Edge-Localized Iterative Reconstruction for Computed Tomography

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***Abstract*—Recently it has been shown that model-based reconstruction (MBR) can greatly improve the quality of computed tomography (CT) images. In particular, MBR can recover fine details and small features in the reconstruction more accurately than conventional algorithms. In order to fully benefit from this higher spatial resolution, MBR reconstruction requires a higher spatial sampling rate, or equivalently smaller voxels, to represent fine details such as edges. However, these higher spatial sampling rates generate many more voxels for a fixed region-of-interest, so the resulting computation required for reconstruction can be greatly increased.**

In this paper, we propose an edge-localized iterative reconstruction algorithm that can reconstruct images at high resolution with computational cost similar to a low resolution reconstruction. The method works by focusing computation only on the regions of the image that contain fine details, such as edges. Experimental results demonstrate that the proposed algorithm can achieve the same visual quality as the full high resolution reconstruction algorithm at significantly reduced computational cost.

***Index Terms*—Computed tomography, model based reconstruction, coordinate descent, multi-resolution, targeted reconstruction.**

I. Introduction

Recent applications of model based reconstruction (MBR) algorithms to computed tomography have shown that MBR can greatly improve the image quality by both reducing noise and increasing resolution [1]–[3]. In particular, MBR can substantially increase spatial resolution through the incorporation of a more accurate model of the scanner. However, in order to fully benefit from this higher spatial resolution, MBR typically requires a higher spatial sampling rate, or equivalently smaller voxels, to represent fine details in the images [4], [5]. One disadvantage of this higher spatial sampling rate is that it can significantly increase the computational cost of MBR since many more voxels need to be reconstructed in the same fixed region-of-interest (ROI). For example, reconstructing the images at twice the resolution increases the number of voxels by a factor of 4 in a 2D plane and by a factor of 8 in a 3D volume.

In this paper, we present an edge-localized update strategy that can reconstruct high resolution images with computational cost similar to a lower resolution reconstruction. The key idea of our algorithm is to focus the computation only on the regions of the image that contain fine details, such as edges. Based on this idea, we first compute a low resolution reconstruction of the ROI. Since the number of voxels in the reconstruction is relatively small, this reconstruction can be achieved with much less computation. The low resolution reconstruction is then used as the initial estimate for the high resolution reconstruction. At that stage, we can detect the regions in the image that contain edges and fine detail, and we use the iterative coordinate descent algorithm to only update voxels in these locations. This approach results in high resolution images that are very close to the results of the conventional high resolution MBR reconstruction, but requires only a fraction of the computation to evaluate.

Although generally speaking, the edge-localized update strategy can be combined with a variety of optimization algorithms, we find that the iterative coordinate descent algorithm (ICD) has several advantages in this application. First, it allows individual voxels to be efficiently updated. Second, it has a relatively fast convergence behavior, especially for the high frequency content and near the edges in the image [6], [7].

This paper is organized as follows. In section II, we provide a brief review of our previous work on the ICD algorithm and the targeted reconstruction framework, which provide the initial estimate for the edge-localized ICD algorithm. In section III, we present the edge-localized ICD algorithm used for fast reconstruction of high resolution images. In section IV, we verify the performance of the proposed algorithm on clinical data.

II. Multi-resolution targeted reconstruction

We use a Bayesian approach for model based reconstruction. Let x denote the vector of the voxels to be reconstructed, and y be the vector of the measurement data. We model the data acquisition by the conditional probability density function $p(y|x)$, and the image by the prior density $p(x)$. The Maximum *a posteriori* estimate is computed by maximizing the *a posteriori* density function $p(x|y)$ which leads to the following minimization problem [6]

$$\hat{x} = \arg\min_{x \geq 0} \left\{ \frac{1}{2} (y - Ax)^T D (y - Ax) + U(x) \right\}, \quad (1)$$

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where A is the forward system matrix, and D is a diagonal weighting matrix which reflects variations in the credibility of data [8]. The term $U(x)$ penalizes large variations in the image domain, while preserving edge characteristics [1].

