(m_rv(t_{k-1})-m_rb_r)\chi_+(w_d(t_{k-1})-B)T\\ &\quad+(m_lv(t_{k-1})-m_lb_l)\chi_-(w_d(t_{k-1})+B)T\end{aligned} \tag{27}$$

which is linearly parametrized by the system parameters. The following linear regression model for parameter estimation is then obtained

$$y_\chi(k)=F^T(k)\theta_b \tag{28}$$

where

$$\begin{aligned}&y_\chi(k)=\chi(w_d(t_{k-1}))\left(x^{(n-1)}(t_k)-x^{(n-1)}(t_{k-1})\right)\\ &F^T(k)=[\chi Y_1(t_{k-1})T,\chi Y_2(t_{k-1})T,\ldots,\chi Y_p(t_{k-1})T,\\ &\quad v(t_{k-1})\chi_+(w_d(t_{k-1})-B)T,\ -\chi_+(w_d(t_{k-1})-B)T,\\ &\quad v(t_{k-1})\chi_-(w_d(t_{k-1})+B)T,\ -\chi_-(w_d(t_{k-1})+B)T]\\ &\theta_b=[a_1,a_2,\ldots,a_p,m_r,m_rb_r,m_l,m_lb_l]^T\end{aligned} \tag{29}$$

Note that (28) is in the standard regression form. With this static model, various estimation algorithms can be used to identify unknown parameters, of which the least square estimation algorithm [13] with forgetting and projection modifications is given below to estimate the values of a_i, m_r, m_rb_r, m_l, and m_lb_l:

$$\begin{aligned}&P(k)=\tfrac{1}{\alpha}P(k-1)\{I-F(k)[\alpha+\\ &\qquad F^T(k)P(k-1)F(k)]^{-1}F^T(k)P(k-1)\}\\ &\varepsilon^0(k)=y_\chi(k)-F^T(k)\widehat{\theta}_b(k-1)\\ &\widehat{\theta}_b(k)=Proj_{\widehat{\theta}_b}\left\{\widehat{\theta}_b(k-1)+P(k)F(k)\varepsilon^0(k)\right\}\end{aligned} \tag{30}$$

in which $\varepsilon^0(k)$ is the *a priori* prediction error and $0<\alpha\le 1$ represents the forgetting factor. The following lemma summarizes the properties of these common estimators [13]:

Lemma 3: With the least squares type adaptation law with projection (30), in the absence of uncertain nonlinearities (i.e., $f_u=0$ in (1)), the following results hold:

(i) In general, $\widehat{\theta}_b(k)\in\overline{\Omega}_{\theta_b},\forall k$, and the *a posteriori* prediction error, $\varepsilon(k)=-F^T(k)\widetilde{\theta}_b(k)$, belongs to l_2.

(ii) In addition, if the following persistent excitation (PE) condition is satisfied:

$$\exists t_0,N,\beta>0,s.t.\sum_{i=k+1}^{k+N}F(i)F^T(i)\ge\beta I_{p+4},\forall t_k\ge t_0 \tag{31}$$

the physical parameter estimates $\widehat{\theta}_b$ converge to their true values, i.e., $\widetilde{\theta}_b(k)\to 0$ as $k\to\infty$ and $\widetilde{\theta}_b\in l_2$.

D. Asymptotic Output Tracking

Theorem 2: [11] In the absence of uncertain nonlinearities (i.e., assuming $f_u=0$ in (1)), when the PE condition (31) is satisfied, an improved steady-state tracking performance – asymptotic output tracking – is also achieved, i.e., $\widetilde{X}(t)\to 0$ and $s\to 0$ as $t\to\infty$.

IV. SIMULATION

The proposed DIARC algorithm is applied to the same example as in [6]:

$$\ddot{x}=a_1\tfrac{1-e^{-x}}{1+e^{-x}}-a_2(\dot{x}^2+2x)sin\dot{x}-0.5a_3xsin3t+w(t) \tag{32}$$

where $w(t)$ is the output of a dead-zone described by (2) with $m_r=1.5$, $m_l=1.2$, $m_rb_r=0.75$, and $m_lb_l=-0.84$ which are assumed not known but within the ranges of $m_r\in[1.2,1.8]$, $m_rb_r\in[0.5,1.1]$, $m_l\in[0.9,1.5]$, and $m_lb_l\in[-1.2,-0.6]$ respectively. The actual values of physical parameters are set as $a_1=a_2=a_3=1$. Let $X_d=[0.5(sint+sin1.6t+sin2.7t+sin3.5t),0.5(cost+1.6cos1.6t+2.7cos2.7t+3.5cos3.5t)]^T$ and initial values of plant states as $X(0)=[-0.5,3.5]^T$. Bounds of system parameters are chosen as $a_1\in[0.7,1.3]$, $a_2\in[0.7,1.3]$, $a_3\in[0.7,1.3]$. The following three control algorithms for system (32) are simulated and compared:

C1: The proposed DIARC adapting the physical parameter estimates $\widehat{a}_1$, $\widehat{a}_2$, and $\widehat{a}_3$ only and without updating the dead-zone parameter estimates.

C2: The proposed DIARC with compensation for the unknown dead-zone presented in section II and III.

C3: The proposed DIARC in which only $\widehat{a}_1$, $\widehat{a}_2$, $\widehat{a}_3$ are updated based on the proposed on-line parameter adaptation algorithm and the dead-zone effect is compensated by (4) with actual dead-zone parameter values.

In C2, k_{s1} in (14) is set as $k_{s1}=2$, and w_{ds2} is calculated using (22), in which $h\ge 1.67|w_{da2}|+0.6|Y_1|+0.6|Y_2|+0.6|Y_3|+0.67|w_{da1}|+1.4+0.84$ from (21), $\varepsilon=5$ and $\delta=0$ as the plant (32) does not have other uncertain nonlinearities except the dead-zone. λ in (12) is selected as $\lambda=4$. γ in (17) is set as $\gamma=1000$ and the bound of $\widehat{d_c}$ is chosen to be $d_{cM}=1.8$. The initial values of all parameter estimates are $\widehat{a}_1(0)=0.85$, $\widehat{a}_2(0)=0.85$, $\widehat{a}_3(0)=0.85$, $\widehat{m_r}(0)=1$, $\widehat{(m_rb_r)}(0)=1$, $\widehat{m_l}(0)=0.9$, $\widehat{(m_lb_l)}(0)=-1$. $P(k)$ in (30) is $P(0)=70000I_7$, and $\alpha=0.9$. For a fair comparison, the controller parameters in cases C1 and C3 are chosen the same as in C2 when they have the same meaning. The nominal values of dead-zone parameters used in the compensation in C1 are set as $\widehat{m_r}(0)=1.4$, $\widehat{(m_rb_r)}(0)=0.8$, $\widehat{m_l}(0)=1.3$, $\widehat{(m_lb_l)}(0)=-0.8$, which are quite close to their true values.

Assuming a sampling period T of $0.001sec$, the simulation results are shown in Fig. 2-4. Specifically, Fig. 2 shows the tracking errors with partially zoomed in steady-state portions shown in Fig. 3 for a better view. From these plots, it is observed that all three cases have almost the same good initial output tracking transient performance, illustrating the strong robust transient performance of the proposed DIARC strategy under the bounded dead-zone compensation errors in general. It is also seen that steady-state output tracking errors of C2 and C3 are almost the same and in the order of 10^{-5}, negligible when compared to that of C1 which is in the order of 10^{-3}. These results agree with the prediction by theory as it is shown in section III that the proposed DIARC in C2 is able to achieve asymptotic convergence of dead-zone parameter estimates to their true values and the perfect dead-zone compensation of the proposed dead-zone inverse (4) when the estimates converge. This is also verified by the history of dead-zone parameter estimates shown in (a) of Fig. 4. (b) of Fig. 4 shows the control inputs $v(t)$ of all three cases, which do not have the chattering problem except the sudden jumps due to the dead-zone inverse (4) when the inputs are across zero. In summary, the good output tracking performance of all three cases demonstrates the robust performance of the proposed DIARC algorithm. The much improved steady-state tracking performance of C2 over C1 validates the need for accurate on-line dead-zone parameter estimates. And the almost same steady-state output tracking performance of C2 and C3 verifies the asymptotic output tracking performance of the proposed scheme.

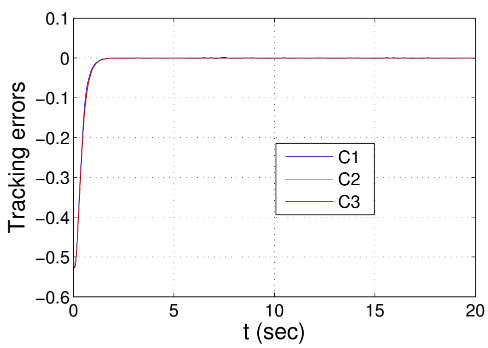

Fig. 2. Output tracking errors of DIARCs with dead-zone

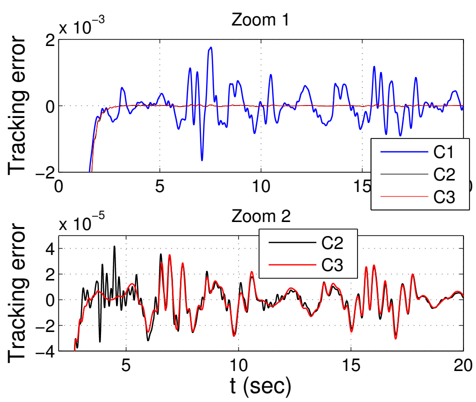

Fig. 3. Partially zoomed in output tracking errors of DIARCs with dead-zone

V. CONCLUSION

In this paper, a DIARC scheme has been developed for uncertain systems preceded by non-symmetrical and non-equal slope dead-zone nonlinearity. The proposed controller makes full use of the dead-zone characteristics that it can be linearly parametrized within certain known working ranges and uses indirect parameter estimation algorithms with on-line condition monitoring for an accurate estimation of the unknown dead-zone nonlinearity. With these accurate estimates of dead-zone parameters, perfect asymptotic dead-zone compensation is then constructed and employed in the development of an integrated direct/indirect adaptive robust control algorithm for the overall system. Consequently, *asymptotic output tracking has been achieved even in the presence of unknown dead-zone nonlinearity, a theoretic result which cannot be attained in all previous researches on adaptive dead-zone compensation*. Furthermore, *the proposed DIARC algorithm also achieves certain guaranteed robust transient performance and final tracking accuracy even when the overall system may be subjected to other uncertain nonlinearities and time-varying disturbances*. Comparative simulation results obtained verify the effectiveness and high-performance nature of the proposed approach.

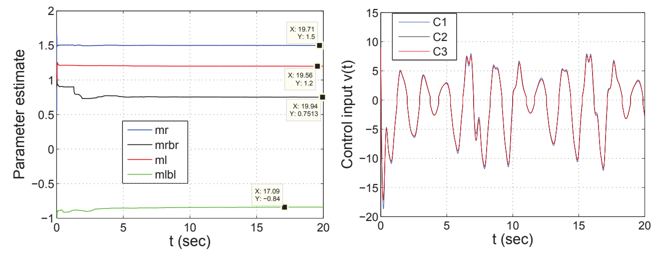

Fig. 4. (a) Dead-zone parameter estimates in C2; and (b) Control input history of all three cases

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