The ICD optimization algorithm works by updating individual voxels to minimize the cost of equation (1). It can be efficiently implemented by keeping a state variable $e = Ax - y$, which we call the residual error sinogram [6]. Conventionally, ICD is implemented so that each voxel is updated exactly once per iteration, but a faster version of the algorithm, which we call non-homogeneous ICD (NH-ICD) updates some voxels more frequently than others in order to speed convergence [7].

Medical imaging typically requires the reconstruction of a targeted ROI smaller than the full size of the scanned object. Multi-resolution approaches are typically used to reduce the computation [9]. In our previous work, we proposed a multi-resolution framework for targeted reconstruction [10]. In this framework, we first perform a low resolution reconstruction that updates all the voxels in the full field of view encompassing all the objects measured by the CT system. Next, a high resolution reconstruction initialized from the low resolution images of the previous stage focuses the computation on the ROI only in order to achieve good image quality without propagating artifacts from the outside to the inside of the target. The residual error sinogram correction method described in [10] reduces the mismatch due to the change in resolution when switching to the high resolution stage.

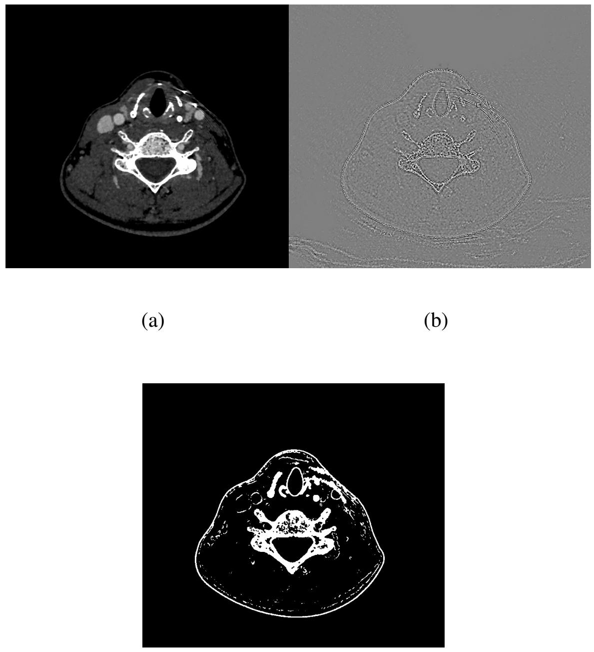
III. Edge-Localized ICD Algorithm

We propose an edge-localized ICD algorithm to perform a fast high resolution reconstruction. The basic idea is to focus the computation on the fine details of the image that are not accurately represented by larger voxels, and update only the voxels near the edges at the highest resolution, while the rest of the voxels are directly estimated from the low resolution reconstruction. In Figure 1(a), we show the initial estimate of the high resolution image, which is interpolated by a factor of 2 from the low resolution reconstruction. Errors in the initial estimate are computed by comparing it to a fully converged high resolution reconstruction. The resulting error image of Fig. 1(b) shows that the edge voxels have significantly larger error magnitude than other voxels. In the high resolution edge-localized reconstruction, the computation is focused on updating the edge voxels depicted in (c), while the rest of the image remains unchanged.

The edge-localized ICD algorithm solves a constrained optimization problem. Let Ω denote the set of edge voxels as shown in Figure 1(c), and $\bar{\Omega}$ be the complement set of Ω, containing all the other voxels. We use x_Ω and $x_{\overline{\Omega}}$ to denote the vector of the voxels in Ω and $\bar{\Omega}$ respectively. Formally, the edge-localized reconstruction algorithm solves the following optimization problem.

$$\begin{cases} \hat{x} = \arg\min_{x \geq 0} \left\{ \frac{1}{2} (y - Ax)^T D (y - Ax) + U(x) \right\} \\ \qquad\qquad \text{Subject to } \hat{x}_{\overline{\Omega}} = \tilde{x}_{\overline{\Omega}} \end{cases} \tag{2}$$

where $\tilde{x}_{\overline{\Omega}}$ is an estimate of the voxels in the set $\bar{\Omega}$. Notice that, assuming $U(x)$ is strictly convex, the objective function



(a) (b) (c)

Fig. 1. Illustration of the edge-localized ICD algorithm. (a) is the initial estimate for the high resolution reconstruction, (b) shows the errors in (a) relative to the true MAP estimate in a [-50,50] HU window, and (c) shows the detected edge voxels that are updated in the final stage.

is also strictly convex. Moreover, the constraints are linear. Therefore, it is easy to verify that the optimization problem in (2) is a convex optimization problem.

We initialize the reconstruction using 512 by 512 sized ROI images reconstructed by the algorithm described in section II. The ROI images are then interpolated by a factor of 2 using bi-cubic interpolation in the axial planes, while the cross-plane resolution remains unchanged. The interpolated images serve two purposes in the edge-localized ICD algorithm. First, for the voxels in Ω, they are used as the initial estimate. Second, for the voxels in $\bar{\Omega}$, the interpolated images are used as $\tilde{x}_{\bar{\Omega}}$ in equation (2), that is, the estimate of the voxels that are not edges.

The edge-localized reconstruction also uses the residual error sinogram resulting from the low resolution reconstruction as its initial residual error sinogram.

To form the set Ω, a robust edge detection algorithm needs to be developed. We choose to perform the edge detection independently on each slice of the interpolated volume, following a four-step process: clipping, edge detection, thresholding, and morphological operation. First, before edge detection, the image is clipped by a lower threshold $T = -400HU$, in order to remove the background objects that are not clinically relevant. Second, we use a Sobel edge detector to compute the gradient image from the clipped image. Third, the gradient image is thresholded using the value T_g. T_g determines the contrast of edge features to be detected. Typically, we use the threshold $T_g = 300$ in order to robustly detect high-

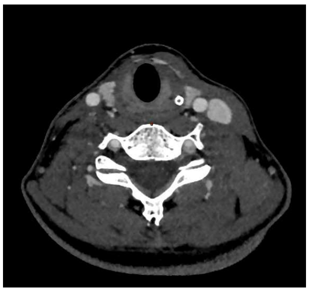


Fig. 2. Clinical reconstruction using MBR, displayed in [-50,350] HU window

contrast edges such as the bone/tissue boundary as well as some clinically important lower-contrast edges such as blood vessel boundaries. After thresholding, a binary image of the selected edge pixels is formed. In the final step, noise and outliers in the edge image are removed with a morphological opening operation followed by a closing operation in order to obtain the final edge map.

The edge-localized ICD algorithm iterates only on the set Ω instead of updating all the voxels in the image volume. The NH-ICD algorithm provides a mechanism to select the order of voxels to update using two steps. First a voxel-line[1] selection algorithm is used to determine a voxel-line for update. Then all the voxels on the voxel-line are updated in sequence. The edge-localized ICD algorithm inherits the voxel-line selection method from the conventional NH-ICD algorithm; however, only the voxels in the set Ω are updated. In order to do so, the voxel-line selection algorithm only selects voxel-lines that have at least one voxel in the set Ω. When a voxel-line is selected for update, the 3D edge mask is first checked and updates are calculated only for the voxels in the set Ω; the rest of the voxels are skipped.

IV. Experimental Results

In this section, we verify the performance of the edge-localized ICD algorithm on clinical data. We compare the images reconstructed by FBP, low resolution MBR, high resolution full update MBR, and high resolution edge-localized MBR. The FBP and low resolution MBR images are reconstructed on 512 by 512 grids over the targeted ROI, whereas the high resolution reconstructions are of size 1024 by 1024. In order to visualize the fine details, the images are interpolated to the size of 2048 by 2048 for comparison.

Figure 2 shows a clinical reconstruction of a patient's neck in 240 mm targeted field of view (tfov). In Figure 3, the images are zoomed in to compare the edges of the bone. The FBP reconstruction using a standard kernel is shown in (a). The low resolution MBR image shown in (b) produces sharper edges than the FBP image. However, the sampling rate of this image is not sufficiently high to support the reconstructed object resolution, so the edges do not appear sufficiently smooth. In (c) and (d) the high resolution reconstructions are shown using full update NH-ICD and edge-localized ICD, respectively.

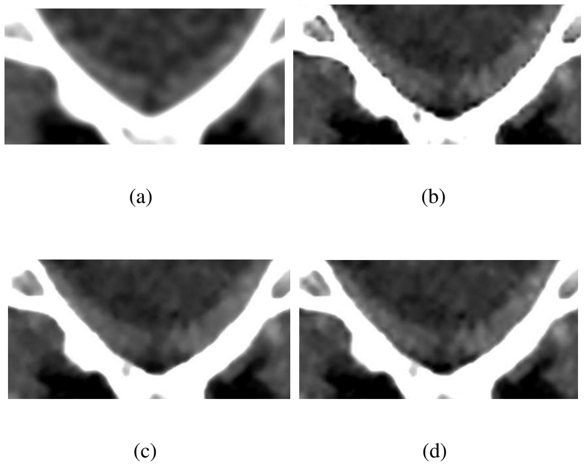


Fig. 3. Zoomed images comparing edge details displayed in [-50,350] HU window. (a) FBP reconstruction (b) low resolution MBR (c) high resolution full update MBR (d) high resolution edge-localized MBR

Both images produce sharp bone edges. There is little visual difference between (c) and (d); however, the edge-localized reconstruction requires substantially less computation than the full update method.

Table I quantitatively compares the image quality and the computational cost of each algorithm. In each row, we compute the root mean squared errors (RMSE)[2] of all the voxels and the RMSE of the edge voxels only, as well as the total computation time of the reconstruction algorithms. The first row shows the low resolution MBR using NH-ICD algorithm. In the low resolution reconstruction, the overall RMSE is relatively low, which implies that low resolution reconstruction provides a good initial estimate in general, while the large RMSE among the edge voxels indicates that they need to be refined. In the second row, by applying the edge-localized ICD algorithm, the RMSE of the edge voxels is reduced by almost half, while the total computation time is increased by only 7.5%. In the third and fourth row, we use two full update methods for the high resolution reconstruction. In row three, we use the NH-ICD algorithm, and in row four, we use the conventional ICD algorithm for both the low resolution and high resolution reconstructions. Both the edge-localized and full update high resolution reconstruction algorithms can achieve similar RMSE for edge voxels, while the computational costs with the two full update methods are substantially higher since both algorithms iterate on all the voxels in the high resolution image grid.

The high resolution reconstruction provides superior image quality not only around bone edges but also when reconstructing small high contrast features. To illustrate this, Figure 4 shows an image of a computed tomography angiography (CTA) study covering a patient's abdomen and part of the chest in 350 mm tfov. In this case, the patient has a stent implant in the abdomen which introduces small high intensity features that must be reconstructed with fine resolution. Figure 5 (a)

[1]A voxel-line is a set of voxels that fall on a line parallel to the axes of the helix in the helical scan mode.

[2]We use a fully converged full update high resolution reconstruction as the reference image for the RMSE calculation.

Method	RMSE of all voxels (HU)	RMSE of edge voxels (HU)	Total computation time [a]
Low resolution MBR (NH-ICD)	10.4	19	1
Edge-localized ICD	8.06	10.9	1.075
Full update MBR (NH-ICD)	6.3	7.8	1.38
Full update MBR (Conventional ICD)	7.0	11.0	2.82

[a]Normalized by the total computation time of the low resolution MBR

TABLE I
TABLE COMPARING THE IMAGE QUALITY AND TOTAL COMPUTATION TIME

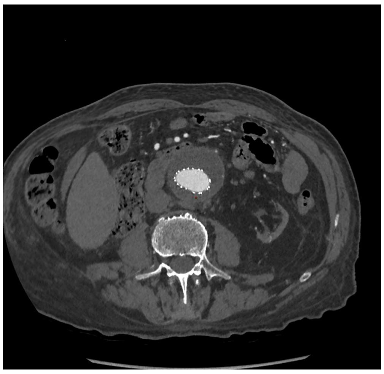


Fig. 4. Abdomen CTA study with a stent implant, displayed in [-200,500] HU window.

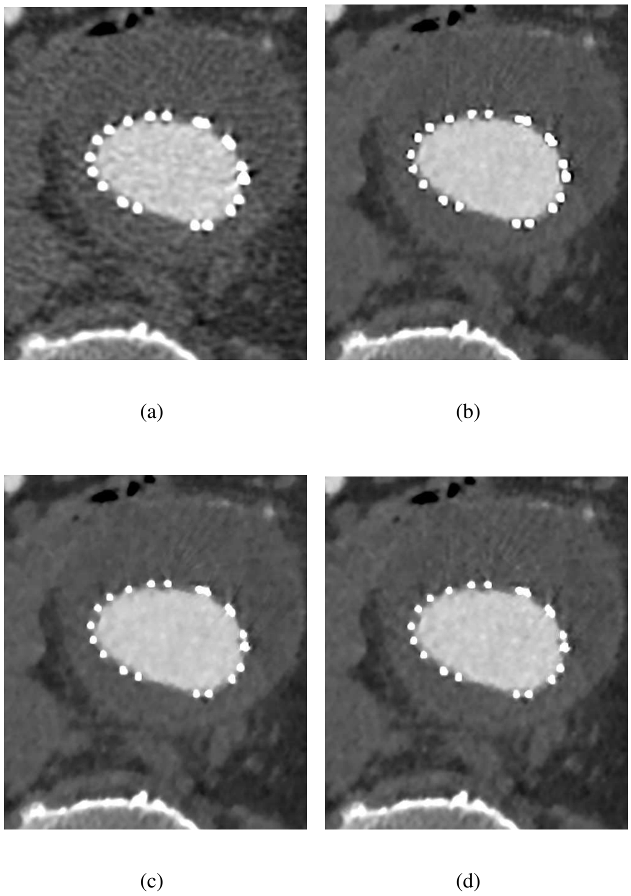


Fig. 5. Zoomed images of the stent implant displayed in [-200,500] HU window. (a) FBP reconstruction; (b) low resolution MBR; (c) high resolution full update MBR; (d) high resolution edge-localized MBR.

shows the FBP image using the standard kernel. The low resolution MBR image shown in (b) significantly reduces the noise in the soft tissue, and the stent has a sharper appearance. The image reconstructed at high resolution using full updates and edge-localized updates are shown in (c) and (d), respectively. By comparing (c) to (b), we notice that the slowly varying area in the image such as the soft tissue remains almost unchanged, whereas the most noticeable difference is in the reconstruction of the stent. Both (c) and (d) show finer details of the stent with less undershoot around the edges. By updating only the edge voxels, the edge-localized ICD reconstruction shown in (d) achieves similar image quality as the full update reconstruction.

V. DISCUSSION AND CONCLUSION

In this paper, we present an edge-localized iterative reconstruction algorithm that can reconstruct images at high resolution with computational cost similar to a low resolution reconstruction. Experimental results with clinical data have demonstrated that the proposed algorithm can achieve the same visual quality as the full high resolution reconstruction algorithm at significantly reduced computational cost.

Although our method is proposed in a multi-resolution framework, the edge-localized update strategy can be implemented differently. For example, a method that would reconstruct the images directly on non-uniform grids to allow the voxel size to vary across different locations as a function of local frequency content would be a natural extension of this work. Although the implementation of such hierarchical approach might be significantly more complex, especially in a regularized environment, it might result in overall improved efficiency.

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